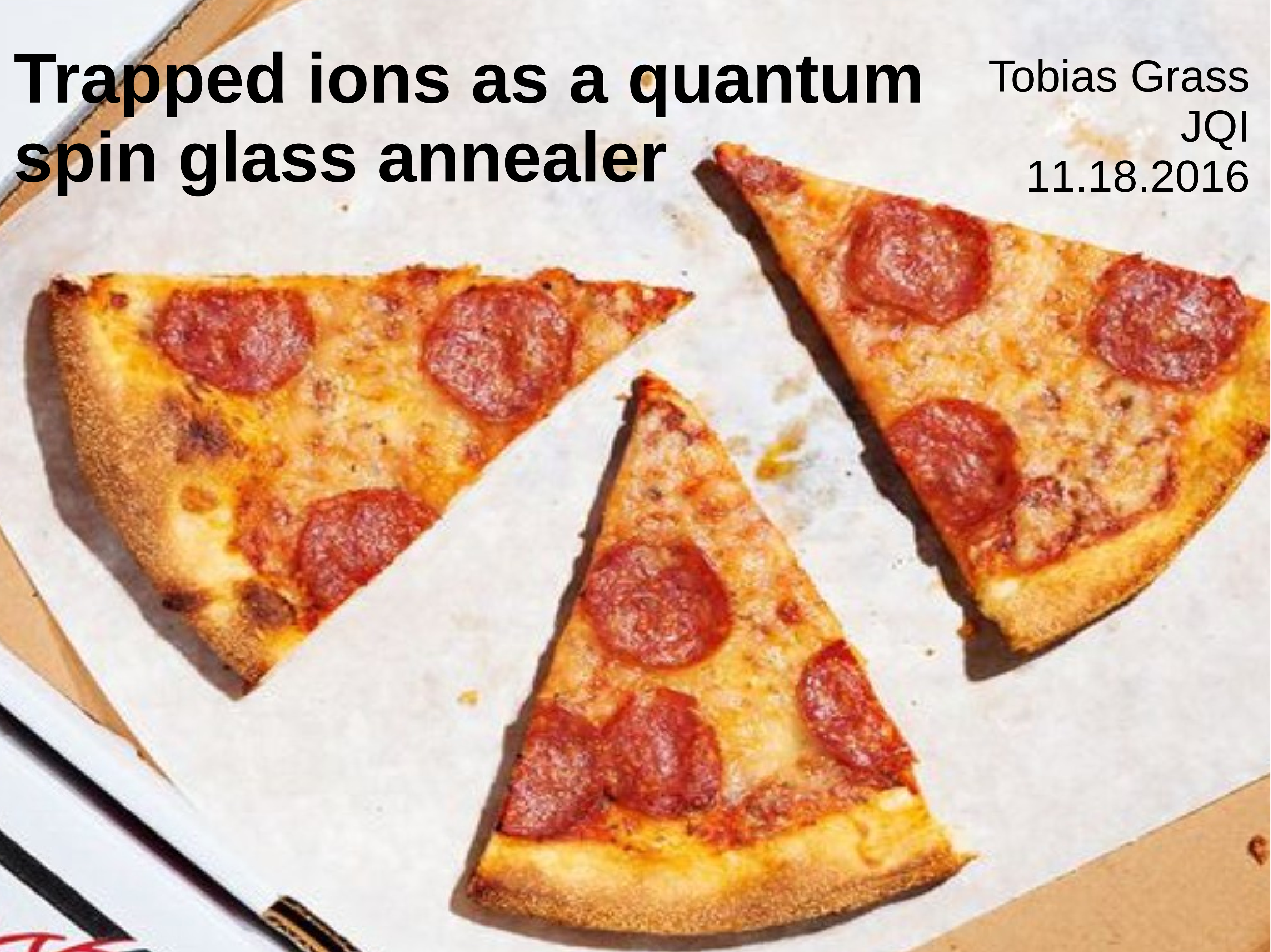
Trapped ions as a quantum spin glass annealer

Tobias Grass
JQI
11.18.2016



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I. Motivation: computational complexity

II. Spin models from ion chains

III. Numerical simulation of annealing dynamics

ICFO, Barcelona, Castelldefels

Castelldefels beach

Institut de Ciencies FOtoniques

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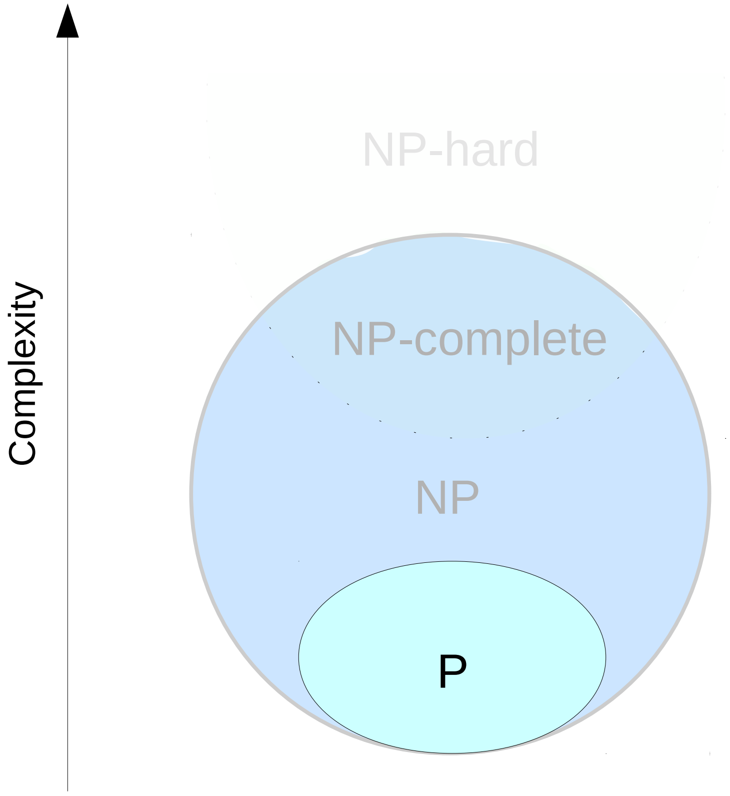
Motivation: Complexity classes



