Quantum Annealing

Enhancing the performance through bias fields

Seminar at Institut de Ciències del Cosmos @ Universitat Barcelona

3.10.2019

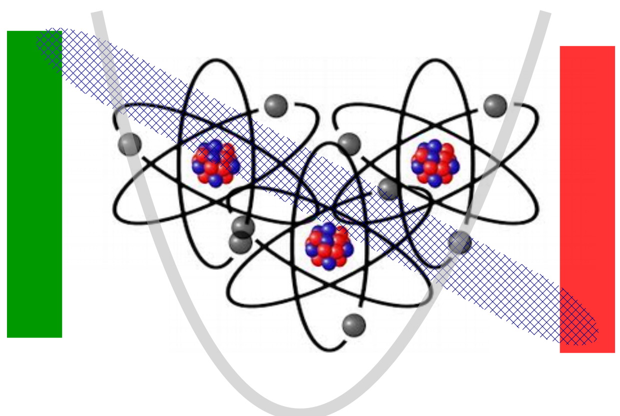
Tobias Grass
ICFO
JQI

"la Caixa" Foundation

JOINT QUANTUM INSTITUTE

Quantum Simulator

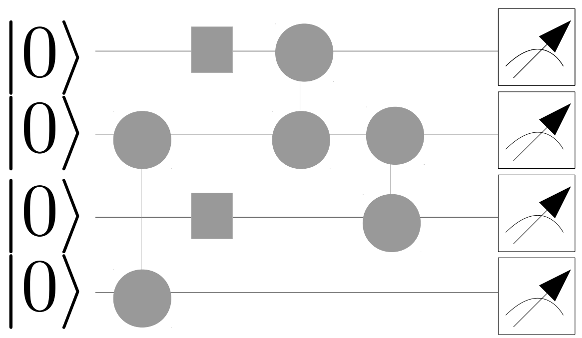
- Tool:
  Engineering of quantum many-body Hamiltonians



- Goal:
  Obtain information about the behavior of the engineered model

Quantum Computer

- Tool:
  Engineering of qubits for storing, and quantum gates for processing quantum information



- Goal:
  Perform universal computations

Quantum Simulator

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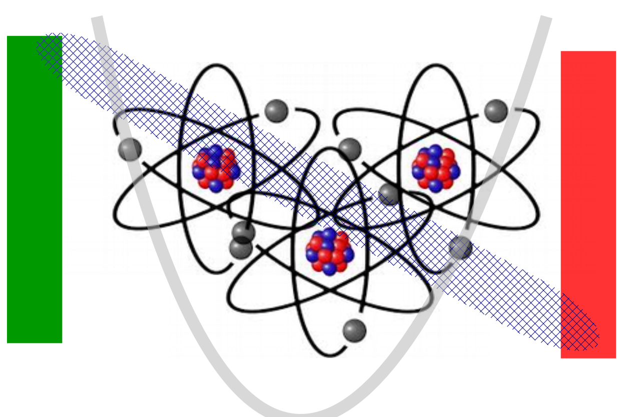

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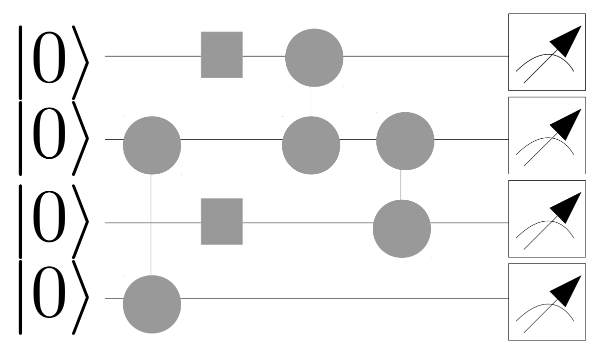

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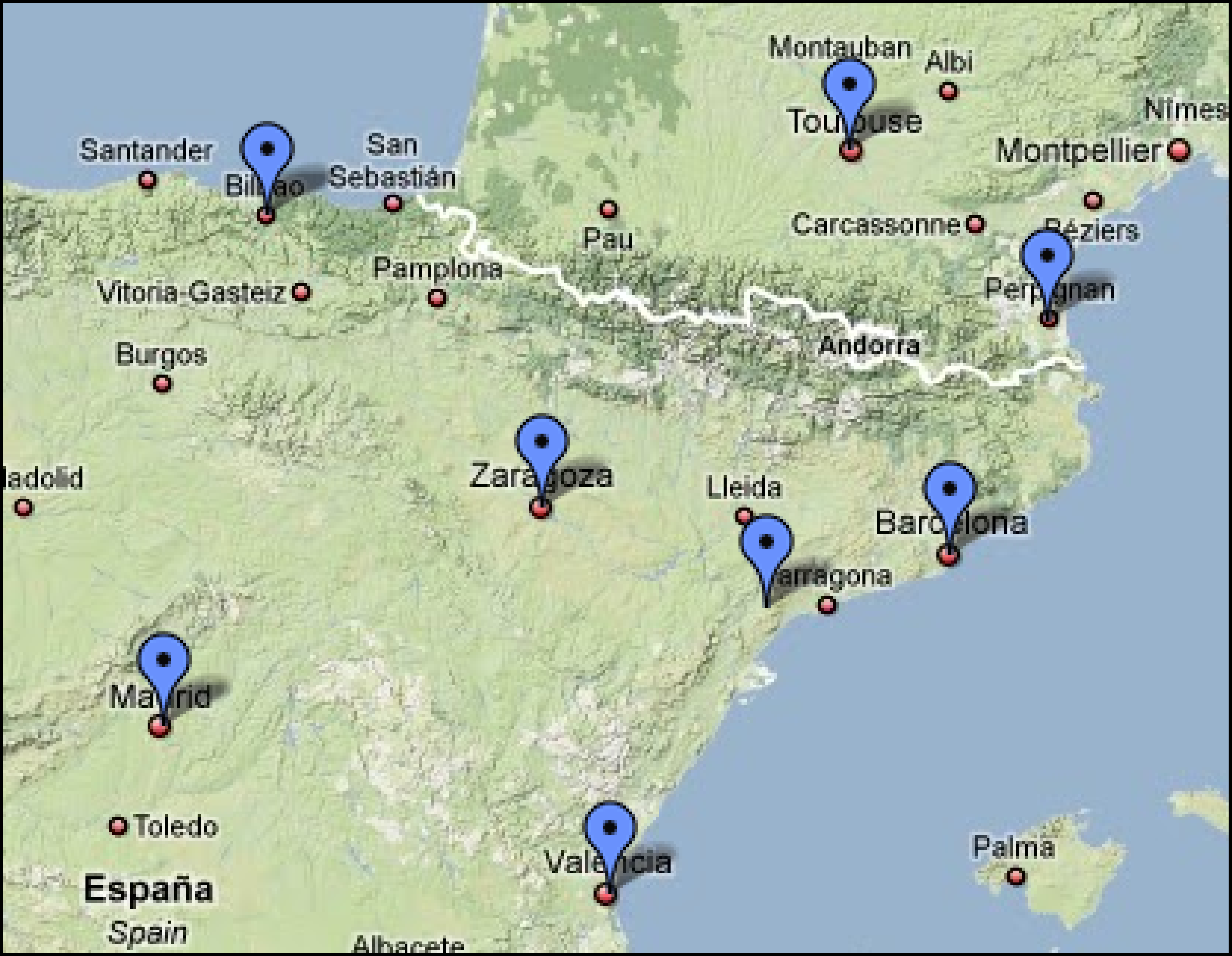
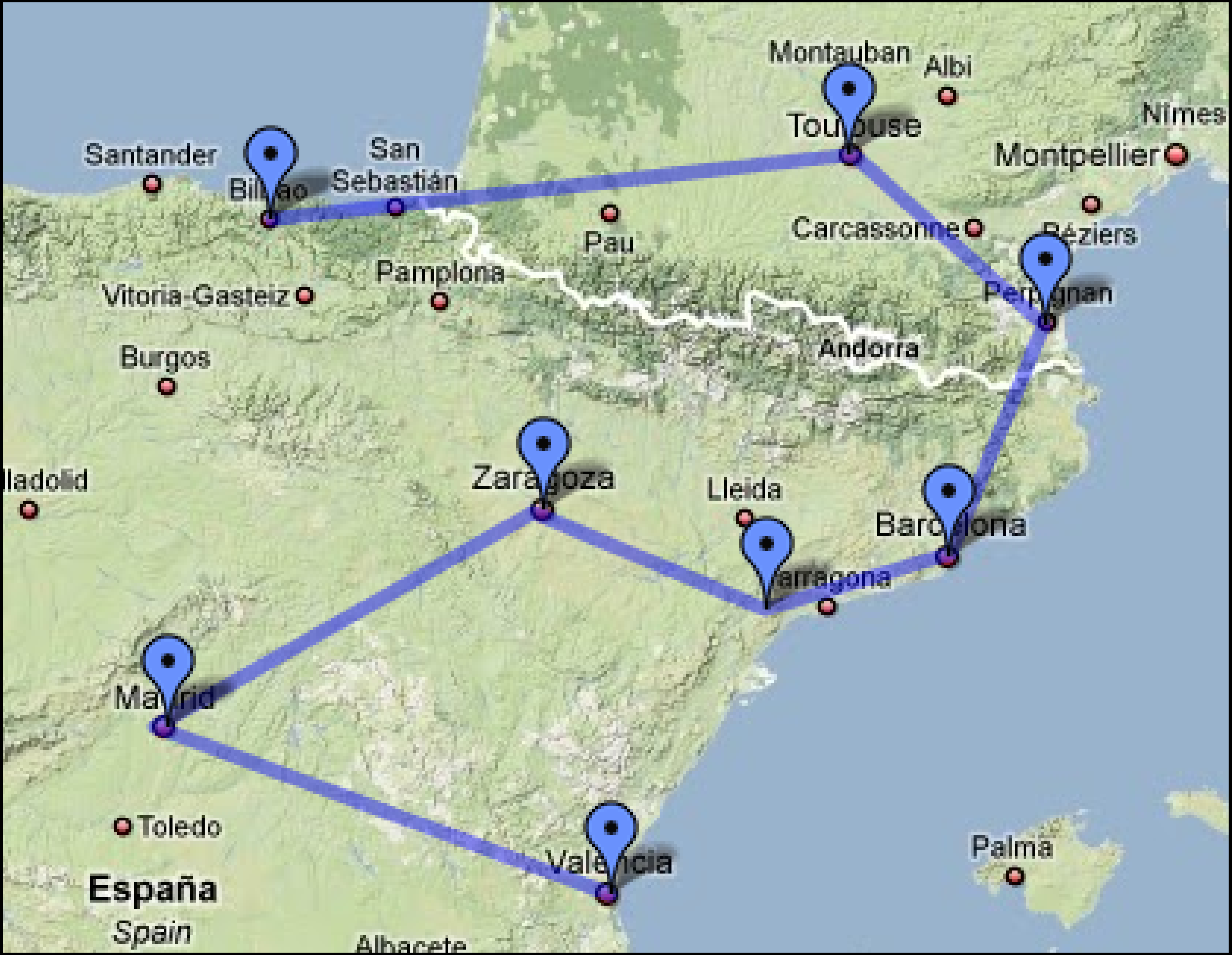


