

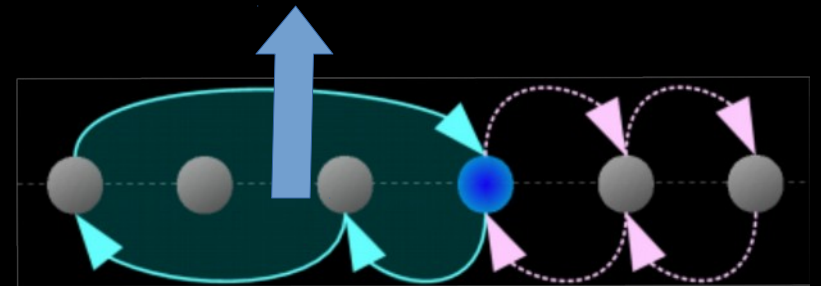
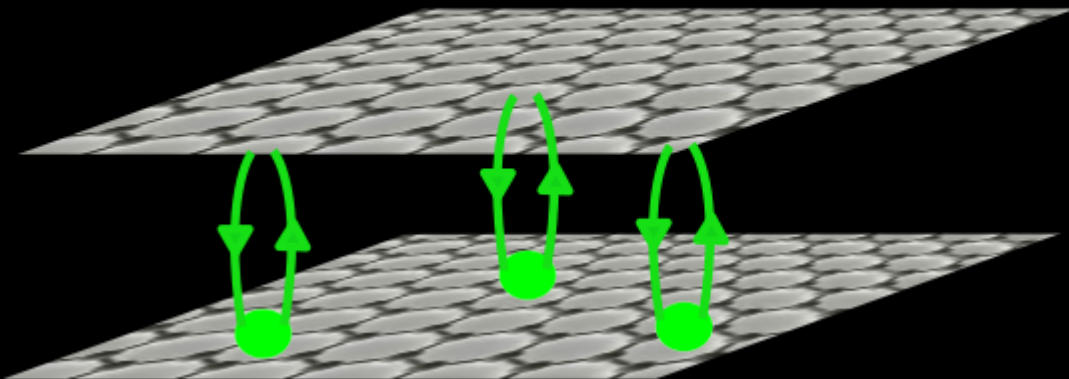
University of Oklahoma

Department of Physics and Astronomy

02/14/2019

Colloquium

# Topological Quantum Simulators



Tobias Grass

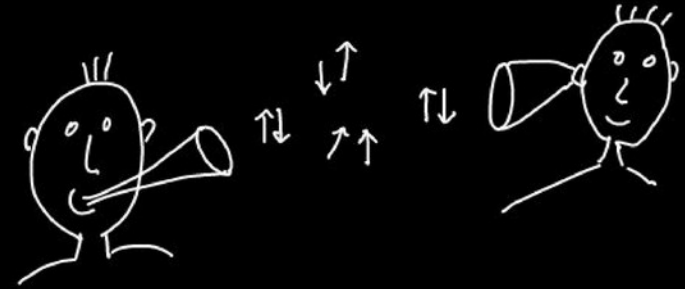
Joint Quantum Institute and University of Maryland

College Park, Maryland, USA



## Quantum Sensing

(precision measurements, atomic clocks, quantum imaging, ...)



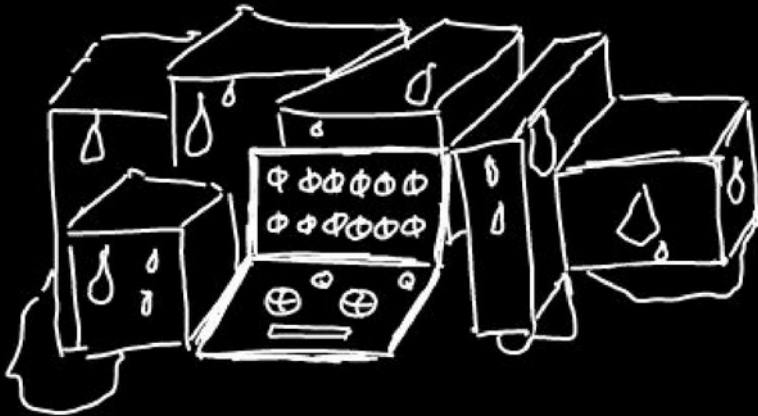
## Quantum Communication

(Quantum cryptography, quantum networks,...)

# Quantum Technologies

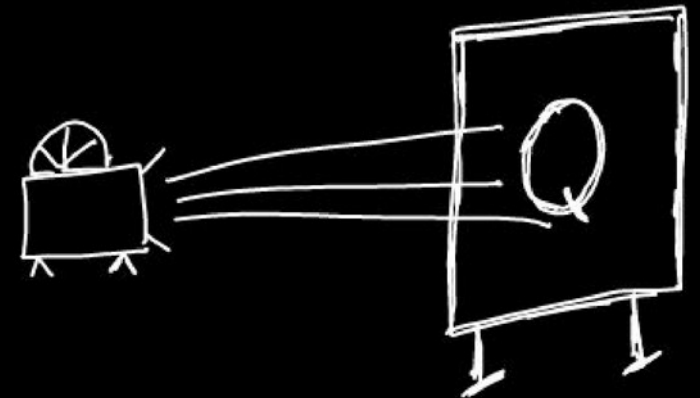
## Quantum Computation

(Quantum processor, quantum algorithms, error-correction,...)



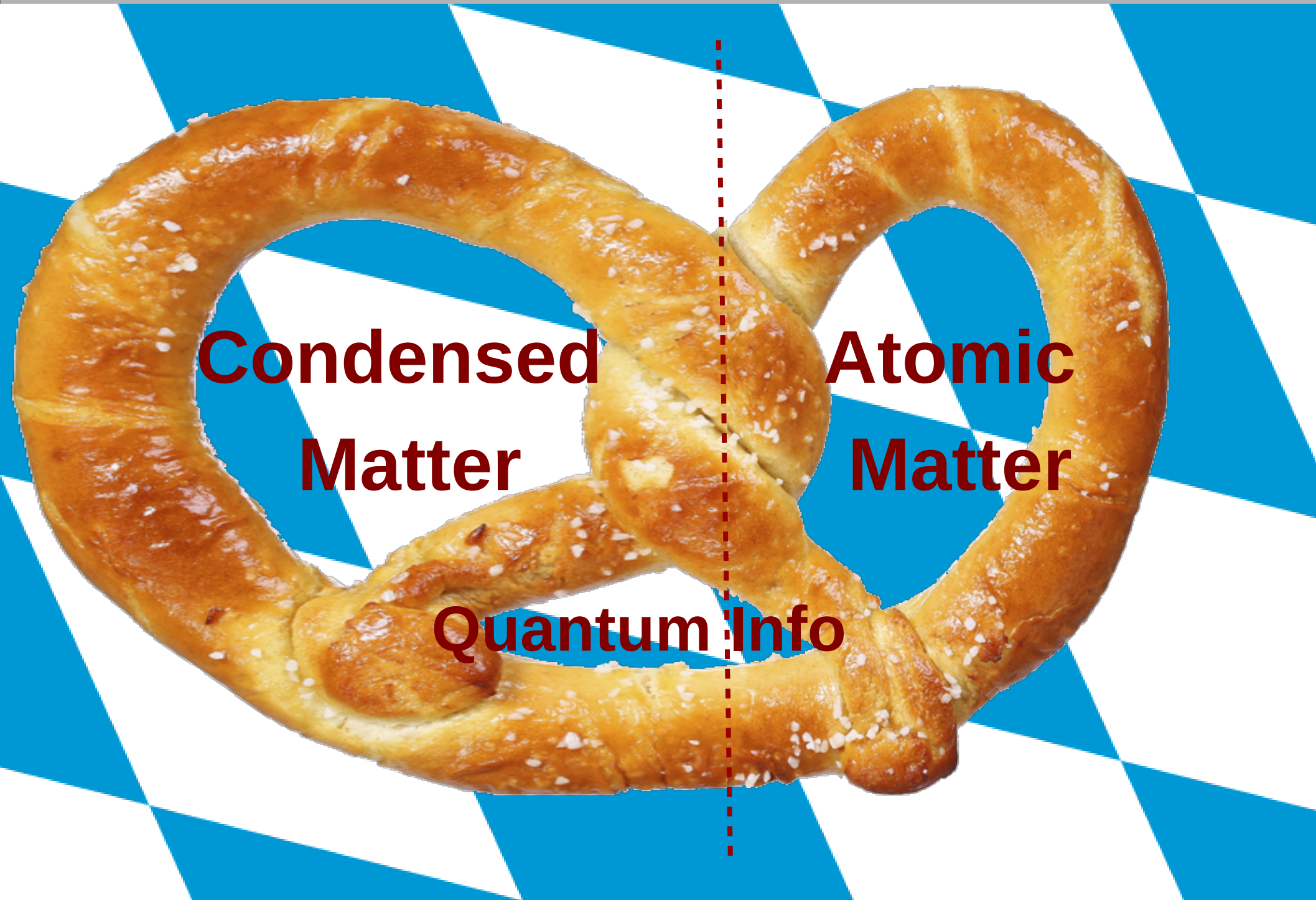
## Quantum Simulation

(quantum many-body physics, model solver, quantum annealing...)