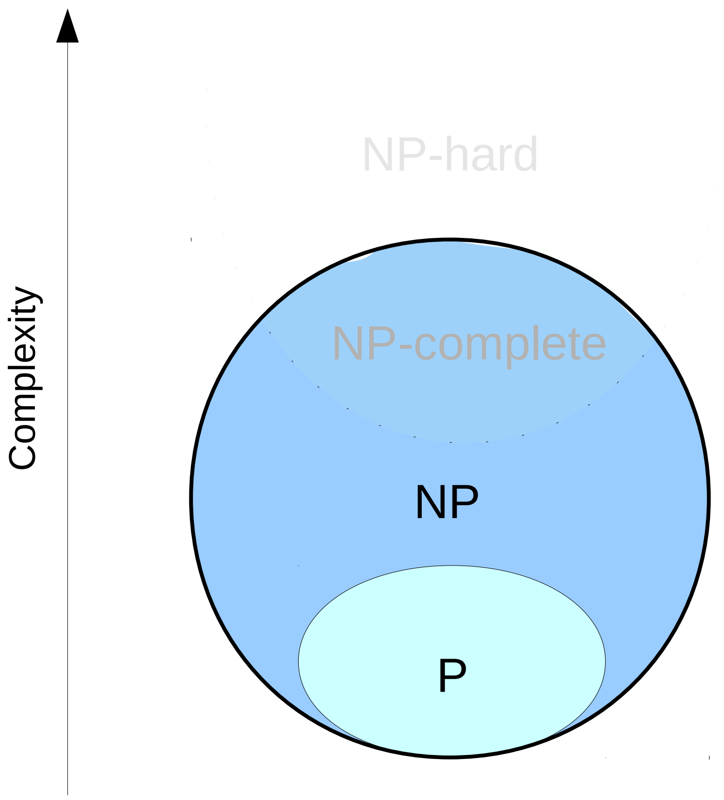
NP-hard: Problems at least as hard as NP-complete problems, but not necessarily in NP

NP-complete: “Hardest” problems in NP (to which any NP problem can be mapped in polynomial time)

NP: Decision problems which can be *solved* on a **non-deterministic** computer (or whose positive answer can be *verified* on a deterministic computer) in polynomial time

P: Decision problems solvable on a deterministic computer in polynomial time

Motivation: Complexity classes



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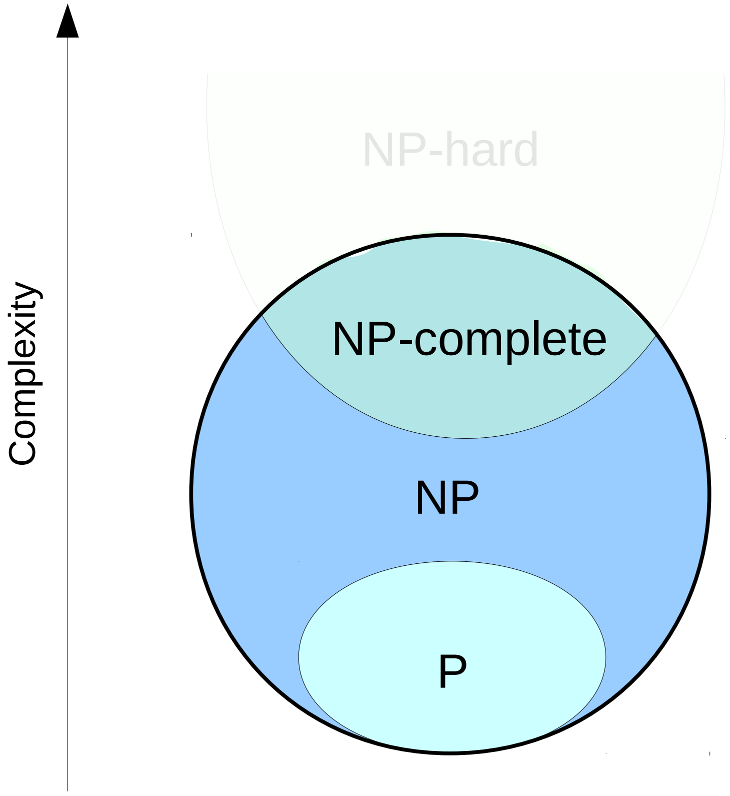
P = NP ?

Motivation: Complexity classes



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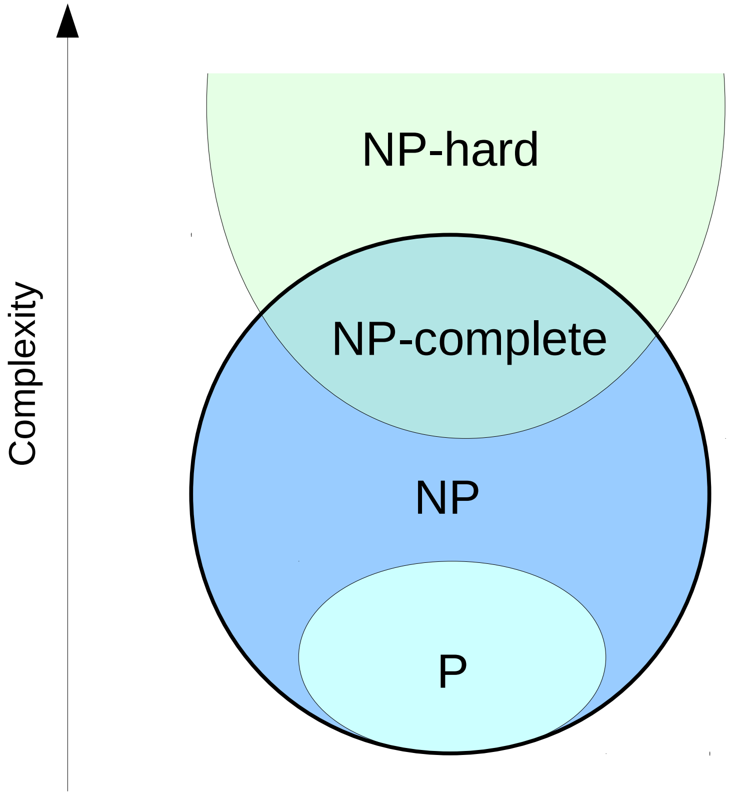

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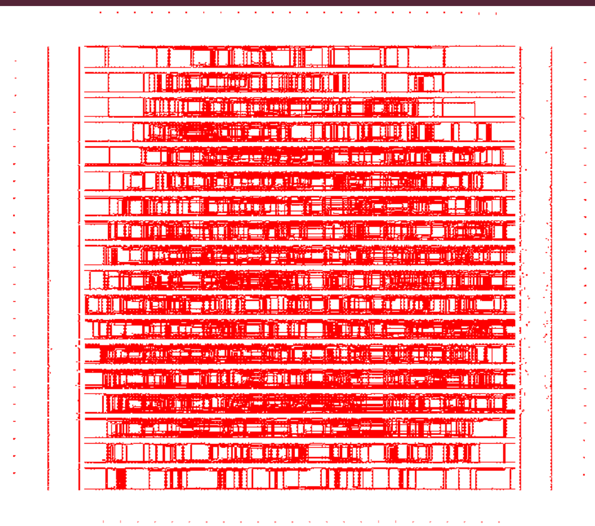
NP-complete problems

Traveling salesman problem

World record:

85,900 connections on a computer chip
computation time: 136 CPU yrs.