https://davidwees.com/sites/default/files/Travelling%20Salesman%20Problem%20-%20Sample.docx



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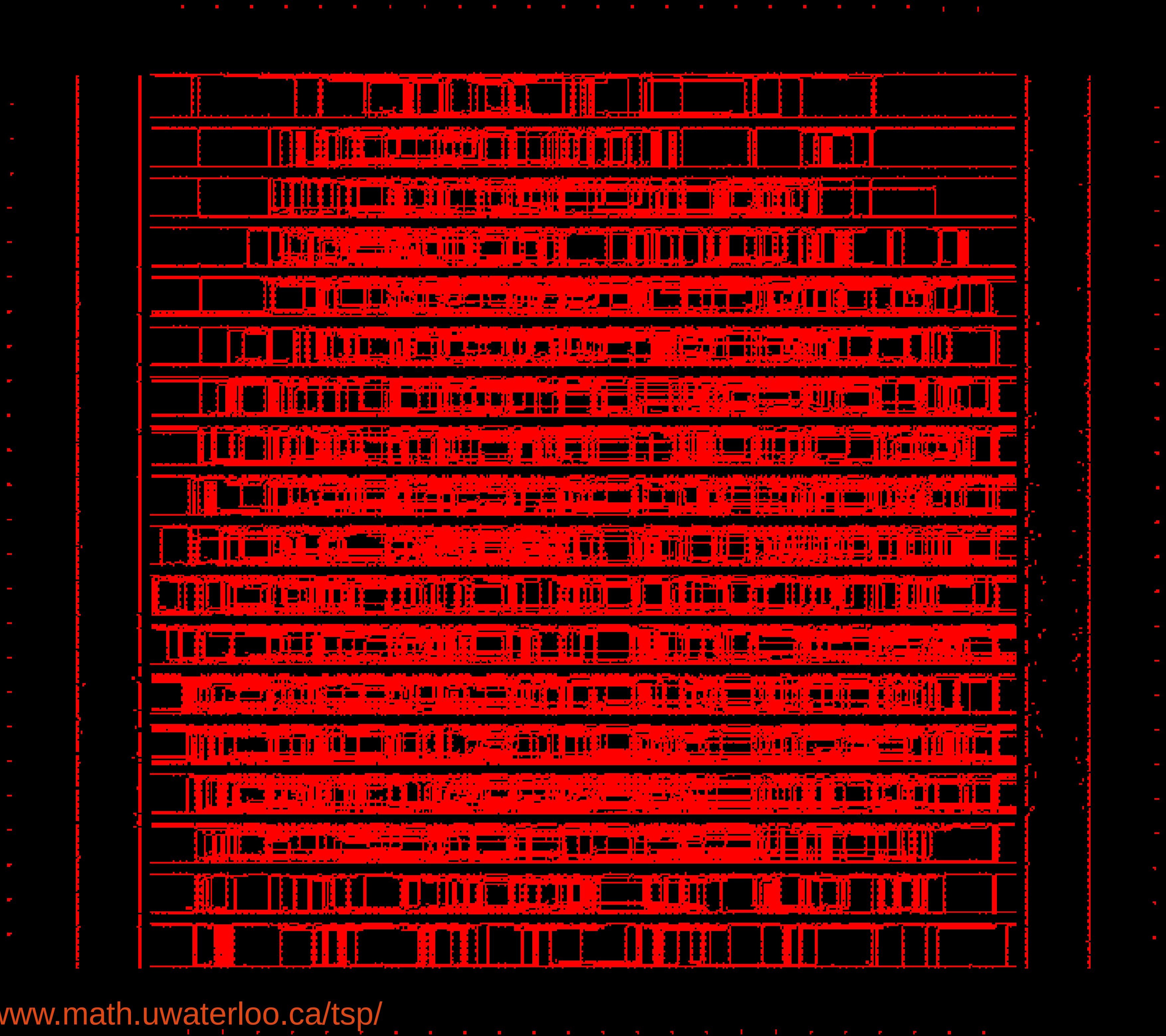


http://www.math.uwaterloo.ca/tsp/

Motivation

1 Optimization problems are very general:

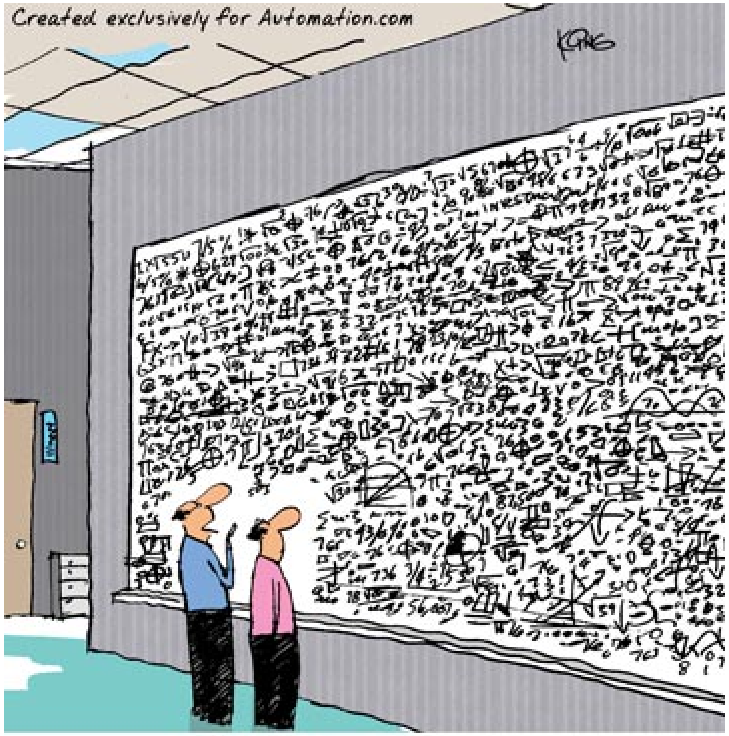
Find the configuration of a system or process which minimizes the associated "cost"

2 Optimization problems can be hard to solve:

If costs are random, also the optimal configuration looks random, and thus, is hard to find

3 Nature solves optimization problems all the time:

Minimizing energy, maximizing entropy



"...and that, in simple terms, is my idea on how to increase factory optimization. any questions?"

https://www.automation.com/library/resources/automation-cartoons-january-28-2009

Different strategies:

1 Simulate Nature!

Mimic the laws of Nature which govern the dynamics during cooling!
Cooling a physical system = dynamical evolution towards configurations with lower energies.
Cooling of an optimization problem = dynamical evolution towards configurations with lower cost functions.