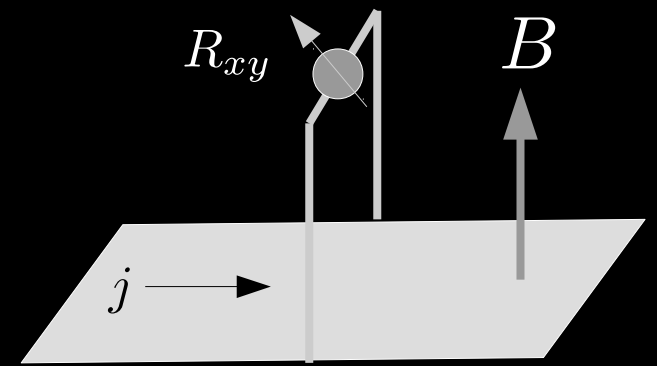
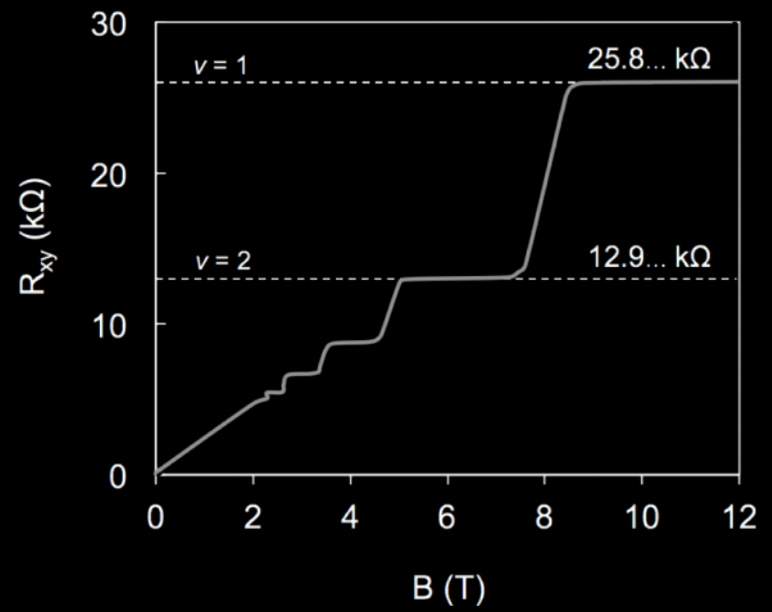
**Condensed  
Matter**


**Atomic  
Matter**

**Quantum Info**

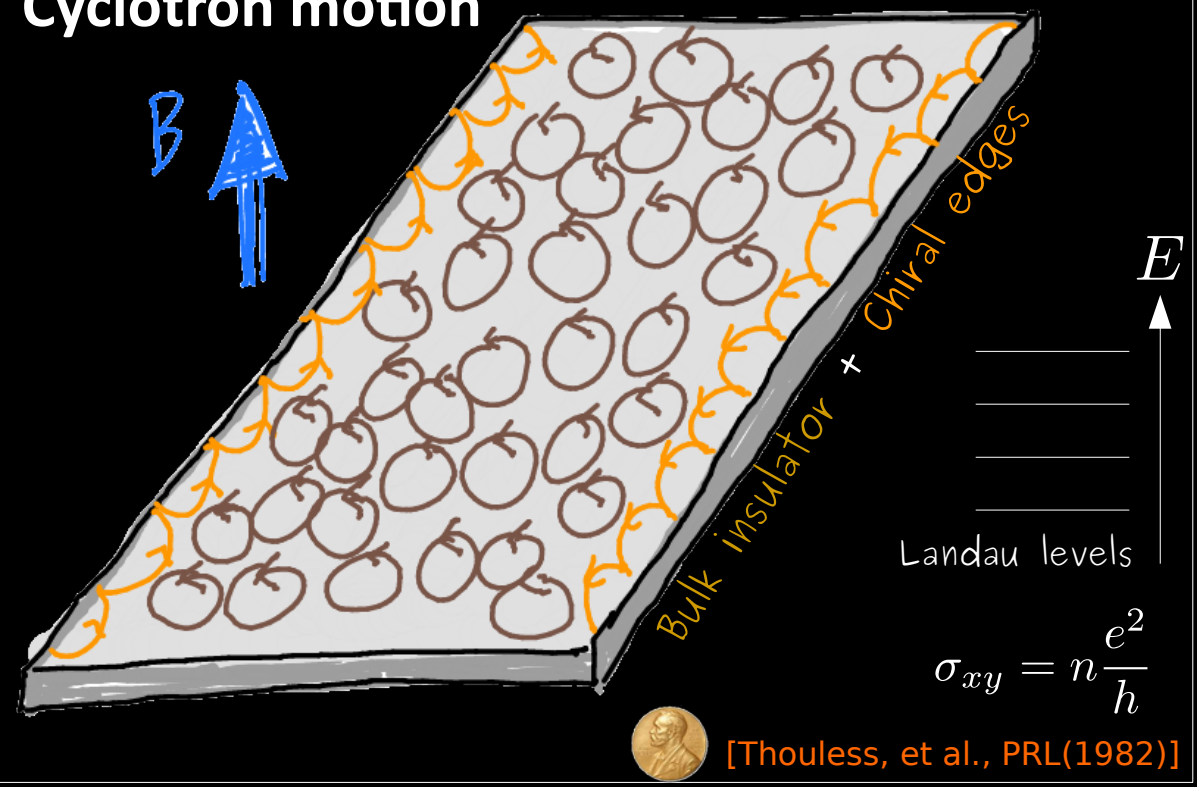
# Integer Quantum Hall Effect

## Experimental observation: Quantized Hall resistance




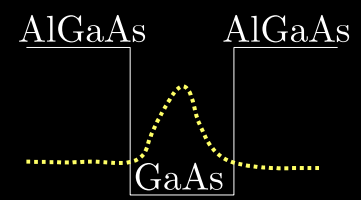
 [von Klitzing, Dorda, Pepper, PRL (1980)]


## 2d transport in magnetic field: Cyclotron motion

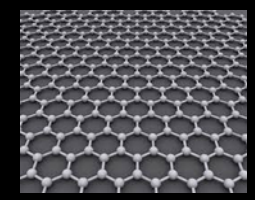



## Experimental systems:

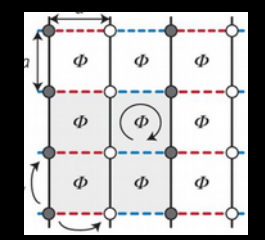
 Semiconductors  
Quantum Wells



 2d materials  
(graphene)