Number partitioning

2 6 7 9 12 13 17 20

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2+9+12+20 -6 -7-13 -17 = 0

2 6 7 9 12 13 17 20
6+17+20 -2 -7 -9 -12-13 = 0

PHYSICAL REVIEW
LETTERS

VOLUME 81 16 NOVEMBER 1998 NUMBER 20

Phase Transition in the Number Partitioning Problem

Stephan Mertens*
Universität Magdeburg, Institut für Physik, Universitätsplatz 2, D-39106 Magdeburg, Germany
(Received 6 July 1998)

Spin glasses

Spin models with random couplings

Journal of Physics A: Mathematical and General

Journal of Physics A: Mathematical and General > Volume 15 > Number 10

On the computational complexity of Ising spin glass models

F Barahona
Show affiliations

F Barahona 1982 *J. Phys. A: Math. Gen.* **15** 3241. doi:10.1088/0305-4470/15/10/028

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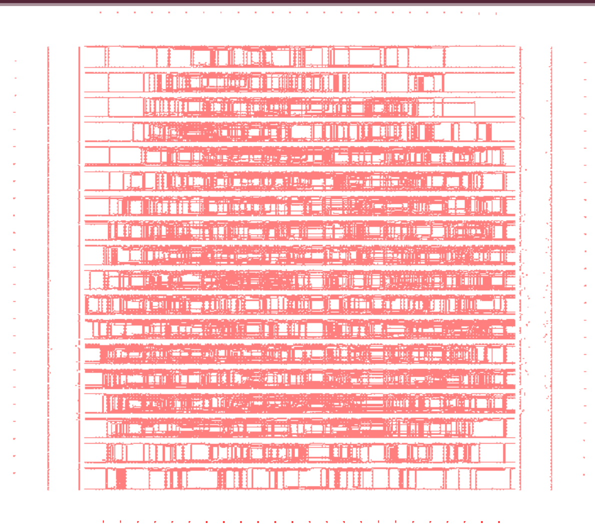

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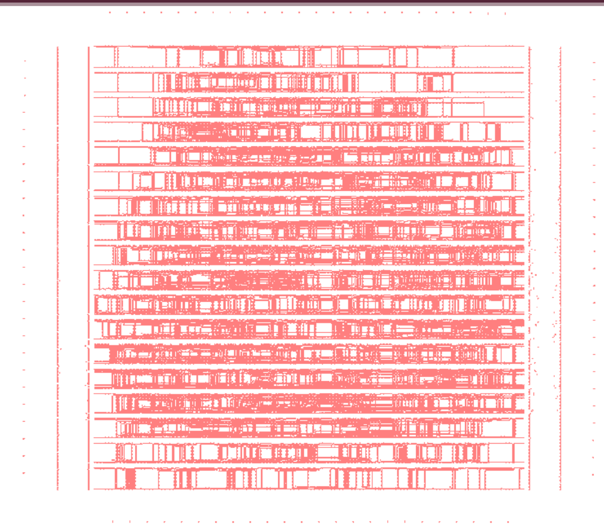

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II.

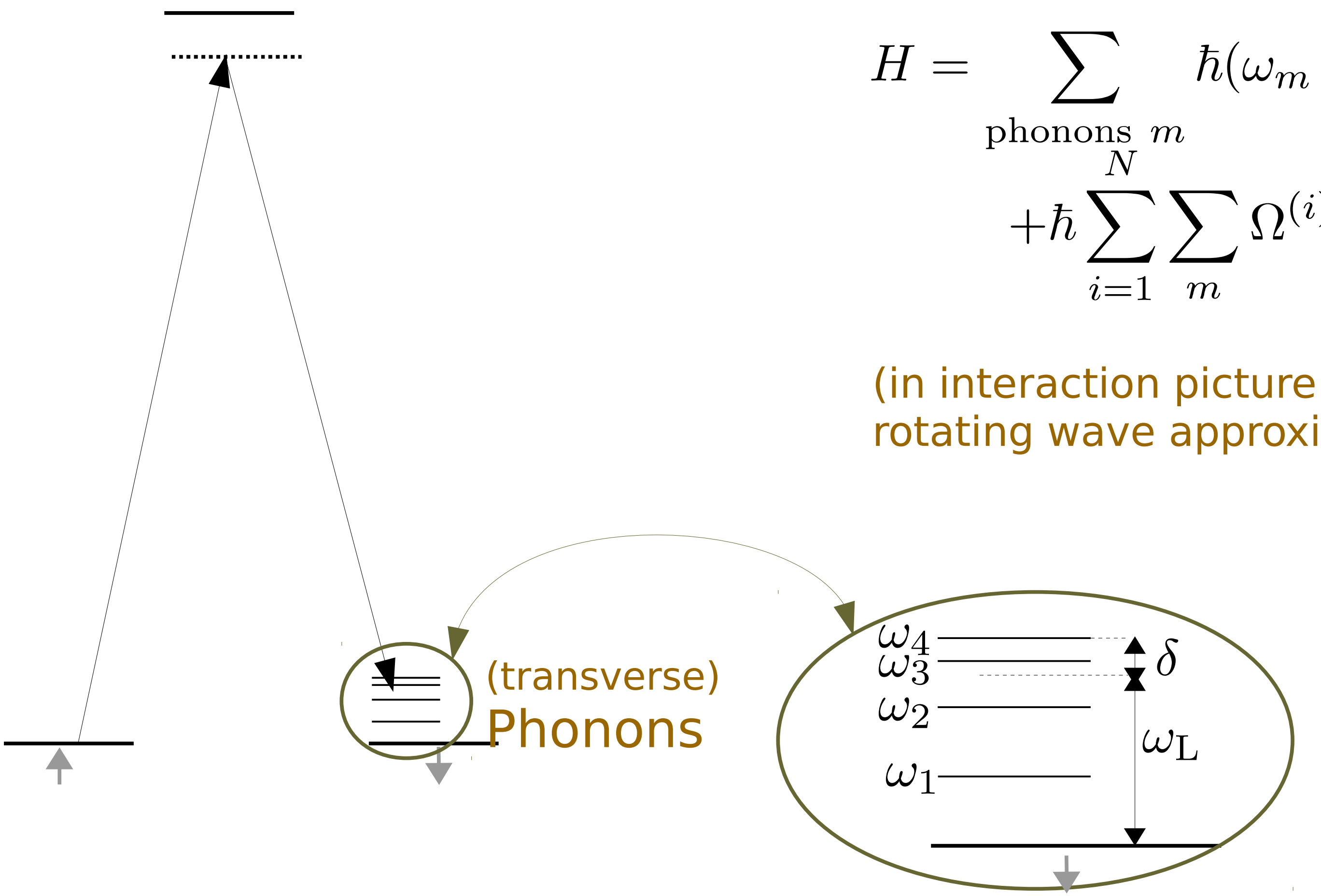
Spin models from ion chains

Trapped ions spin models

Spin-phonon coupling

$$H = \sum_{\text{phonons } m} \hbar(\omega_m - \omega_\mathrm{L})\hat{a}_m^\dagger \hat{a}_m + \hbar \sum_{i=1}^{N} \sum_{m} \Omega^{(i)} \eta_m^{(i)} (\hat{a}_m + \mathrm{H.c.}) \sigma_x^{(i)}$$