2 Let Nature simulate!

Build a system which mimics the optimization problem, and, by the laws of physics, let it cool!
Configuration space of the system must map to the configuration space of the problem, with energies corresponding to the cost function of the problem ("Problem Hamiltonian").

Different strategies:

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Thermal fluctuations

- controlled by a temperature
- dynamics governed by master equation

$$\frac{\mathrm{d}}{\mathrm{d}t} P_i(t) = \mathcal{L}_{ij}(t) P_j(t)$$

Cooling can be classical or quantum!

Quantum fluctuations

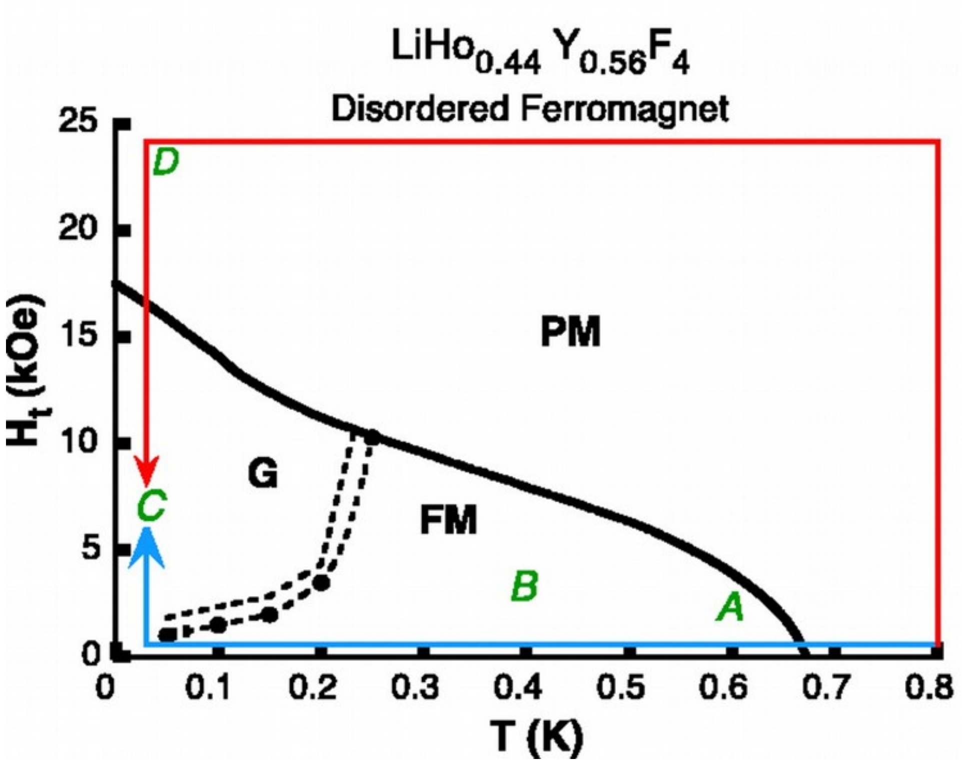
- controlled by a quantum (i.e. non-commuting) field
- Dynamics governed by Schroedinger equation

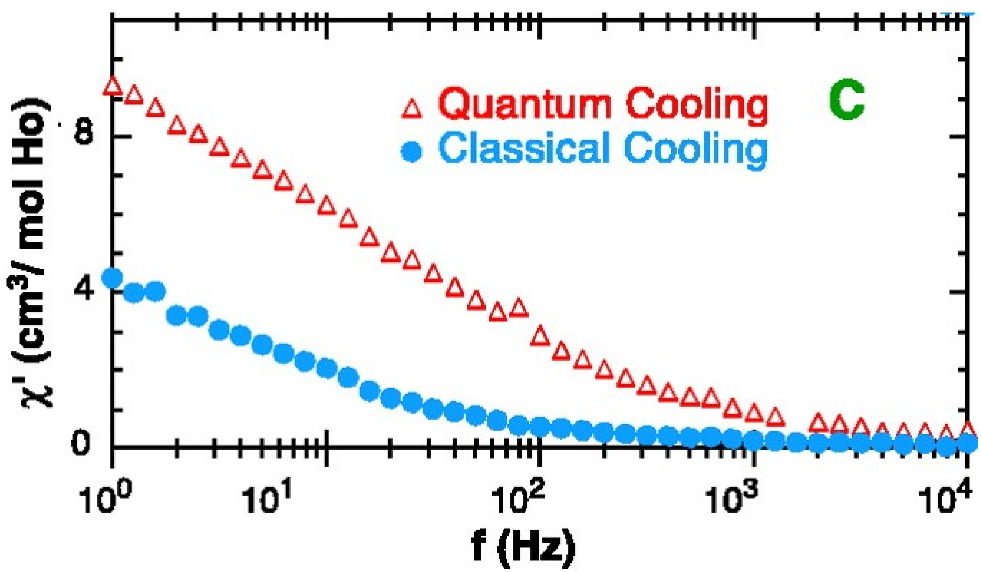
$$i\hbar \partial_t \Psi(t) = \mathcal{H}(t) \Psi(t)$$

2 Let Nature simulate!

Build a system which mimics the optimization problem, and, by the laws of physics, let it cool!
Configuration space of the system must map to the configuration space of the problem, with energies corresponding to the cost function of the problem ("Problem Hamiltonian").

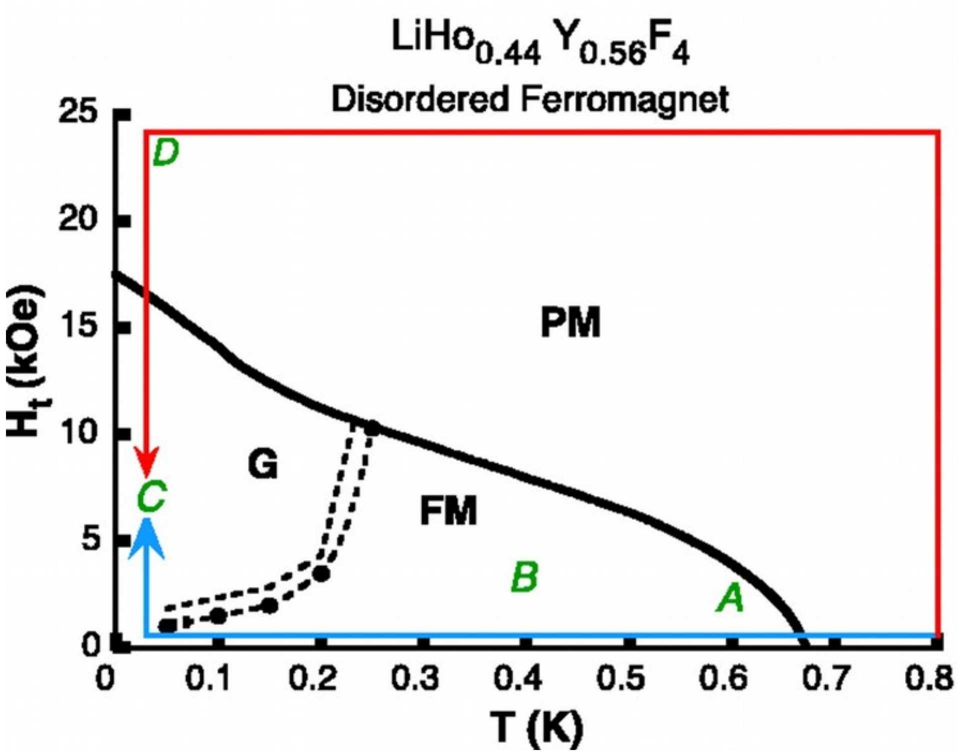
Classical vs. quantum cooling

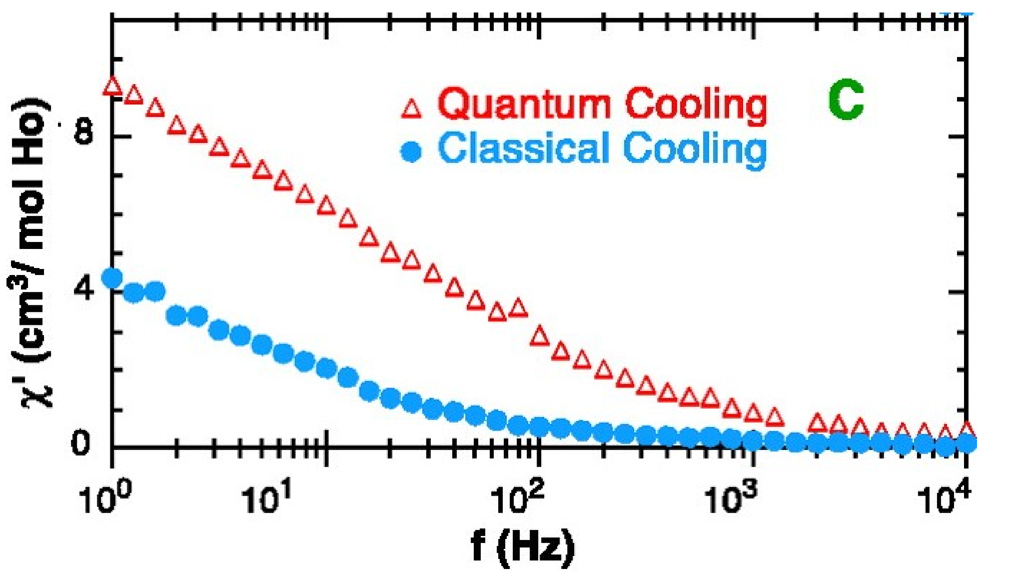




J. Brooke, D. Bitko, T. F. Rosenbaum, G. Aeppli, Science 284, 779 (1999)

Classical vs. quantum cooling





J. Brooke, D. Bitko, T. F. Rosenbaum, G. Aeppli, Science 284, 779 (1999)

Simulated cooling:

PHYSICAL REVIEW E VOLUME 58, NUMBER 5 NOVEMBER 1998

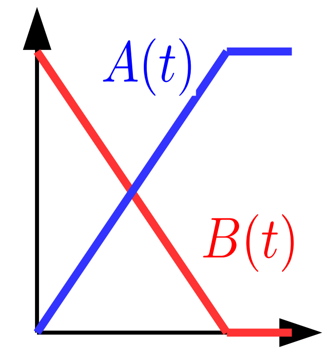
Quantum annealing in the transverse Ising model

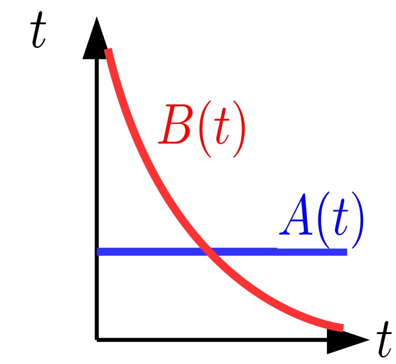
Tadashi Kadowaki and Hidetoshi Nishimori
Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551, Japan
(Received 30 April 1998)

Standard Quantum Annealing Protocol

- General form of time-dependent Hamiltonian:

$$H(t) = A(t)H_{\text{problem}} + B(t)H_{\text{driver}}$$





- How to choose the Hamiltonian?

$$H_{\text{problem}} = \sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z$$

$$H_{\text{driver}} = \sum_i \sigma_i^x$$

Journal of Physics A: Mathematical and General

Journal of Physics A: Mathematical and General > Volume 15 > Number 10

On the computational complexity of Ising spin glass models

F Barahona
Show affiliations

F Barahona 1982 *J. Phys. A: Math. Gen.* **15** 3241. doi:10.1088/0305-4470/15/10/028

REPORT

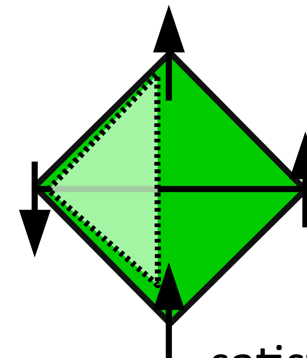
A Quantum Adiabatic Evolution Algorithm Applied to Random Instances of an NP-Complete Problem

Edward Farhi[1,*], Jeffrey Goldstone[1], Sam Gutmann[2], Joshua Lapan[3], Andrew Lundgren[3], Daniel Preda[3]
+ See all authors and affiliations

Science 20 Apr 2001:
Vol. 292, Issue 5516, pp. 472-475
DOI: 10.1126/science.1057726

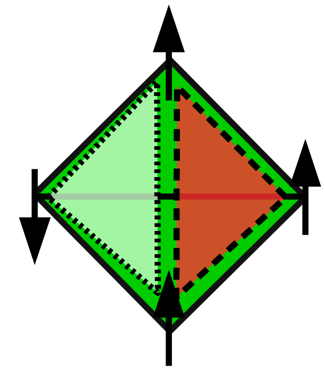
Example: Exact Cover

A problem instance for N bits $\{z_1,\ldots,z_N\}$ is defined by M clauses C_{ijk}. Each clause shall select three bits $\{z_i,z_j,z_k\}$, and demand $z_i+z_j+z_k=2$.



satisfiable

Decision problem:
Is there a bit assignment which satisfies all clauses simultaneously?

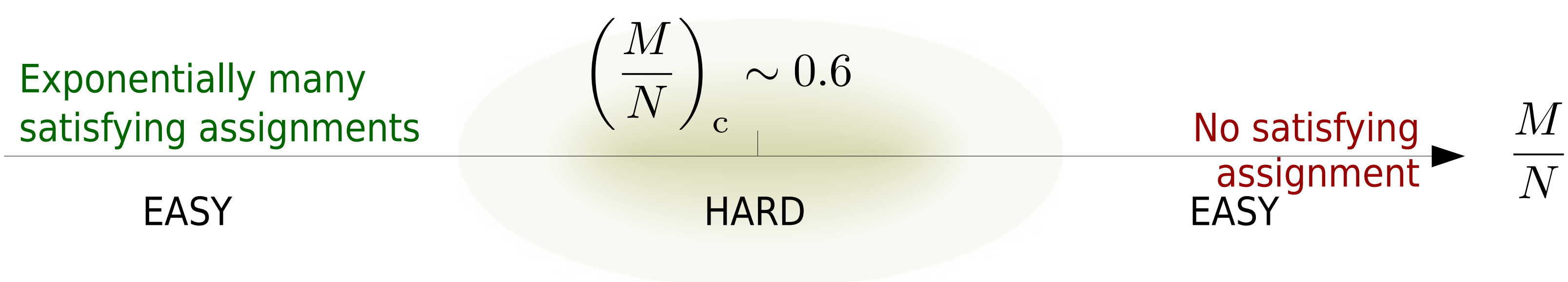


not satisfiable

Formulation of the problem as an Ising model:

$$H_\mathrm{p} = \sum_\mathrm{C} h(C) \quad \text{with} \quad h(\mathrm{C}_{ijk}) = (\sigma_z^i + \sigma_z^j + \sigma_z^k - 1)^2$$

Is the ground state energy E=0 or E>0 ?



Programmable quantum systems:

- Engineering of Ising Hamiltonians
- Initial state preparation
- Time-dependent control
- Detection of final state

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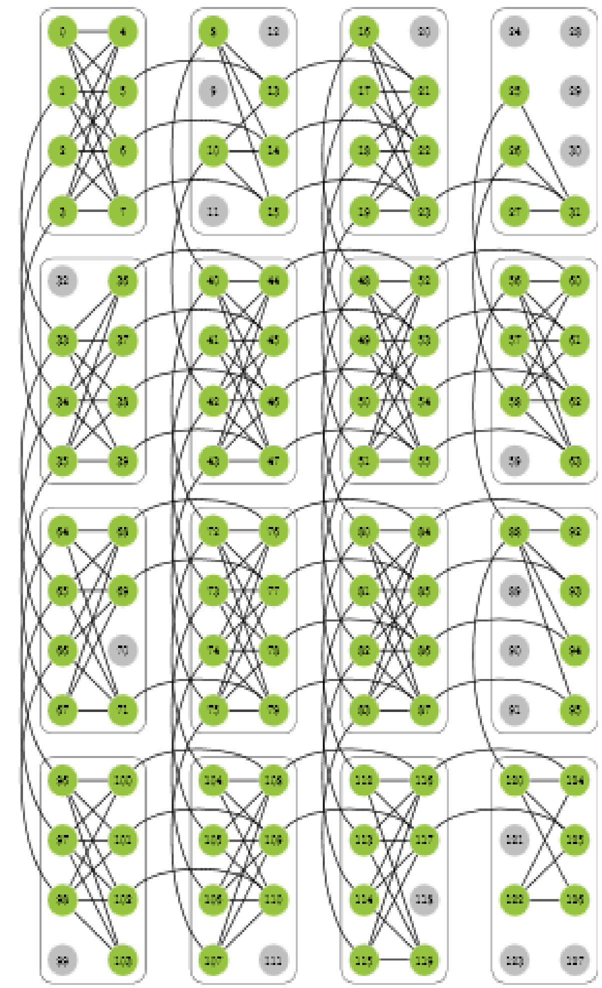

D-Wave device:

State of art quantum annealer of 2048 superconducting flux qubits with programmable Josephson couplings

Programmable quantum systems:

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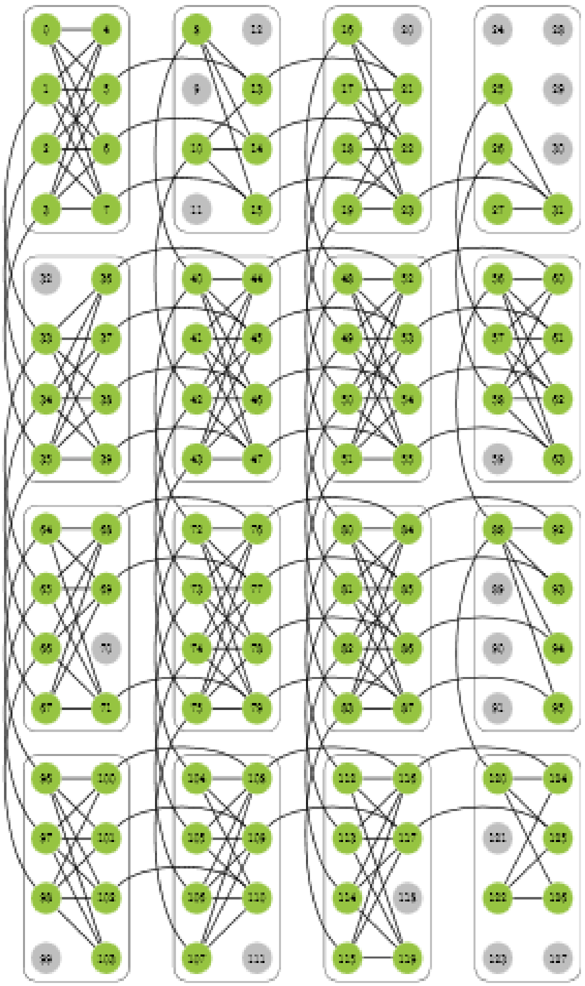
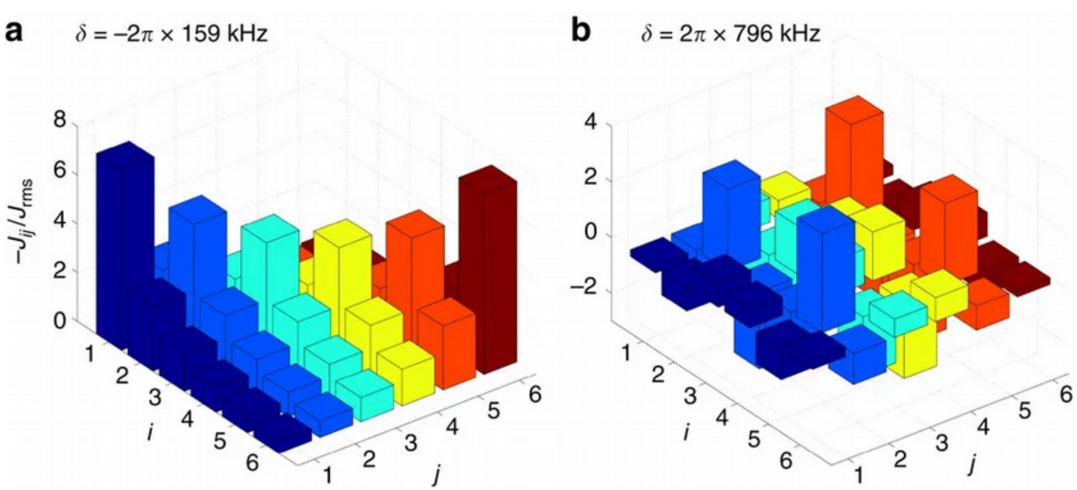


Trapped ions:

- routinely used as quantum simulators of spin chains [Monroe group in Maryland, Blatt group in Innsbruck]
- tunable phonon-mediated Ising interactions
- complicated (i.e. quasi-random) interactions can occur quite naturally, and directly represent a number partitioning problem



TG, D. Raventos, B. Julia-Diaz, C. Gogolin, M. Lewenstein, Nat. Commun. 7 11524 (2016)

D-Wave device:

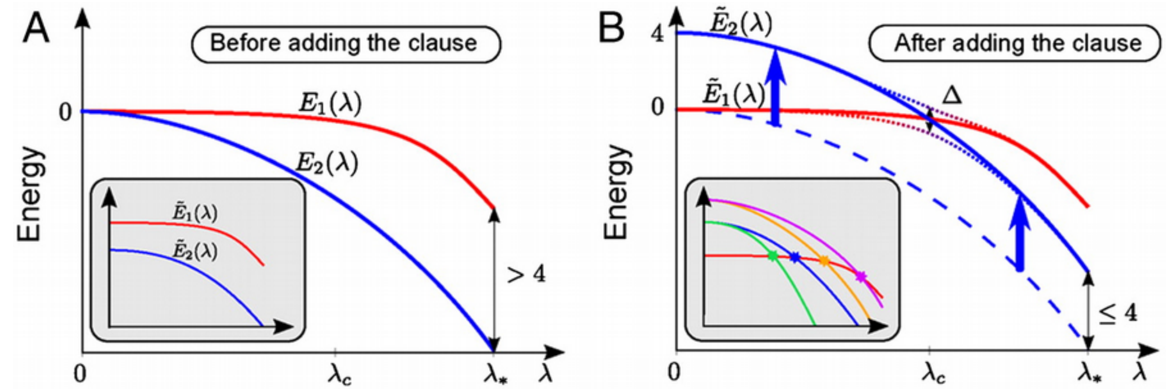
State of art quantum annealer of 2048 superconducting flux qubits with programmable Josephson couplings

The bottleneck of quantum annealing

Anderson localization makes adiabatic quantum optimization fail

Boris Altshuler, Hari Krovi, and Jérémie Roland

PNAS July 13, 2010. 107 (28) 12446-12450; https://doi.org/10.1073/pnas.1002116107



System undergoes a first-order phase transition: Δ becomes exponentially small!

Numerical studies: T. Joerg, F. Krzakala, J. Kurchan, A.C. Maggs, Phys. Rev. Lett. 101, 147204 (2008)

Adiabatic condition: Annealing has to be slow compared to inverse gap!

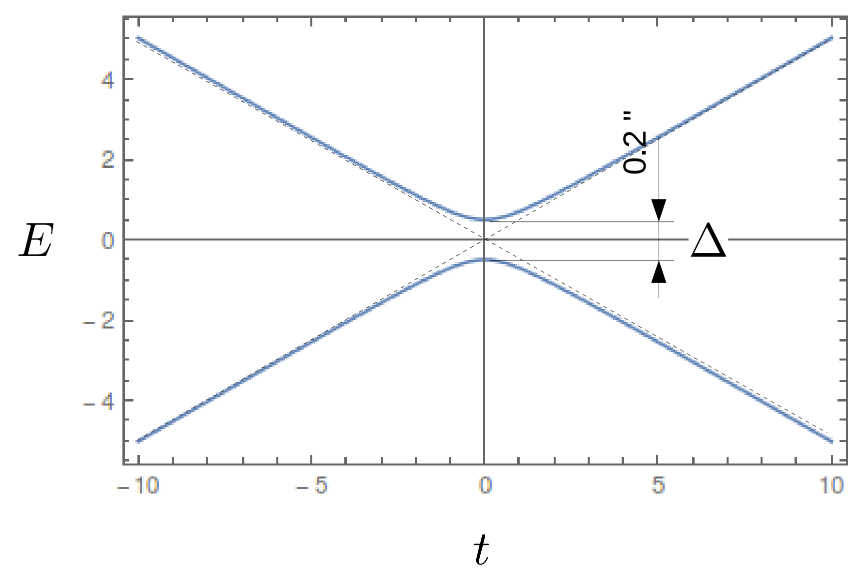
Example: Landau-Zener problem

$$H(t) = \frac{1}{2}\gamma t \sigma_z + \frac{1}{2}\Delta\sigma_x$$

$$P_{\rm LZ} = {\rm e}^{-\pi\Gamma^2/2}, \quad \Gamma \equiv \frac{\Delta}{\sqrt{\hbar\gamma}}$$

$$\Gamma \gg 1 \Rightarrow \Delta^2 \gg \hbar\gamma$$

$$T \sim \frac{1}{\Delta^2}$$



How to avoid small gaps?