 Quantum  
simulators

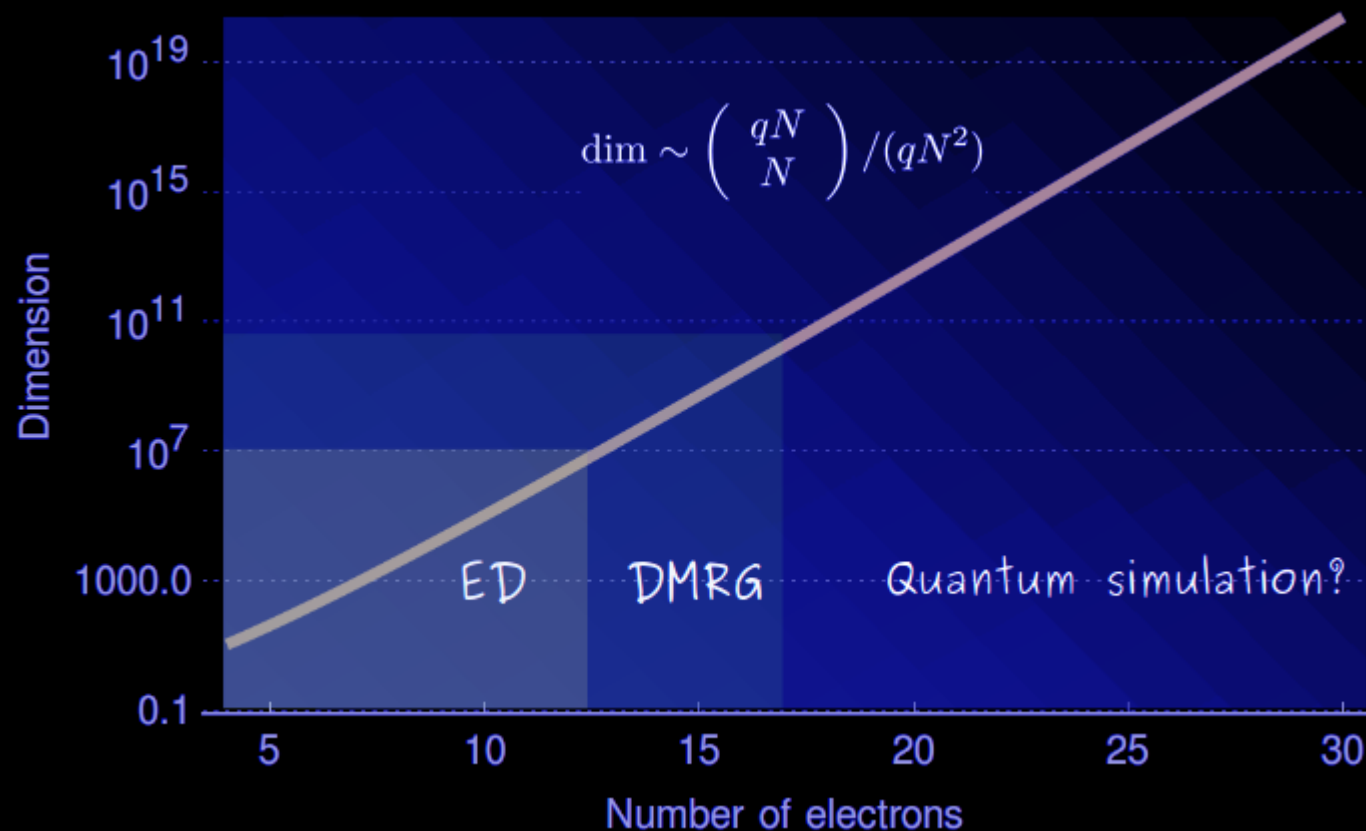
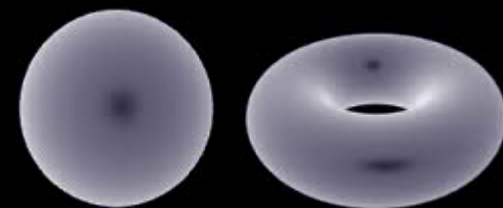


**Remarkable observation:** Robust Hall conductances also for fractionally filled Landau levels  [Tsui, Stormer, Gossard, PRL (1982)]

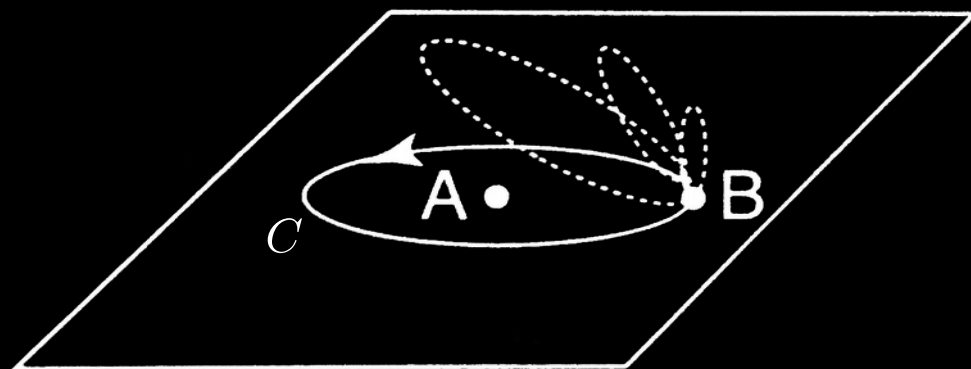
**Explanation:** Gapped liquid due to interactions  [Laughlin, PRL (1982)]

→ Non-perturbative many-body problem

→ Numerical methods: Exact diagonalization, DMRG



## Particle exchange in 3 dimensions:



$$\Psi_{AB} \rightarrow e^{i\gamma(C)} e^{i2\varphi_{\text{stat}}} \Psi_{AB}$$

Topological equivalence between double-exchange loop and no loop

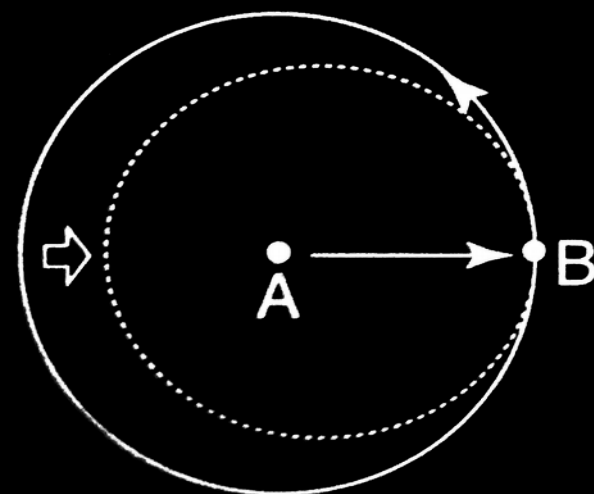
$$e^{i2\varphi_{\text{stat}}} = 1$$

→ Statistical phase restricted:

$$\varphi_{\text{stat}} = 0 : \quad \Psi_{AB} = +\Psi_{BA} \quad \rightarrow \text{bosons}$$

$$\varphi_{\text{stat}} = \pi : \quad \Psi_{AB} = -\Psi_{BA} \quad \rightarrow \text{fermions}$$

## Particle exchange in 2 dimensions:



Double-exchange loop cannot be shrunk away

→ Particle exchange can produce an arbitrary phase

$$\Psi_{AB} = e^{i\varphi_{\text{stat}}} \Psi_{BA} \quad \rightarrow \text{anyons}$$

Quasiparticle excitations in FQH systems exhibit anyon statistics!



## Non-Abelian anyons: degenerate states characterized by “fusion rules”

Example: Fibonacci anyons

$$1 \otimes \tau = \tau$$

$$1 \otimes 1 = 1$$

$$\tau \otimes \tau = 1 \oplus \tau$$

$\Rightarrow$

$$\tau \otimes \tau \otimes \tau = 1 \oplus 2 \cdot \tau$$

$$\tau \otimes \tau \otimes \tau \otimes \tau = 2 \cdot 1 \oplus 3 \cdot \tau$$

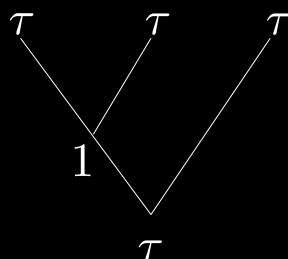
$$\tau \otimes \tau \otimes \tau \otimes \tau \otimes \tau = 3 \cdot 1 \oplus 5 \cdot \tau$$

$\vdots$

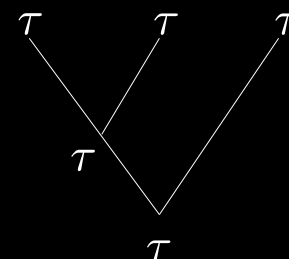
Number of states which fuse to  $\tau$  follows Fibonacci series: 1, 2, 3, 5, 8...

Topological qubit:

$$|0\rangle \equiv$$



$$|1\rangle \equiv$$



Topological quantum computing:

Process information by braiding



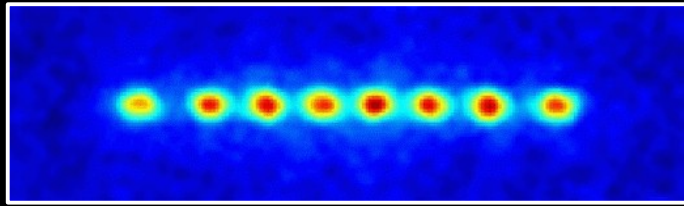
Technically involved



Robust against local noise



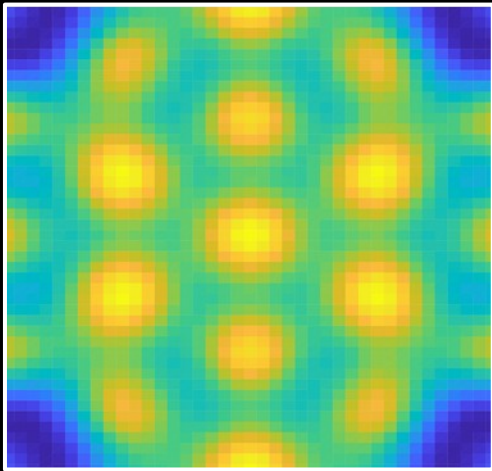
**THIS TALK:** How to engineer Fibonacci phase in graphene!