(in interaction picture after rotating wave approximation)



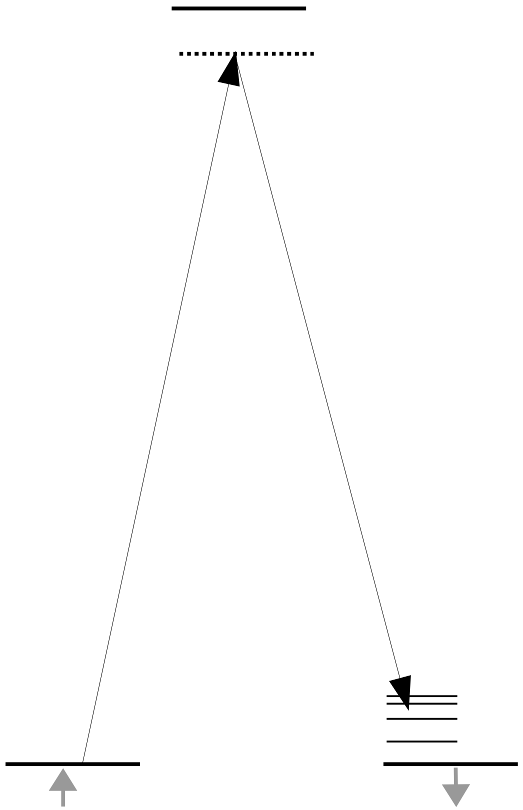
$\Omega^{(i)}$: Rabi frequency (at ion *i*)

$\eta_m^{(i)}$: Lamb-Dicke parameter (ion *i* to mode *m*)

ω_L : laser beatnote frequency

Trapped ions spin models

Spin-phonon coupling

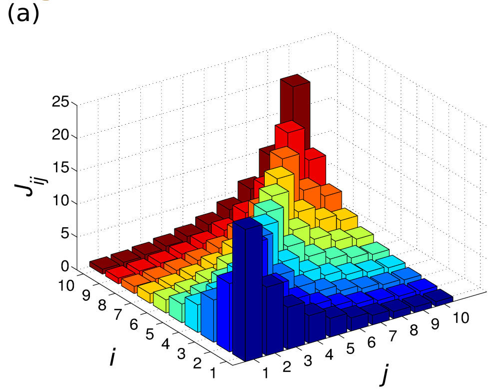


Effective Hamiltonian:

$$H_{\text{eff}} = -\sum_{ij}\sum_{m}\frac{\hbar\Omega_i\Omega_j\eta_m^{(i)}\eta_m^{(j)}}{\omega_m-\omega_{\text{L}}}\sigma_x^{(i)}\sigma_x^{(j)} = -\sum_{ij}J_{ij}\sigma_x^{(i)}\sigma_x^{(j)}$$

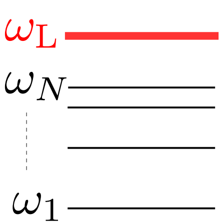
Ising-like model with controllable long-range coupling:

(a)

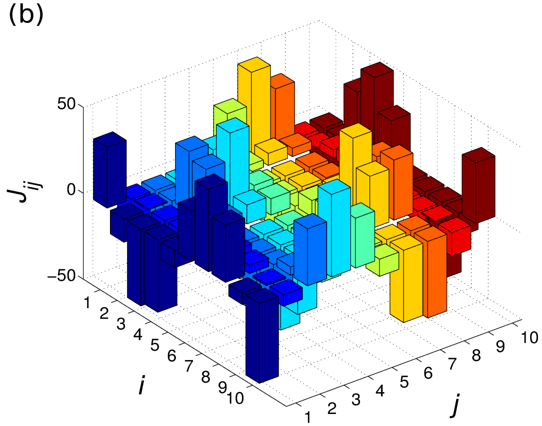


$\omega_{\text{L}} > \omega_N$:

- antiferromagnetic
- power-law decay

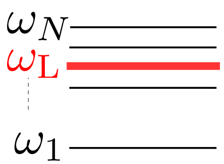


(b)



$\omega_1 < \omega_{\text{L}} < \omega_N$:

- mixed signs
- glassy



$\Omega^{(i)}$: Rabi frequency (at ion *i*)

$\eta_m^{(i)}$: Lamb-Dicke parameter

ω_{L} : laser beatnote frequency

Near resonance: Mattis model

At *l*-th phonon resonance: $J_{ij} \propto \sum_m \frac{\xi_m^{(i)} \xi_m^{(j)}}{\omega_m - \omega_{\mathrm{L}}} \;\Rightarrow\; J_{ij} \propto \pm \xi_\ell^{(i)} \xi_\ell^{(j)}$

$H_{\mathrm{eff}} \propto \pm \sum_{ij} \xi_\ell^{(i)} \xi_\ell^{(j)} \sigma_x^{(i)} \sigma_x^{(j)}$ factorizes: $H_{\mathrm{eff}} \propto \pm \left(\sum_i \xi_\ell^{(i)} \sigma_x^{(i)} \right)^2$

Minus: Ferromagnetic coupling

Two-fold degenerate ground state defined by the mode pattern:

$$\langle \sigma_x^{(i)} \rangle = \pm \mathrm{sign}(\xi_\ell^{(i)})$$

ω_4 — $\xi_4 = (0.5, 0.5, 0.5, 0.5)$
GS1= ↑↑↑↑ GS2= ↓↓↓↓

ω_3 — $\xi_3 = (0.66, 0.26, -0.26, -0.66)$
GS1= ↑↑↓↓ GS2= ↓↓↑↑

ω_2 — $\xi_2 = (0.5, -0.5, -0.5, 0.5)$
GS1= ↑↓↓↑ GS2= ↓↑↑↓

ω_1 — $\xi_1 = (-0.26, 0.66, -0.66, 0.26)$
GS1= ↑↓↑↓ GS2= ↓↑↓↑

Plus: Antiferromagnetic coupling

Energy is cost function of *number partitioning problem*:

$$E = \left(\sum_{i \in \uparrow} \xi_\ell^{(i)} - \sum_{i \in \downarrow} \xi_\ell^{(i)} \right)^2$$

ω_4 — ↑↑↓↓ ↓↓↑↑ ↑↓↑↓ ↓↑↓↑
$\xi_4 = (0.5, 0.5, 0.5, 0.5)$

ω_3 — ↑↓↓↑ ↓↑↑↓ ↑↑↑↑ ↓↓↓↓
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ω_2 — ↑↑↓↓ ↓↓↑↑ ↑↓↑↓ ↓↑↓↑
$\xi_2 = (0.5, -0.5, -0.5, 0.5)$