- Inhomogeneous driving:

Standard protocol: $H(t) = (1-\gamma(t))H_{\text{driver}} + \gamma(t)H_{\text{problem}}$ with $H_{\text{driver}} = B\sum_i \sigma_i^x$

Modified protocol: $H_{\text{driver}} = B\sum_i \mu_i \sigma_i^x$

[E. Farhi, J. Goldstone, D. Gosset, S. Gutmann, H. Meyer, P. Shor, Quant. Inf. Comp. 11, 181 (2011)]

- Non-stoquastic driver Hamiltonian:

Stoquastic: No sign problem. All off-diagonal elements (in z-basis) are positive (e.g. transverse Ising)

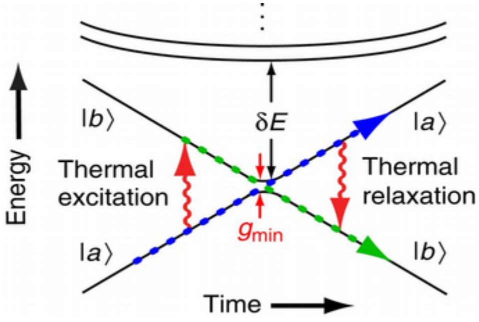
Introducing terms which render the Hamiltonian non-stoquastic can turn 1st order phase transitions to 2nd order transitions:

Example: $H(\gamma,\lambda) = \gamma H_{\text{p}} + (1-\gamma)H_{\text{driver}} + \gamma(1-\lambda)H_{\text{XX}}$ with $H_{\text{XX}} \sim \sum_{ij} \sigma_i^x \sigma_j^x$

[H. Nishimori and K. Takada Front. ICT 4:2 (2017)]
[L. Hormozi, E. W. Brown, G. Carleo, and M. Troyer, Phys. Rev. B 95, 184416 (2017)
[I. Ozfidan *et al.*, arXiv 1903.06139]

- Thermal assisted quantum annealing:

[N. Dickson *et al.*, Nat. Commun. 4, 1903 (2013)]

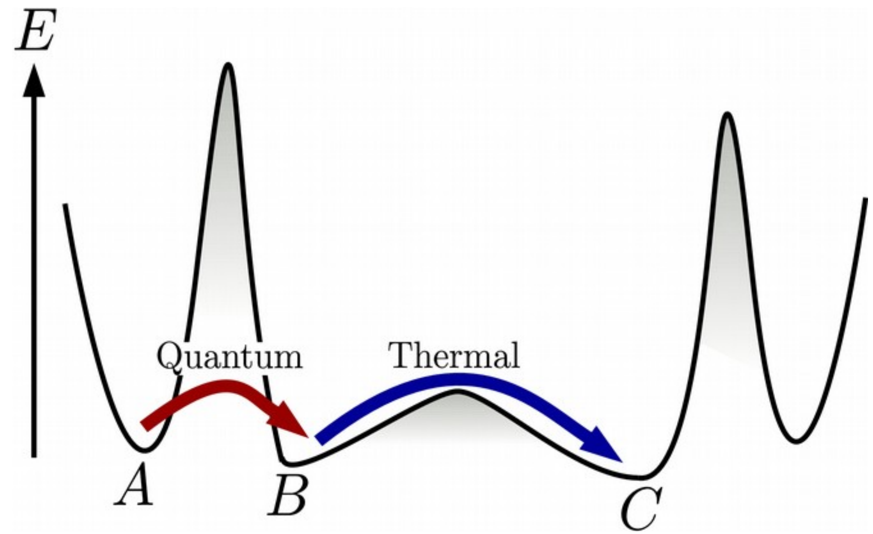


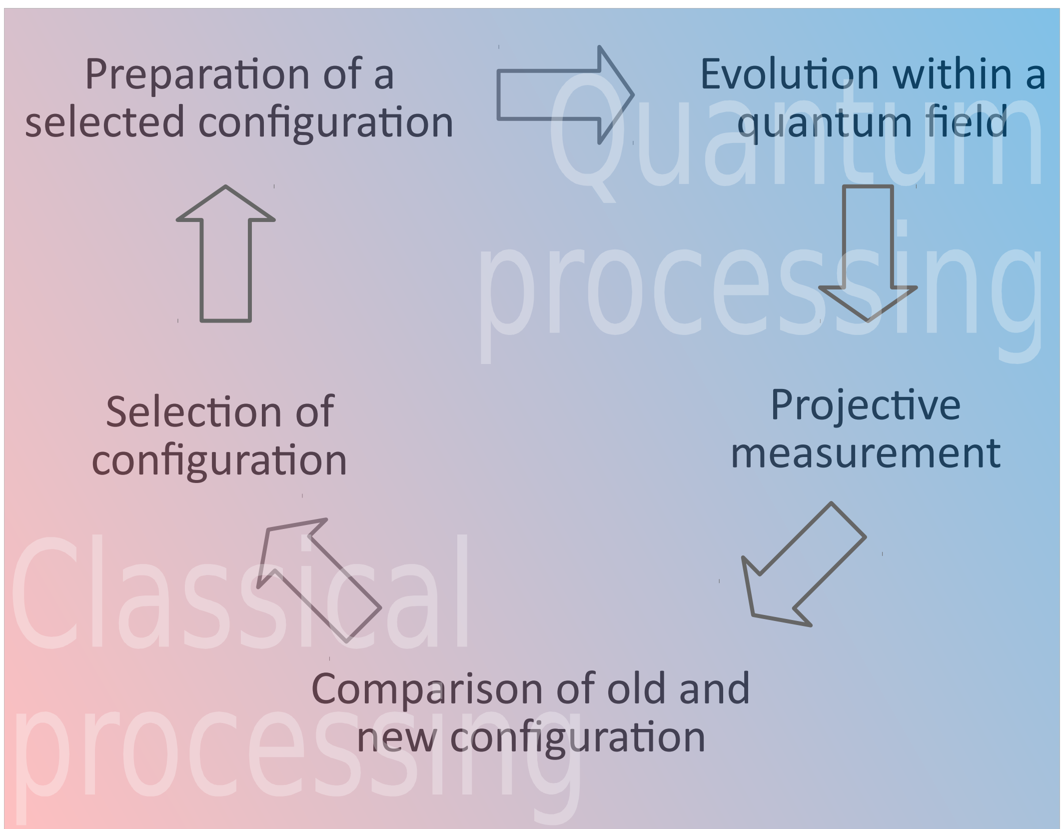
Hybrid algorithms

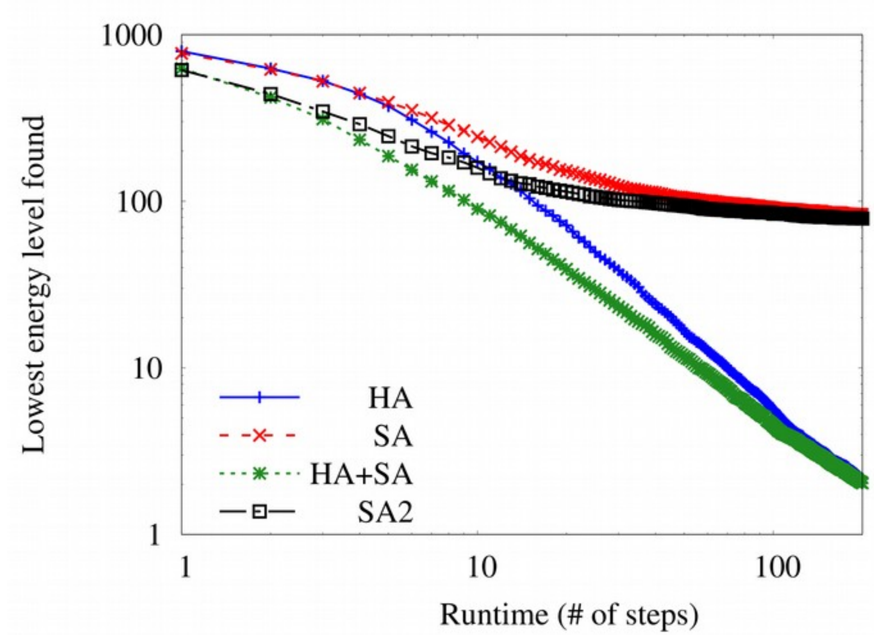
Combining quantum and classical search strategies

N. Chancellor, New J. Phys. 19 023024 (2017)

T. Grass and M. Lewenstein, Phys. Rev. A 95, 052309 (2017)



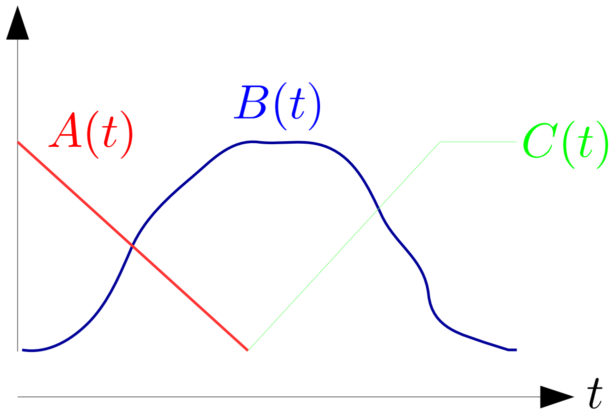




Performance of different search strategies, tested with random energy model of 11 spins