## (1) QUANTUM SIMULATIONS WITH TRAPPED IONS:

Synthetic Hofstadter ladder (IQH system)

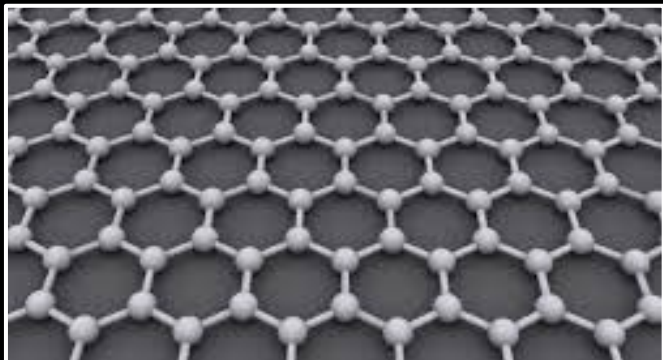
- Microscopic System (individual control)
- Naturally long-ranged interactions
- 1-D



## (2) QUANTUM SIMULATION WITH COLD ATOMS:

Fractional Wigner Crystal

- Mesoscopic System
- Long-range interactions through Rydberg dressing



## (3) QUANTUM SIMULATION IN REAL MATTER

Non-Abelian Fibonacci phase

- Macroscopic System

# (1)

# QUANTUM SIMULATION OF TOPOLOGICAL MATTER WITH TRAPPED IONS

Synthetic Hofstadter ladder

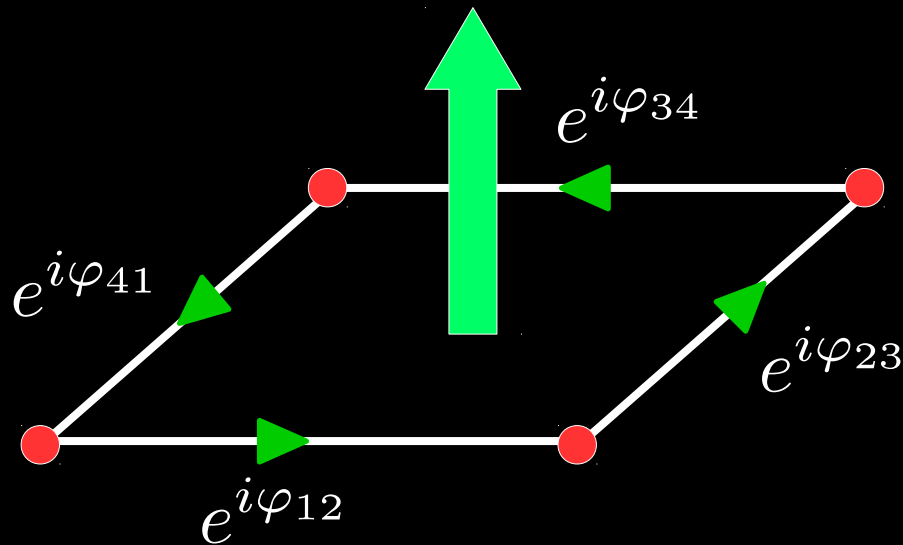
[Grass, Muschik, Celi, Chhajlany, Lewenstein PRA (2015)]

Triangle with flux

[Grass, Celi, Pagano, Lewenstein, PRA (Rapid) (2018)]

Magnetic field  $\rightarrow$  wave function picks up Berry phase when moving along closed loop

$$\Phi = \varphi_{12} + \varphi_{23} + \varphi_{34} + \varphi_{41} \quad (= \text{encircled flux})$$

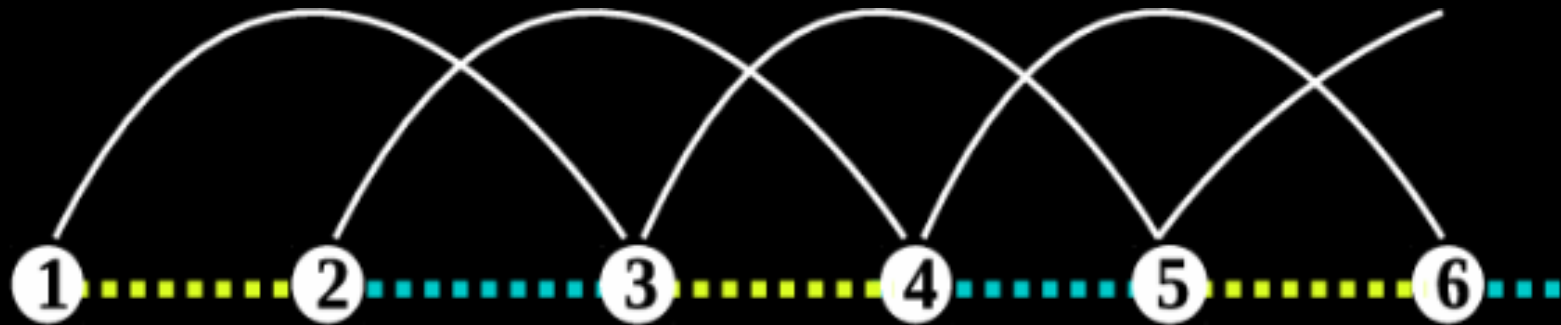


In 1D: Closed loops are along straight lines

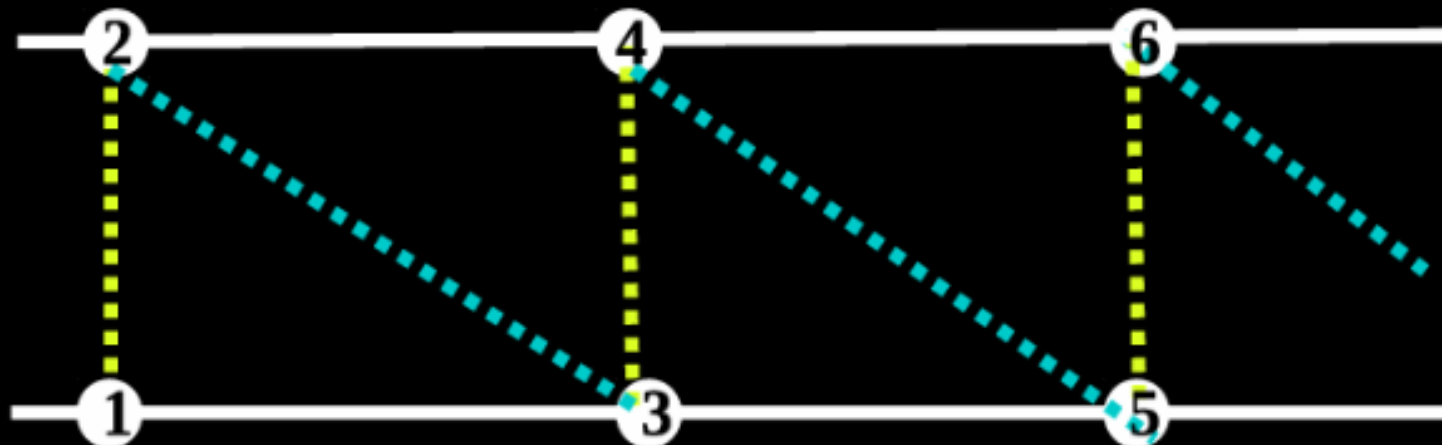


Can we still pick up a non-zero Berry phase?