ω_1 — ↑↓↓↑ ↓↑↑↓ ↑↑↑↑ ↓↓↓↓
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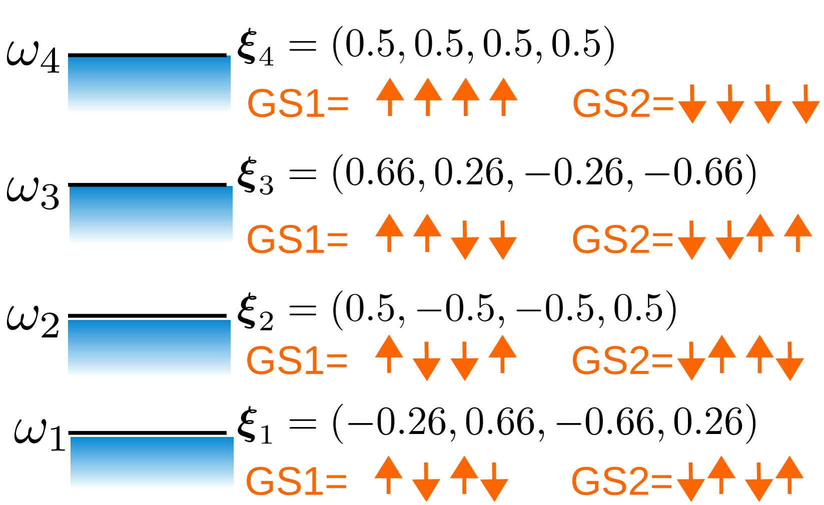

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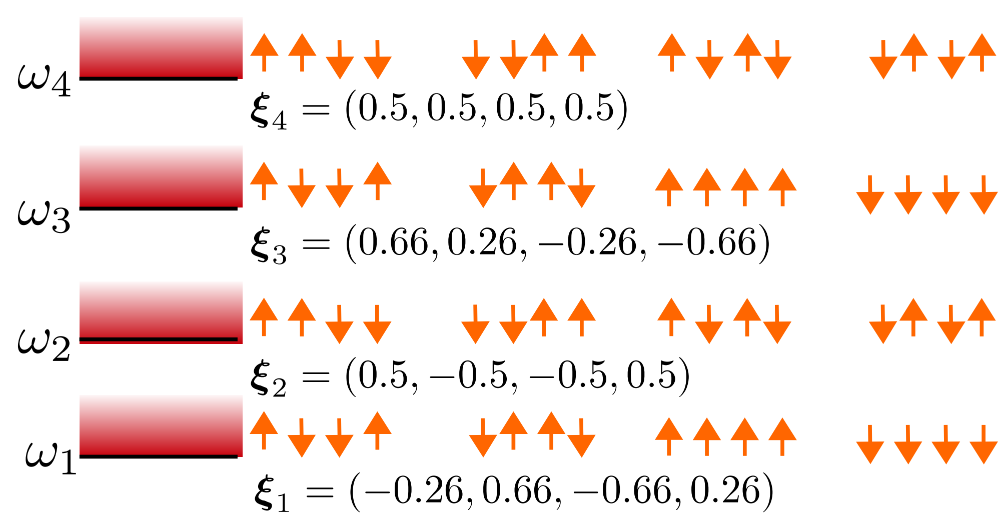

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factorizes:

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Energy is cost function of *number partitioning problem*:

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$2^{N/2}$ ground states due to parity symmetry

→ Number partitioning is trivial.
→ Problem becomes hard if degeneracy is lifted.
→ Off-resonant modes select unique ground state.

Solution via quantum annealing?

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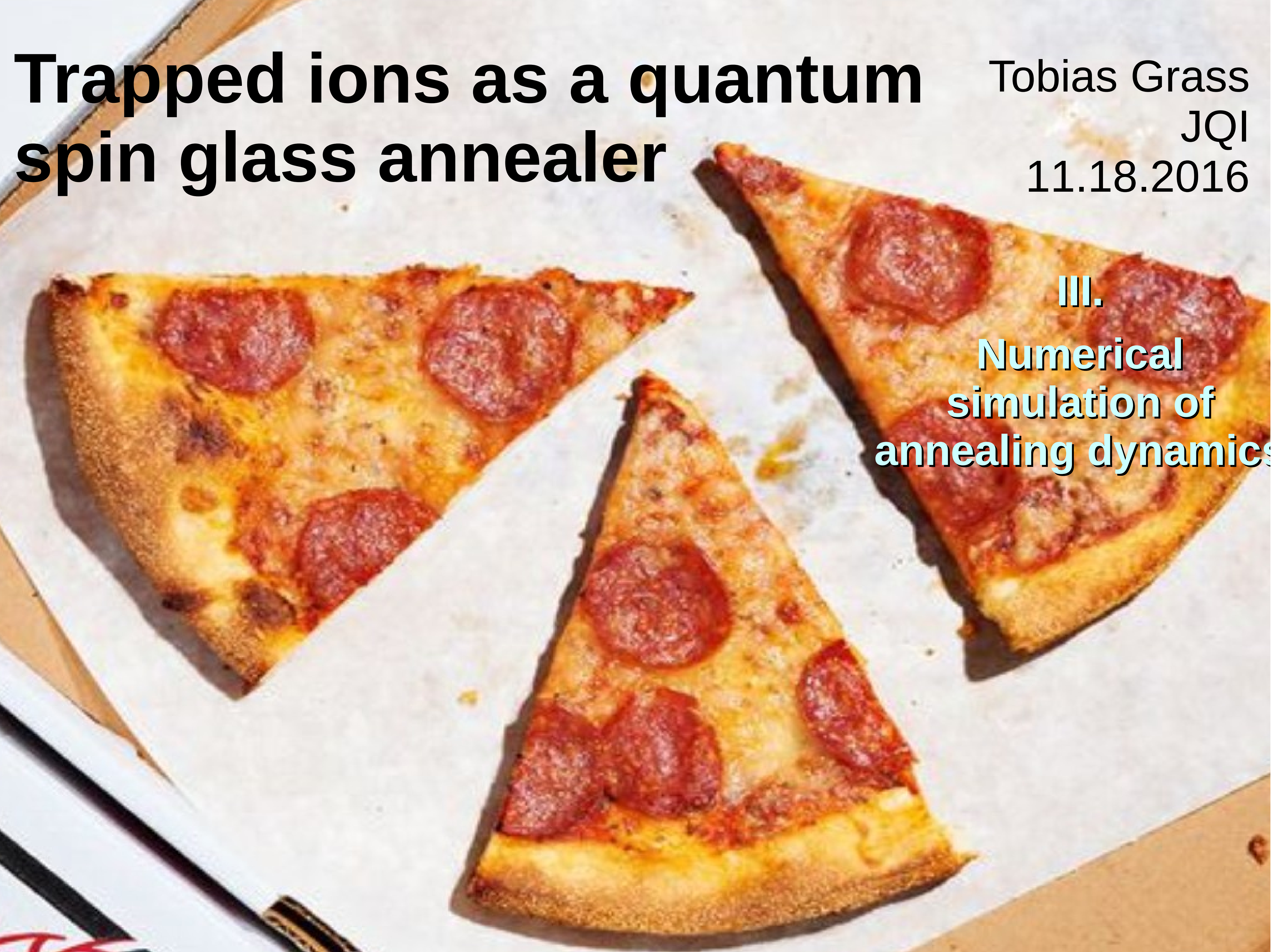
III.