Reverse annealing

Annealing starts with classical configuration, the quantum fluctuations are then switched on and off



$$H(t) = A(t)H_{\text{init}} + B(t)H_{\text{driving}} + C(t)H_{\text{problem}}$$

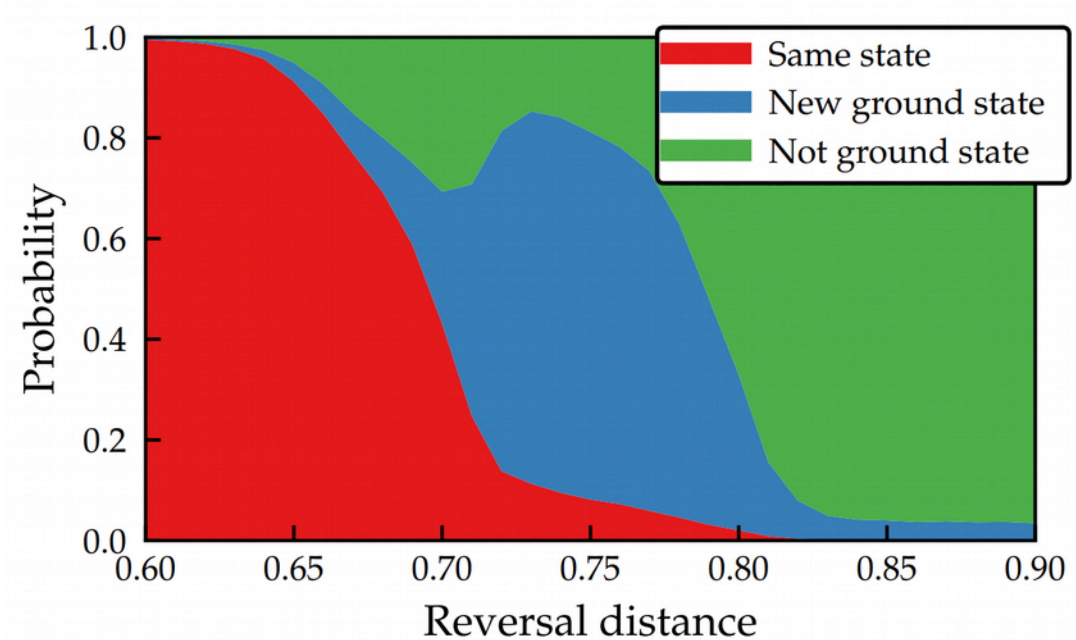
$$H_{\text{init}} = \sum_i h_i \sigma_i^z \qquad H_{\text{driving}} = \sum_i \sigma_i^x \qquad H_{\text{problem}} = \sum_{ij} J_{ij}\sigma_i^z \sigma_j^z$$

Adiabatic versions:

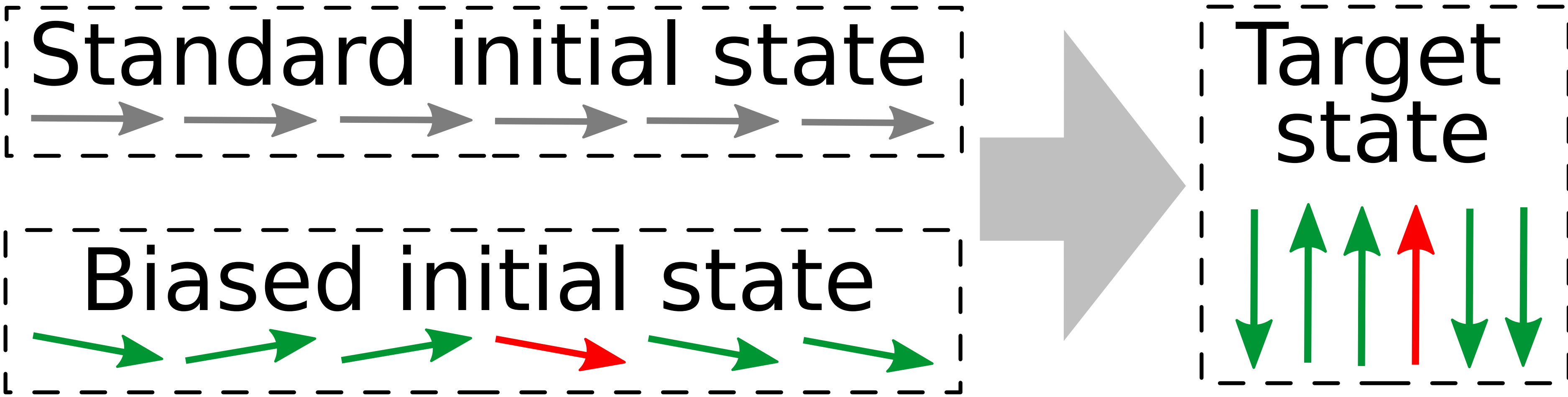
[Perdomo-Ortiz, A., Venegas-Andraca, S.E. & Aspuru-Guzik, A. Quantum Inf Process 10, 33 (2011)]

Non-adiabatic versions:

[D-wave white paper series 14-1018A-A (2017)]

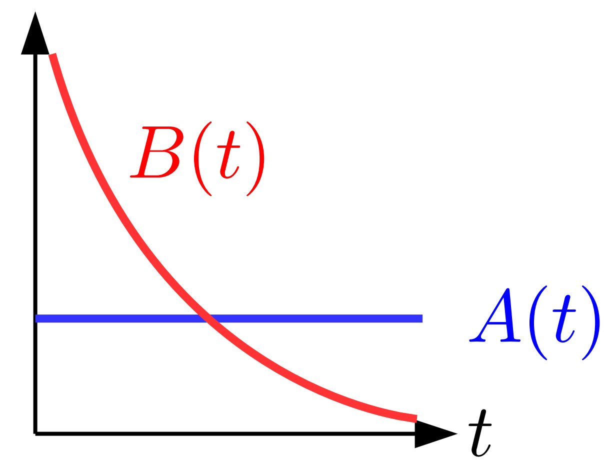


Annealing with bias field



General form or Hamiltonian as in standard annealing:

$$H(t) = A(t)H_{\text{problem}} + B(t)H_{\text{driver}}$$



But the driver Hamiltonian shall not be agnostic to the problem.

T. Grass, Phys. Rev. Lett. 123, 052309 (2019)

How to implement the bias?

Add longitudinal field component to the driver Hamiltonian:

$$H_{\text{driver}} = \sum_m (\sigma_m^x + h_m \sigma_m^z), \quad h_m = \pm 1$$

The biased initial state is

- still an easy-to-prepare ground state of the driver Hamiltonian
- still a superposition of all *z*-polarized states (with non-equal weights)

If the driver Hamiltonian is *adiabatically* switched off, the ground state of the problem Hamiltonian will be reached.

When does the longitudinal field enhance "adiabaticity"?

T. Grass, Phys. Rev. Lett. 123, 052309 (2019)

Success rates with and without bias

For randomly generated instances of the Exact Cover 3 problem (with unique satisfying assignment), the annealing outcome for fixed annealing times is obtained (through numerical simulation of the dynamics).

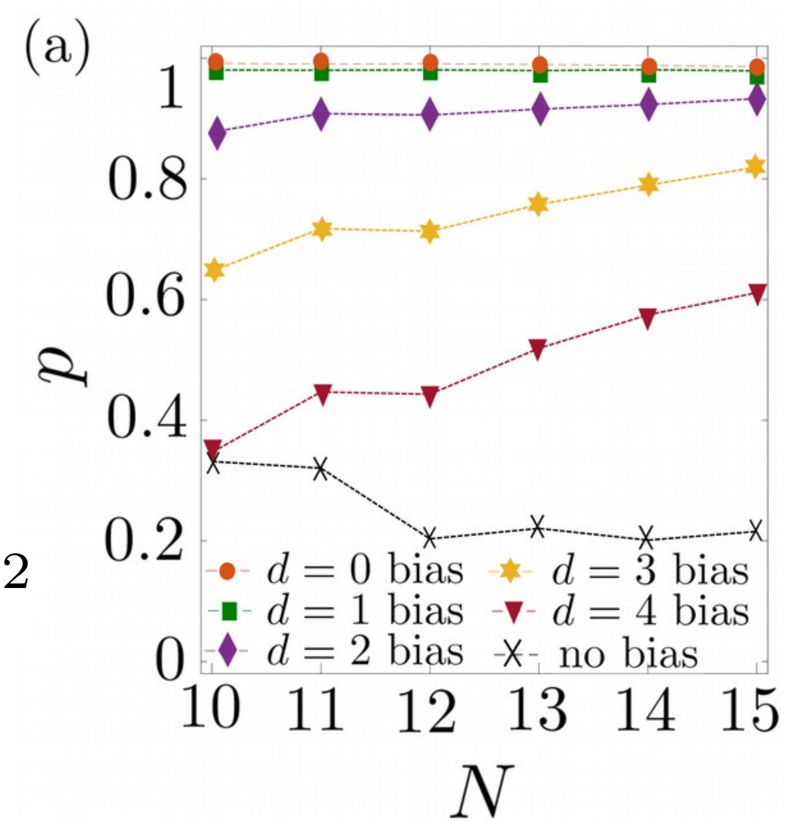
Bias fields are generated randomly with fixed Hamming distance to the optimal solution.