Chain with nearest and next-nearest neighbor interactions:

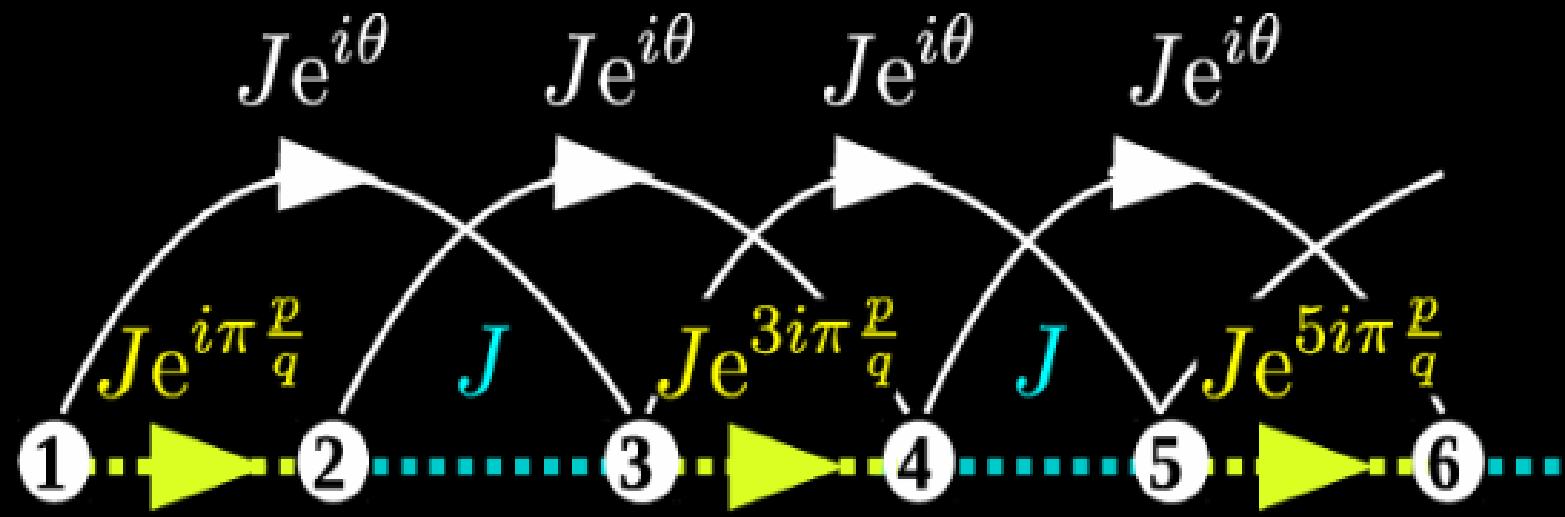


Mapping onto triangular ladder (“synthetic ladder”):

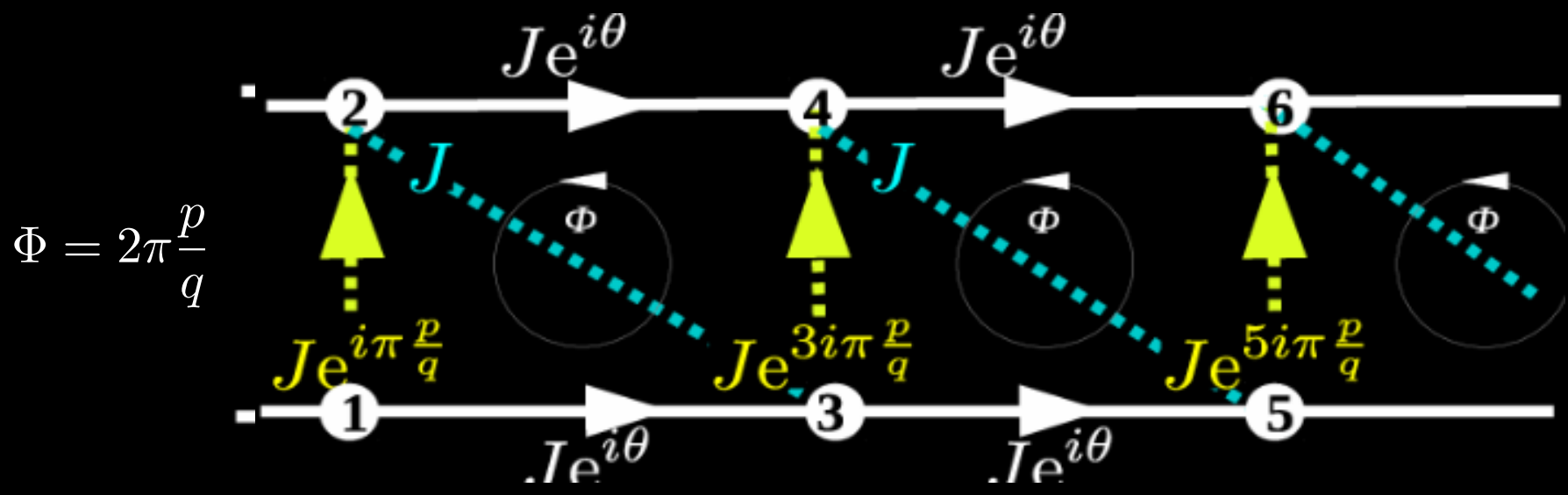


[Grass, Muschik, Celi, Chhajlany, Lewenstein, PRA (2015)]

Chain with nearest and next-nearest neighbor interactions:



Mapping onto triangular ladder ("synthetic ladder"):

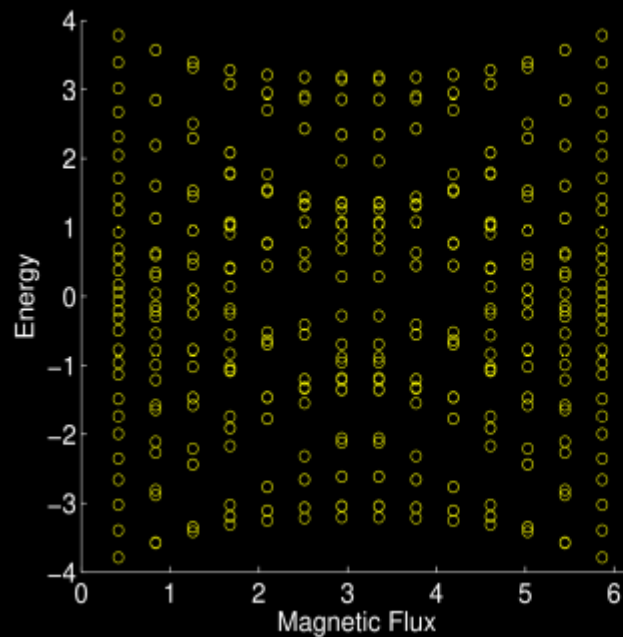


[Grass, Muschik, Celi, Chhajlany, Lewenstein, PRA (2015)]

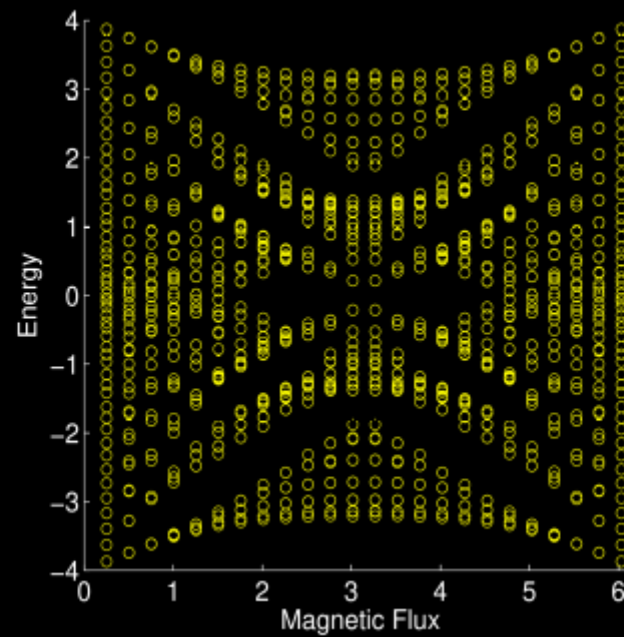
Hofstadter model: Lattice model for integer quantum Hall effect

Famous feature: Fractal energy spectrum  $\rightarrow$  “Hofstadter butterfly”