Numerical simulation of annealing dynamics



From classical to quantum

Classical Hamiltonian

Quantum Hamiltonian

$$H = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)} \longrightarrow H = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)} + B\sum_i \sigma_z^{(i)}$$

Quantum fluctuations due to transverse field

Phase diagram

Spin glass
or
“ferromagnet”

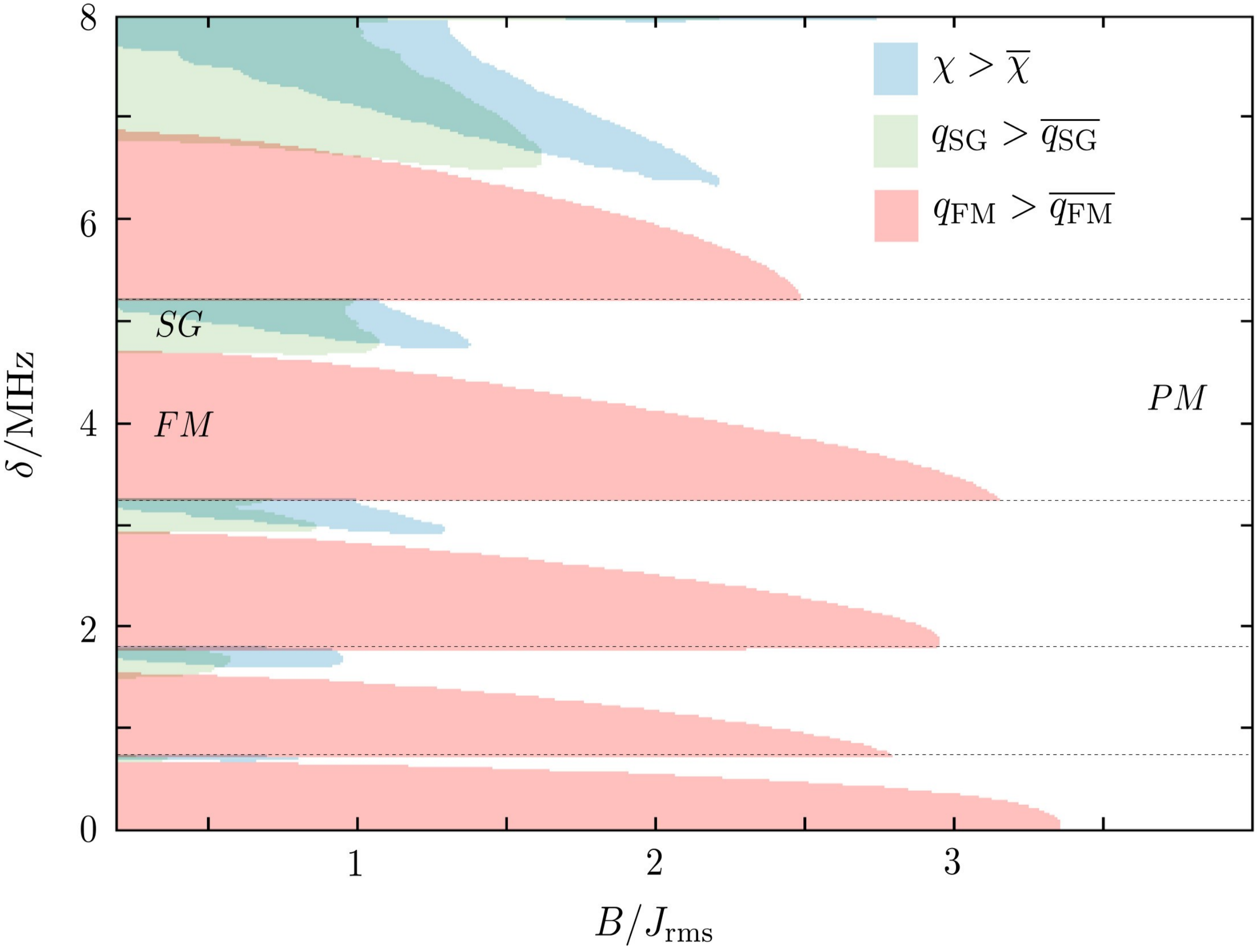
Quantum spin glass
or “ferromagnet”
or paramagnet

Dynamics

Quantum annealing:
Dynamics in a slowly decaying transverse field

“Phase diagram”

System properties upon varying detuning and transverse field for N=6:



Useful thermal averages:

$$q_{\mathrm{FM}} = \frac{1}{N}\sum_i \langle\langle\sigma_x^i\rangle\rangle_T^2$$

$$q_{\mathrm{EA}} = \frac{1}{N}\sum_i \langle\langle\sigma_x^i\rangle^2\rangle_T$$

$$q_{\mathrm{SG}} = q_{\mathrm{EA}}/q_{\mathrm{FM}}$$

(should be calculated for $k_{\mathrm{B}}T \approx J$ in the presence of a Z_2 breaking field)