Optimal solution: $S^0 = \{s_1, \ldots, s_N\}$

Bias configuration: $H = \{h_1, \ldots, h_N\}$

Hamming distance: $d = \sum_i |s_i - h_i|/2$

Success rate: $p = |\langle \Psi_{\mathrm{final}} | \Psi_{\mathrm{ground\ state}} \rangle|^2$



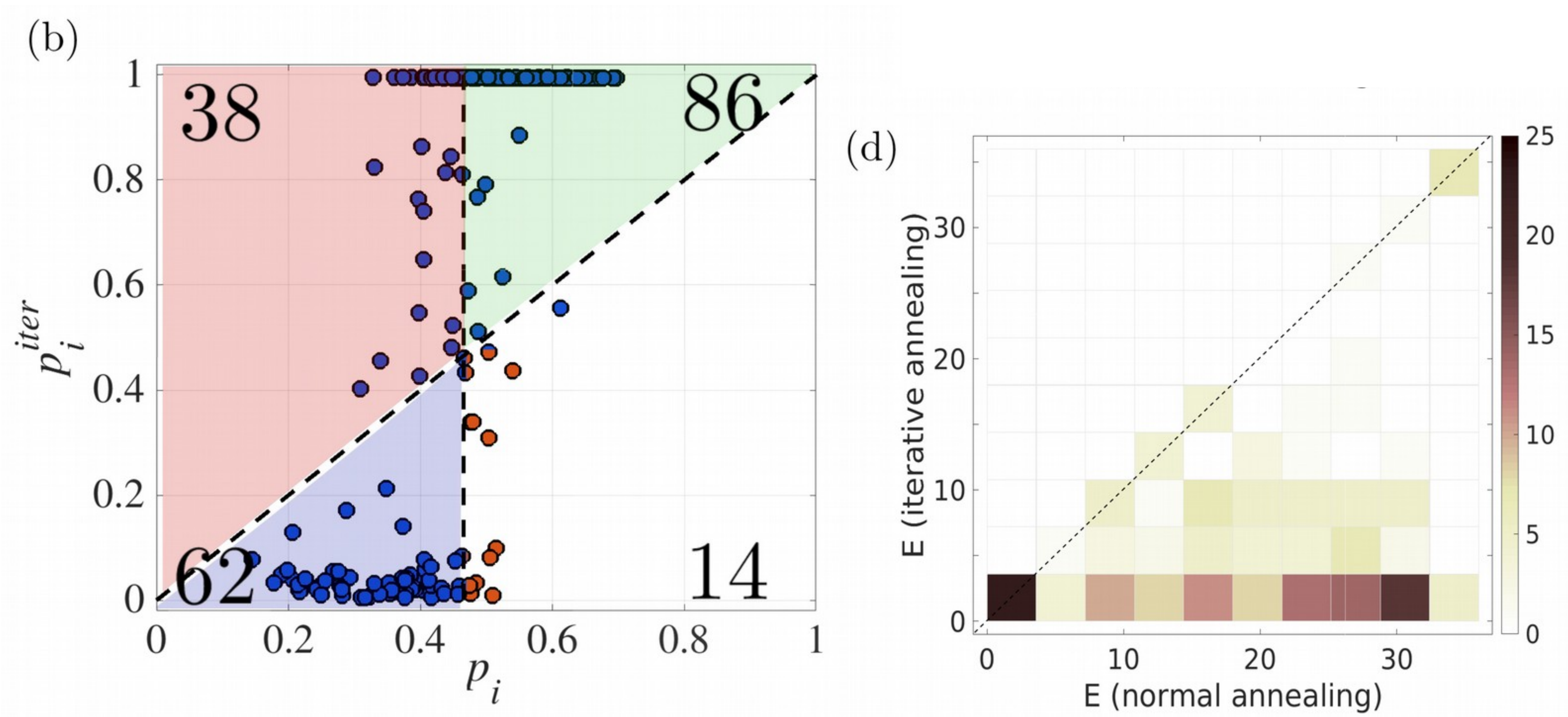
T. Grass, Phys. Rev. Lett. 123, 052309 (2019)

Iterative scheme

First run: Unbiased quantum annealing.

Next runs: Biased annealing, bias defined from previous outcome.

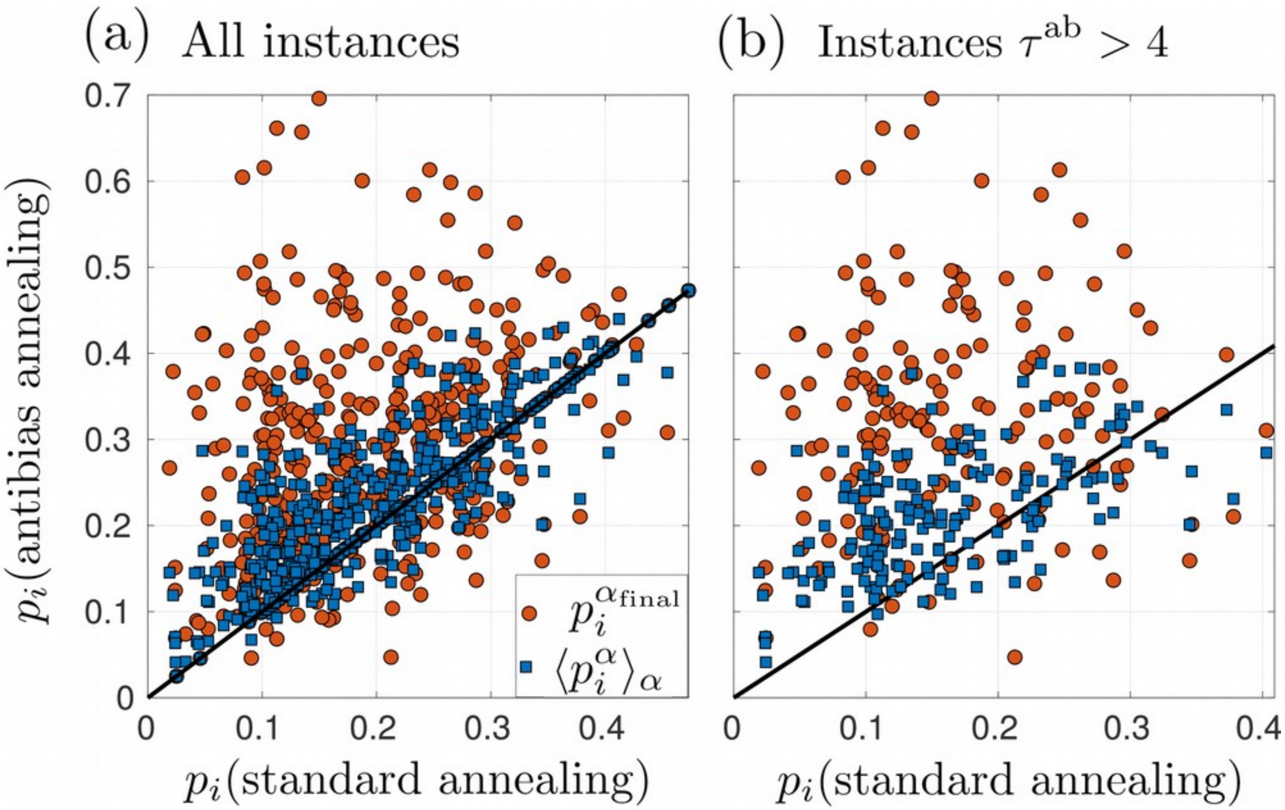
Stop condition: Input and outcome coincide.



T. Grass, Phys. Rev. Lett. 123, 052309 (2019)

Iterative "anti"-bias scheme

- Iterative bias scheme fails for the hardest instances
- Anti-bias scheme: apply bias which is anti-aligned with previous outcomes (accumulatively)
- New runs will be biased away from the previous (wrong) outcomes, thus have higher likelihood to find the correct
- Stop when a state with *E*=0 is measured



T. Grass, Phys. Rev. Lett. 123, 052309 (2019)

Concluding Remarks

- Quantum Annealing: solve optimization problem by implementing them in a physical system.
- Quantum "cooling" process: adiabatic evolution from an easy-to-prepare ground state with many quantum fluctuations to problem ground state.
- Bottleneck: small gaps along the annealing path.
- Possible solutions:
  - Inhomogeneous fields
  - Non-stoquastic driver Hamiltonian
  - Hybrid approaches (thermal/classical + quantum cooling)
  - Reverse Annealing
  - Biased Annealing:
    - Longitudinal fields in the driver Hamiltonian favor configurations which are "similar" to this bias.
    - Good bias =>, significantly enhanced success probability.
    - Iteratively generated bias: for the hardest instances it typically favors only a sub-optimal solution.
    - Anti-bias fields can push the system away from wrong states.

Thank you!