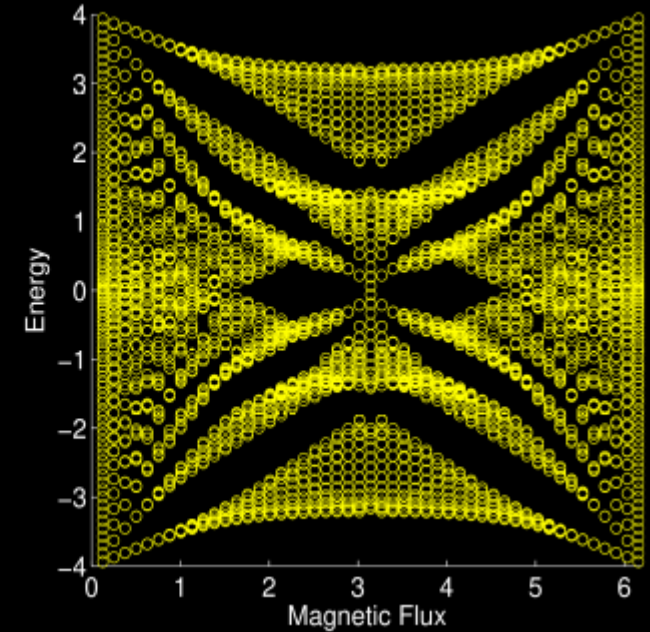
## Energy vs. flux per plaquette



$N=30$



$N=50$

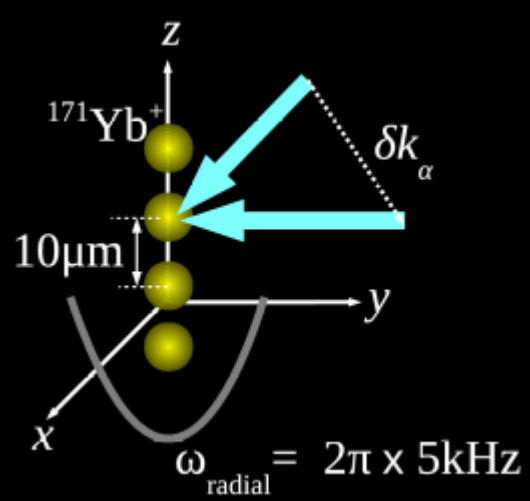


$N=100$

Also: Localized edge states, non-zero Chern numbers, ...

[Grass, Muschik, Celi, Chhajlany, Lewenstein, PRA (2015)]

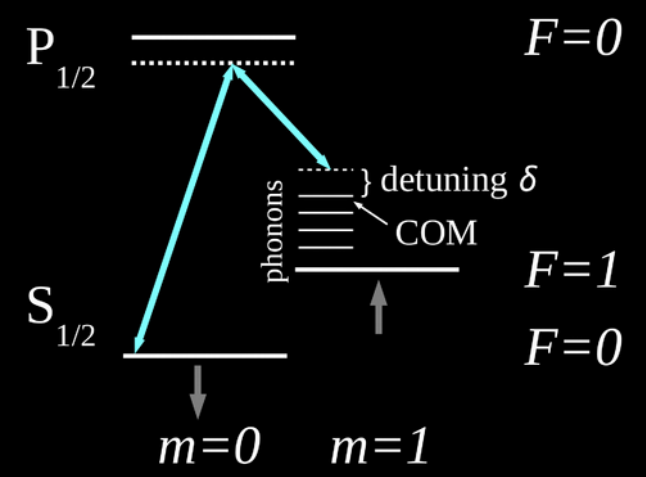
## Engineering spin-spin interactions



$$H_{\text{eff}} = \sum J_{ij} \sigma_j^x \sigma_i^x$$

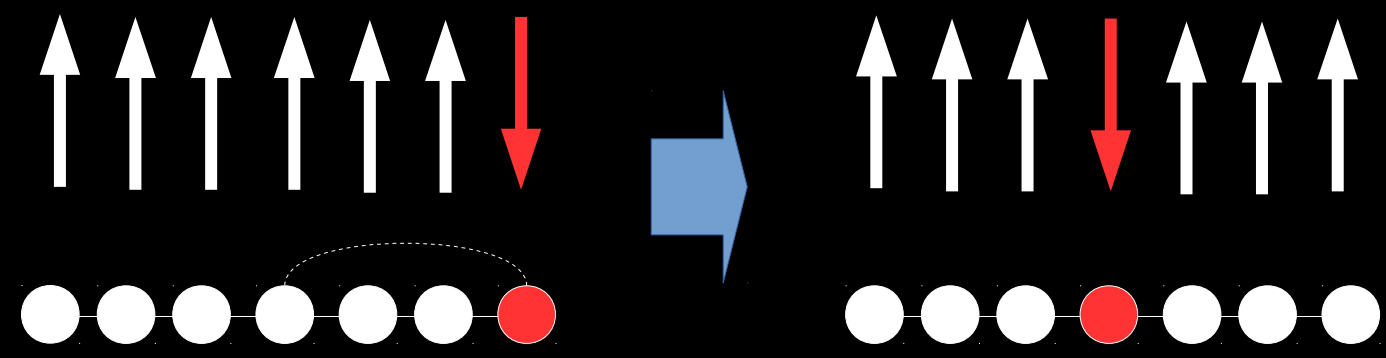
2<sup>nd</sup> order perturbation theory for spin-phonon model (i.e. virtual phonons)

[Porras, Cirac, PRL (2004)]



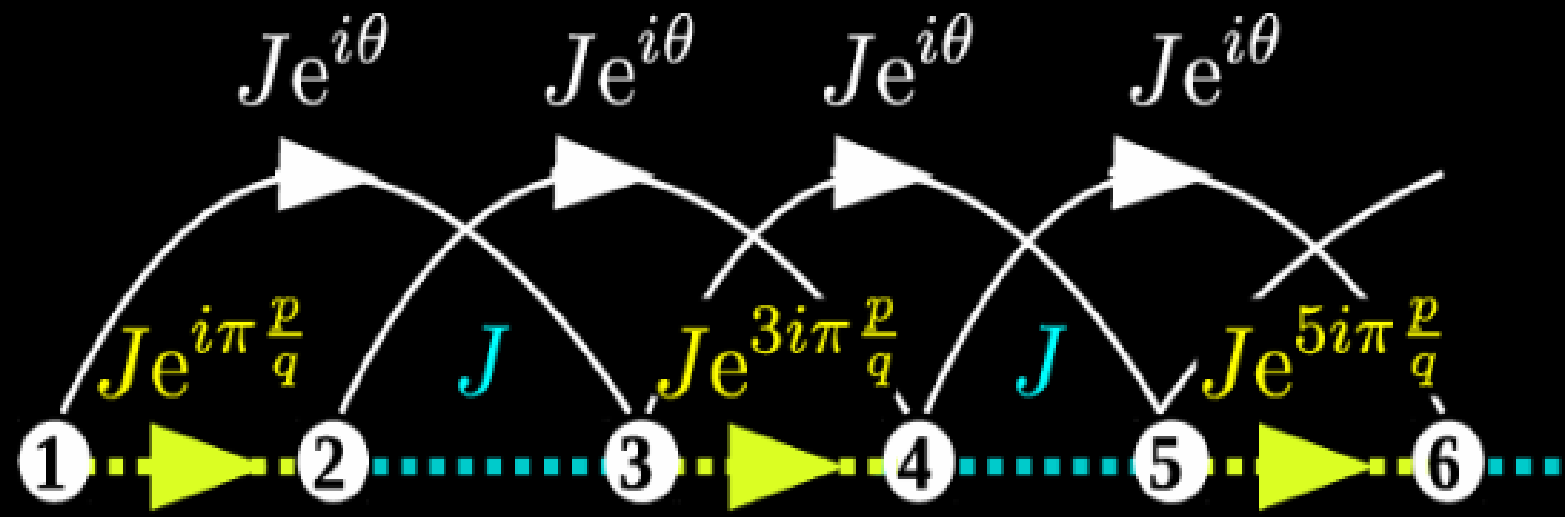
Early experiments:  
 [Friedenauer, Schmitz, Glueckert, Porras, Schaetz, Nat. Phys (2008)] [Kim, Chang, Islam, Korenblit, Duan, Monroe, PRL (2009)]

Equivalence spin flip ↔ hopping:

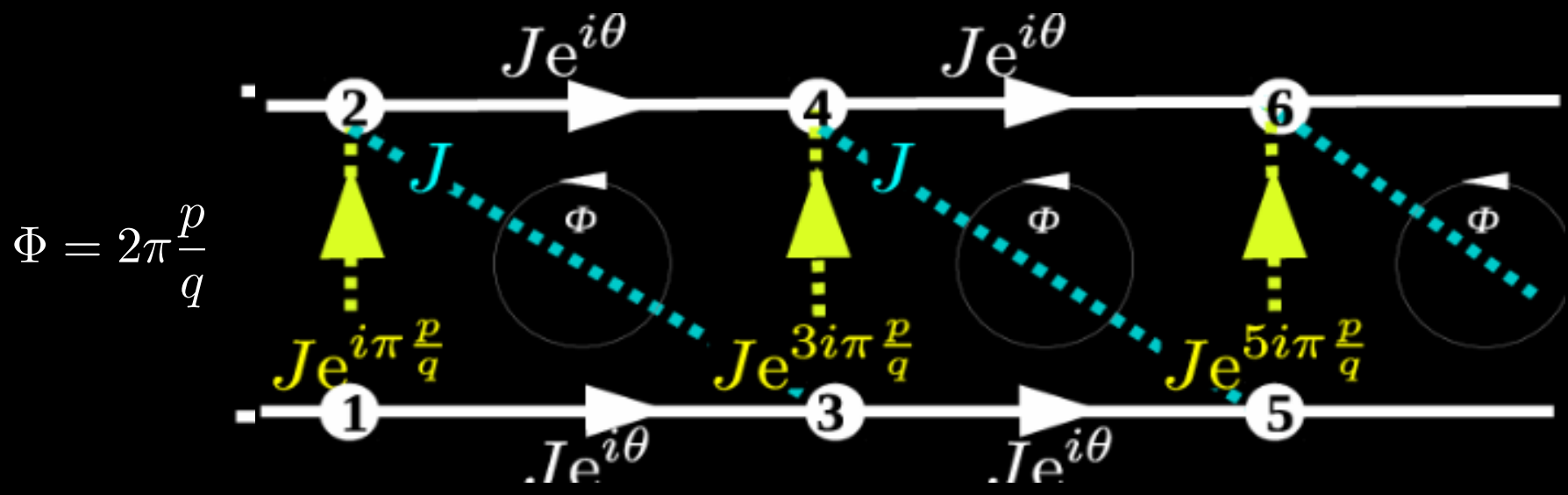




Chain with nearest and next-nearest neighbor interactions:



Mapping onto triangular ladder ("synthetic ladder"):



[Grass, Muschik, Celi, Chhajlany, Lewenstein, PRA (2015)]

## Periodic driving to engineer gauge potentials

[Cold atom experiments: Pisa (Arimondo), Hamburg (Sengstock), MIT (Ketterle), Munich (Bloch), Zurich (Esslinger)...]

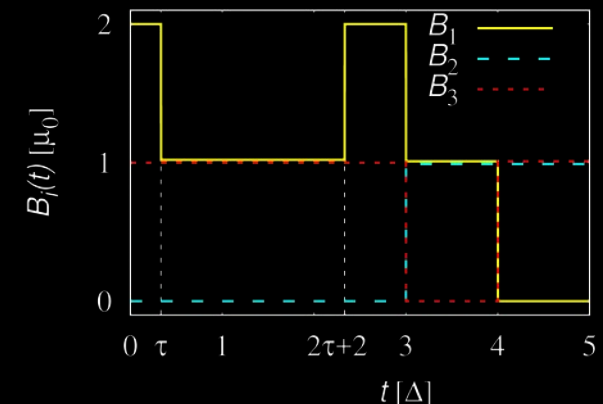
### Recipe for spin chain (XY model):

Add “shaken” field:  $H(t) = H_{XY} + \sum_i B_i(t) \sigma_i^z$  with  $H_{XY} = \sum_{i<j} J_{ij} (\sigma_i^+ \sigma_j^- + \text{h.c.})$

Gauge transform (Floquet basis):  $U(t) = e^{-i \sum_i \chi_i(t) \sigma_i^z}$  with  $\chi_i(t) = \int_0^t dt' B_i(t')$

In new basis, average over shaking period:  $H_{\text{eff}} = \sum_{i<j} J_{ij}^{\text{eff}} (\sigma_i^+ \sigma_j^- + \text{h.c.})$  where  $J_{ij}^{\text{eff}} = \frac{\bar{J}_{ij}}{T} \int_0^T dt e^{2i[\chi_i(t) - \chi_j(t)]}$

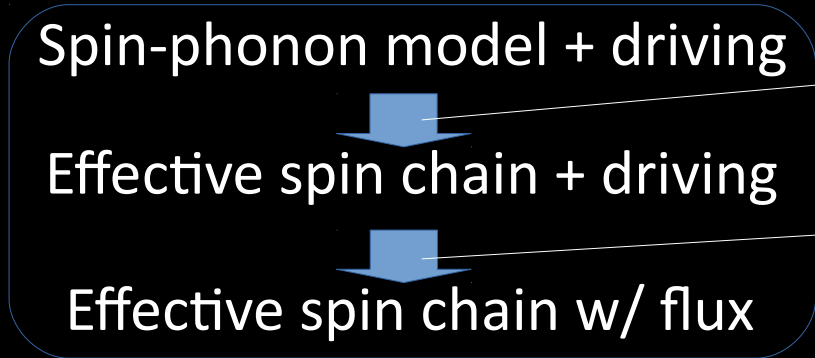
Effective Hamiltonian is again XY model, with renormalized interaction parameters



[Grass, Muschik, Celi, Chhajlany, Lewenstein, PRA (2015)]

[Grass, Celi, Pagano, Lewenstein, PRA Rapid (2018)]

# Effective model of the effective model...



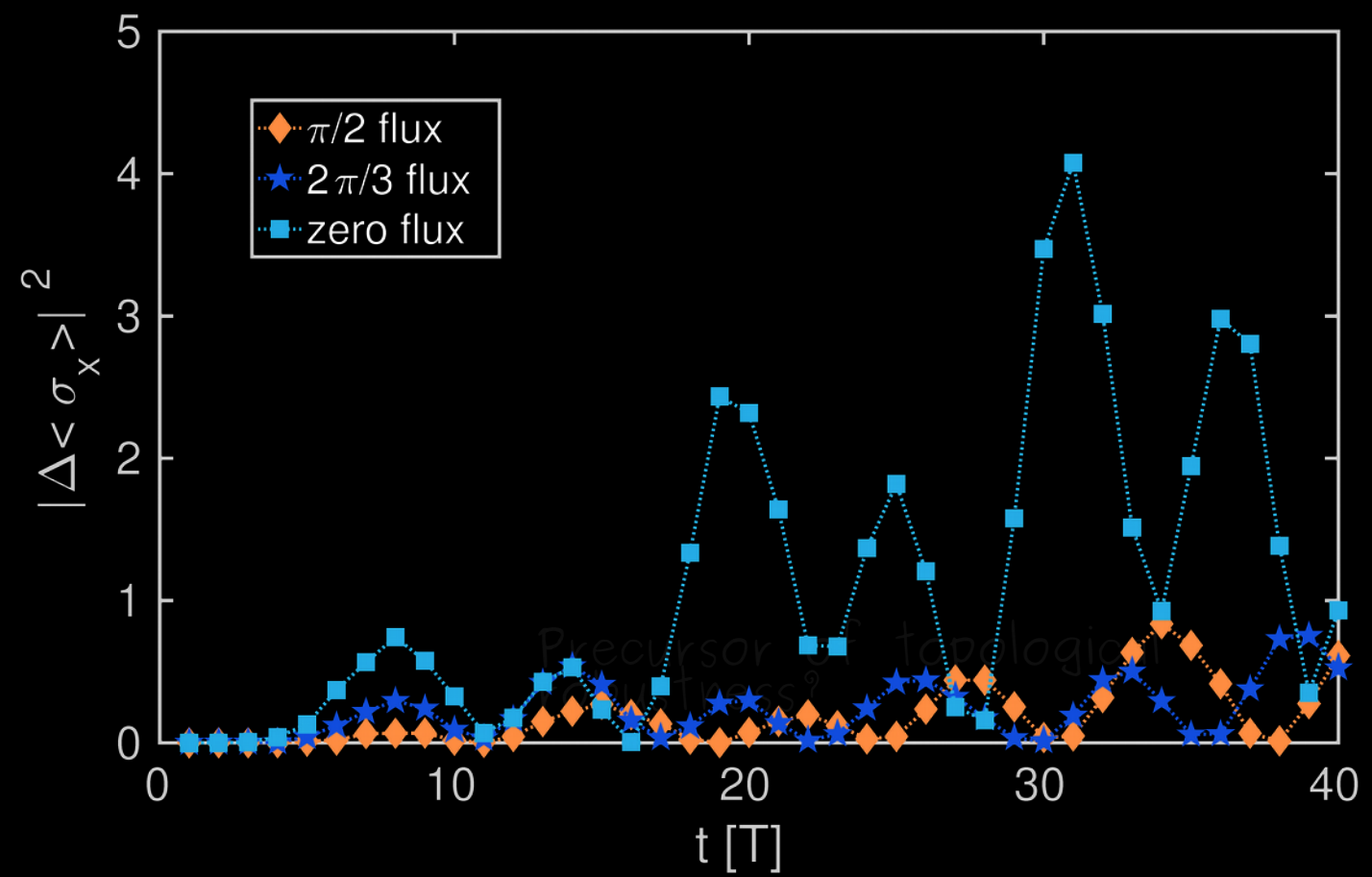
First approximation:  
Problematic for strong driving

Second approximation:  
Problematic for weak driving

$\Phi = \pi/2$   
Good fidelity

$\Phi = 0$   
Low fidelity

Onset of topological robustness?