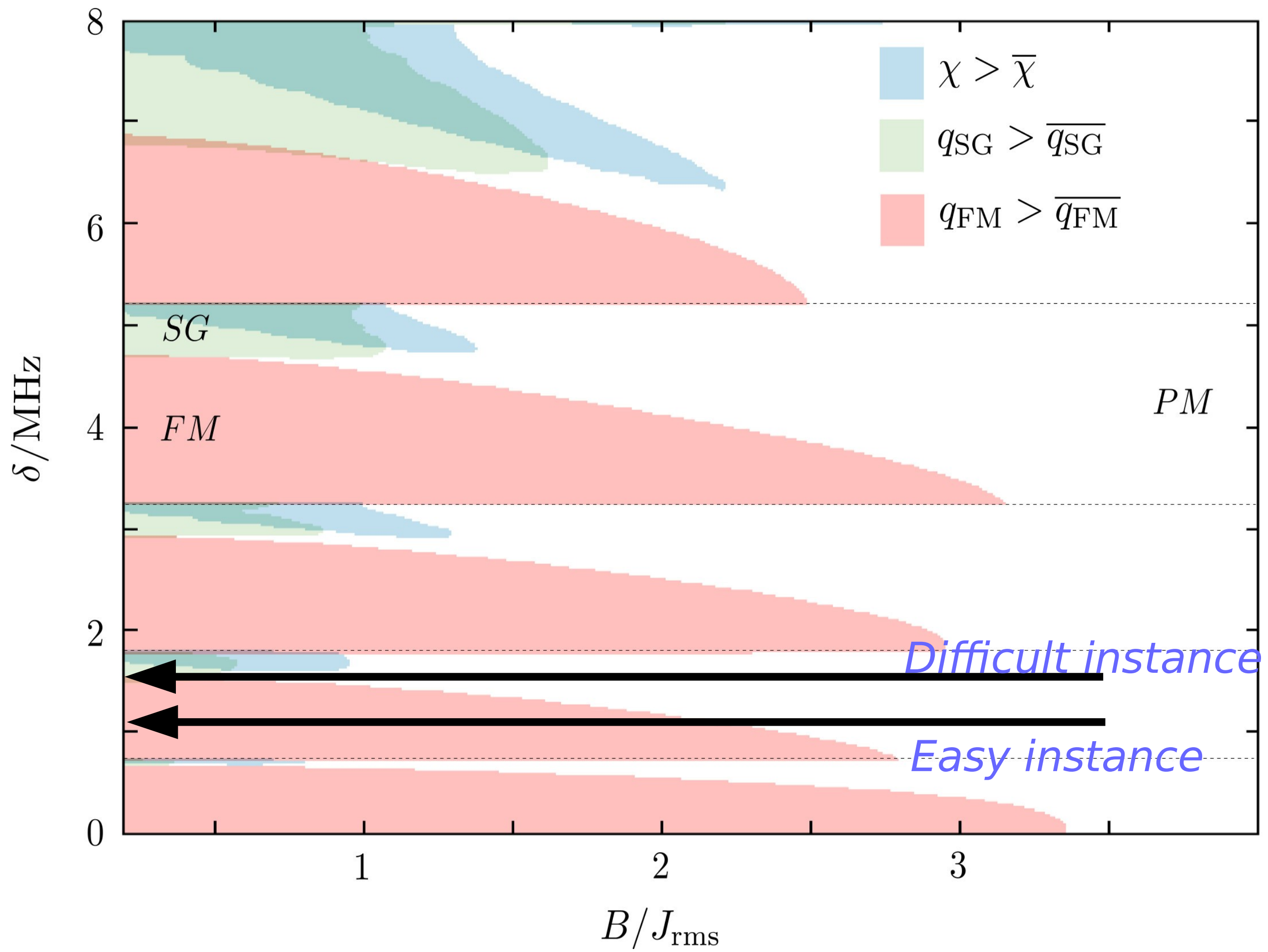
Magnetic susceptibility:

$$\chi = \frac{1}{N}\sum_{ij}\left(\frac{\partial\langle\sigma_x^i\rangle}{\partial h_x^j}\right)^2$$

(small longitudinal field *h* plus Z_2 breaking field)

Quantum annealing

System properties upon varying detuning and transverse field for *N*=6:



Using Krylov methods we simulate dynamics in a time-dependent Hamiltonian for 6 spins and including the phonons:

$$H(t) = H_0(t) + B(t)\sum_i \sigma_z^{(i)} + \epsilon_{\mathrm{bias}}\sigma_x^{(1)}$$

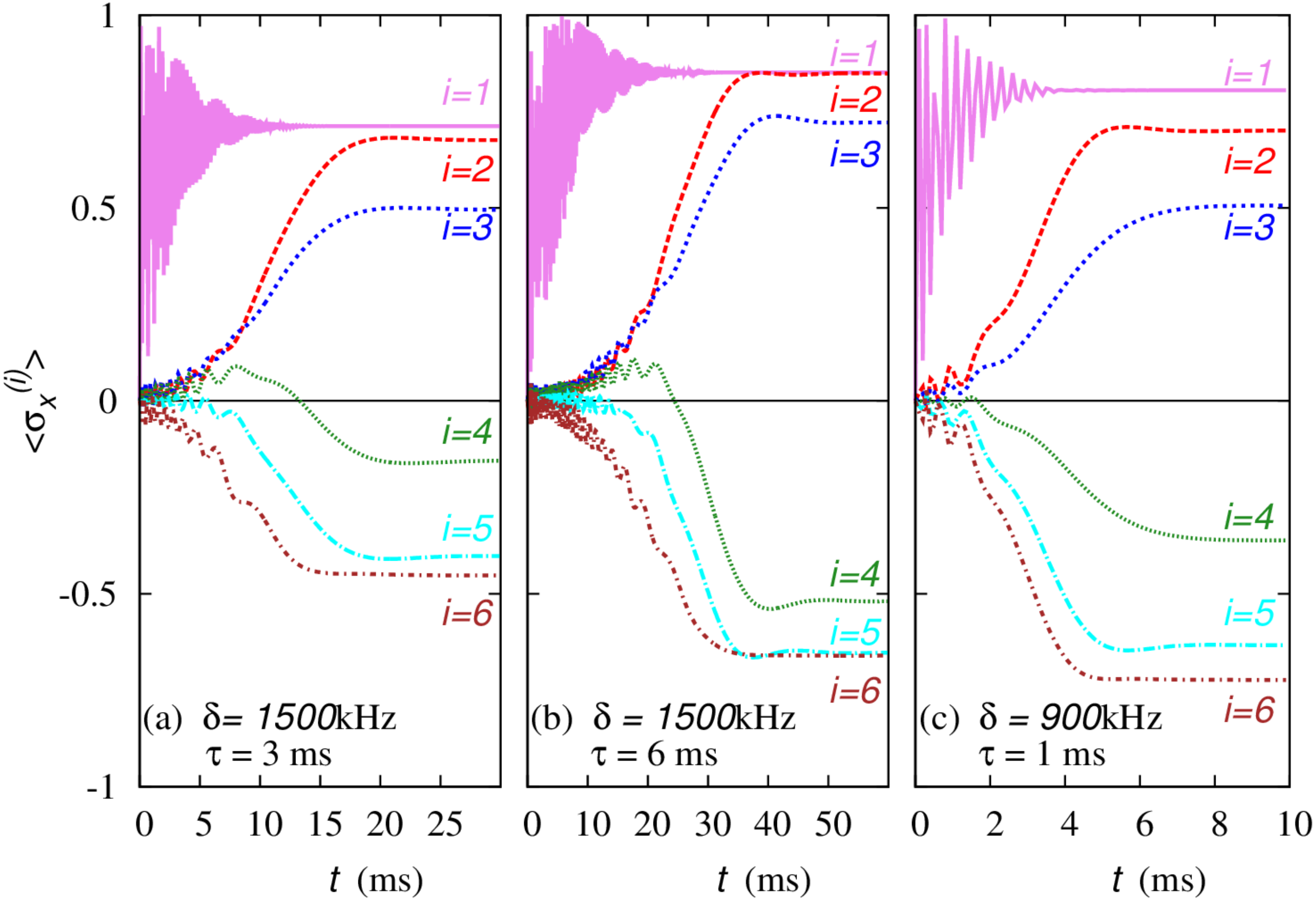
Spin-phonon coupling:

$$H_0(t) = \sum_m \hbar\omega_m a_m^\dagger a_m + \hbar\Omega\sin(\omega_{\mathrm{L}}t)\times$$
$$\times \sum_{i,m} \xi_m^{(i)} \sqrt{\frac{\omega_{\mathrm{recoil}}}{\omega_m}}\sigma_x^{(i)}(a_m + a_m^\dagger)$$

Decaying transverse field:

$$B(t) = B_0 \exp(-t/\tau)$$

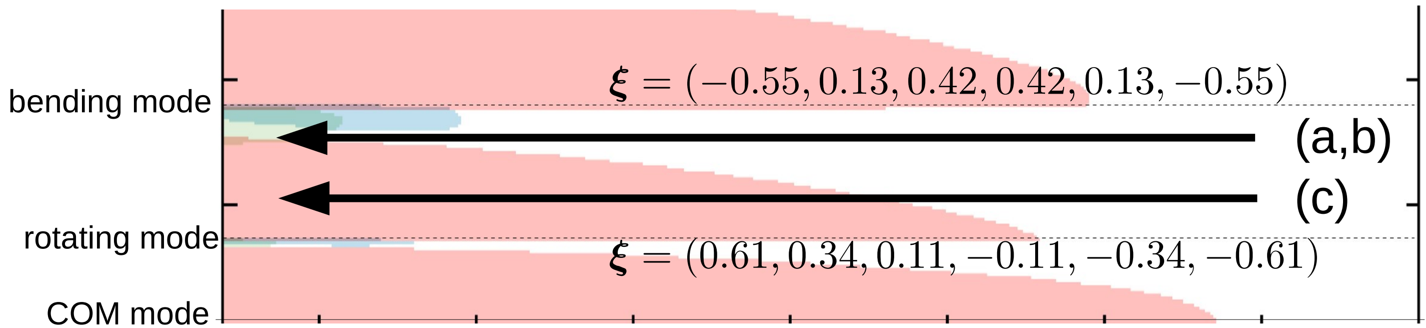
Exact dynamics (6 ions)



Local magnetization vs time:

(a,b) glassy target state
(c) ferromagnetic target state

For all instances, the correct sign (defined by pattern of the rotating mode) is produced during the annealling



Semiclassical approximation

Heisenberg equations of motion:

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\langle a_m\rangle = \langle[a_m, H(t)]\rangle$$

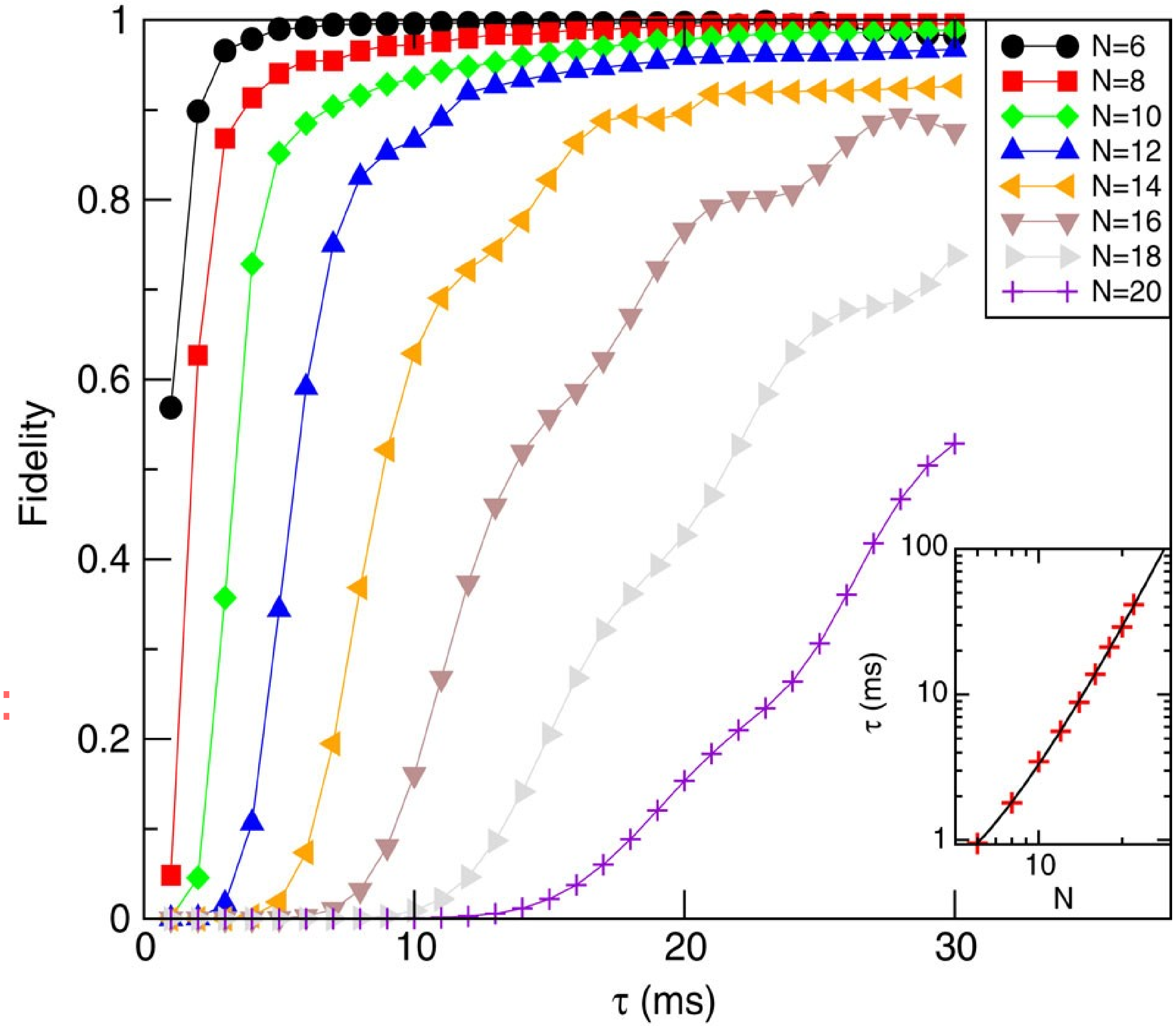
$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\langle\sigma_\alpha^i\rangle = \langle\left[\sigma_\alpha^i, H(t)\right]\rangle$$

Mean-field decoupling:

$$\langle a_m\sigma_x^i\rangle \approx \langle a_m\rangle\langle\sigma_x^i\rangle$$

Scaling up to 22 ions: Polynomial increase of annealing time:

$$\tau \propto N^4$$



Summary

[T. Grass *et al.*, Nat. Commun. **7** 11524 (2016)]

David Raventós (ICFO)

Bruno Juliá-Díaz (UB, ICFO)

Christian Gogolin (ICFO,MPQ)

Maciej Lewenstein (ICFO, ICREA)

→ Ion chains can naturally incorporate spin glass physics.

→ Setup directly relates to (NP-hard) number partitioning problem.

→ Feasibility of quantum annealing shown for small system (6 spins plus phonons) using exact diagonalization.

→ Scales well within a semiclassical approximation: Polynomial increase of annealing time.

Thank you!