[Grass, Celi, Pagano, Lewenstein, PRA Rapid (2018)]

# (2)

## QUANTUM SIMULATION OF TOPOLOGICAL MATTER WITH COLD ATOMS

Synthetic gauge fields through laser dressing

[Julia-Diaz, Dagnino, Guenther, Grass, Barberan, Lewenstein, Dalibard, PRA (2011)]

Non-Abelian spin singlet phases in bosonic FQH system

[Grass, Julia-Diaz, Barberan, Lewenstein, PRA (Rapid) (2012)]

Engineering fermionic Laughlin state through confinement-induced resonance

[Julia-Diaz, Grass, Dutta, Lewenstein, Nat. Commun. (2013)]

Integer quantum Hall phase of bosons

[Grass, Raventos, Lewenstein, Julia-Diaz, PRB (2014)]

Fractional Wigner crystal through Rydberg dressing

[Grass, Bienias, Gullans, Lundgren, Maciejko, Gorshkov, PRL (2018)]

Atoms: Electro-neutrality = insensitivity to magnetic fields

Synthesize the effect of a magnetic field:

- **rotation** (Coriolis force  $\leftrightarrow$  Lorentz force)
- **laser-dressing** (imprinted Berry phase)
- **laser-assisted tunneling** (in optical lattices)
- **Floquet engineering** (complex hopping term)

Experimentally achieved phases:

- vortices and vortex lattices

[Matthews, Anderson, Haljan, Hall, Wieman, and Cornell, PRL (1999)]

[(\*) Abo-Shaeer, Raman, Vogels, Ketterle, Science (2001)]

...

- integer quantum Hall phases (Hofstadter model)

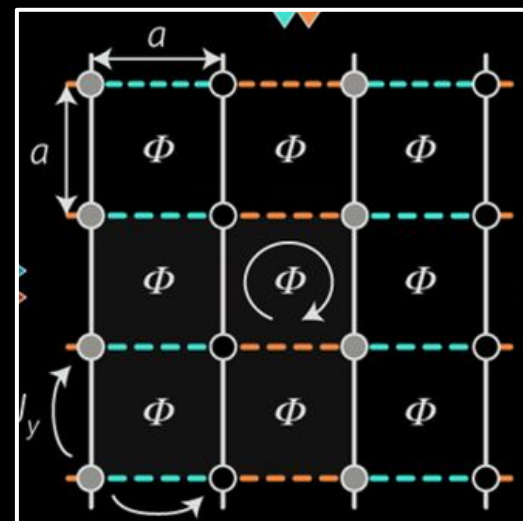
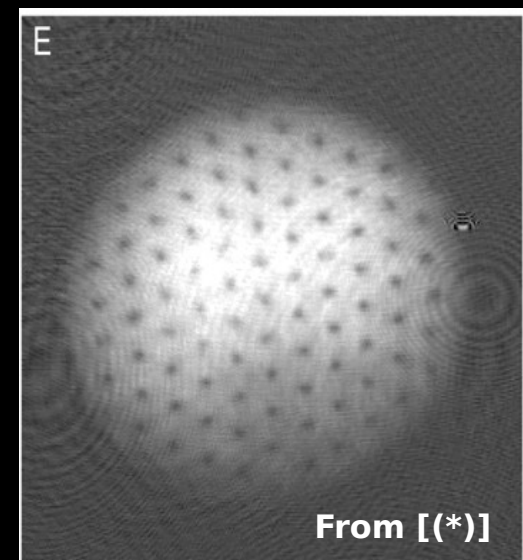
[(\*\*) Aidelsburger, ..., Bloch & Goldman, Nat. Phys. (2015)]

[Stuhl, ..., Spielman, Science (2015)]

[Mancini, ..., Fallani, Science (2015)]

...

Still outstanding: Synthesis of fractional quantum Hall phase

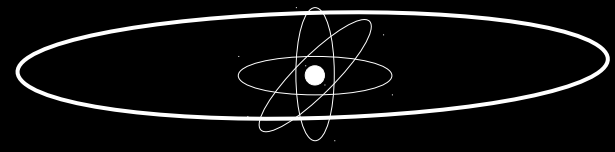


# Engineering interactions by Rydberg dressing

**GOAL:** Engineer long-ranged atom-atom interactions

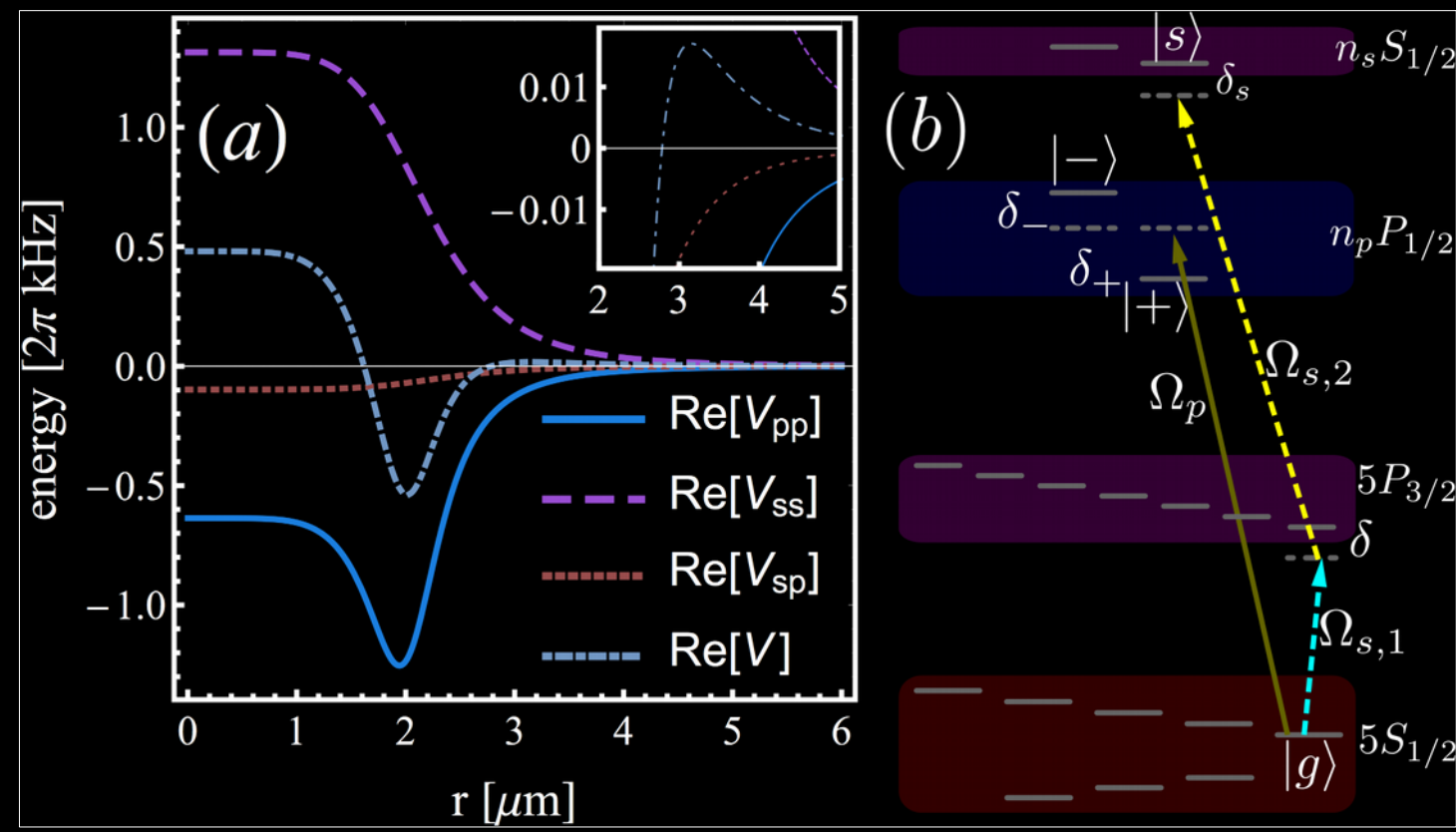
**HOW:** Using Rydberg states (van der Waals interactions)

Short-lived => only Rydberg **dressing**



[Jau, Hankin, Keating, Deutsch, Biedermann, Nat. Phys. (2016); Zeiher,..., Bloch, Gross, Nat. Phys. (2016)]

**APPLICATION:** Atomic quantum Hall system with tunable pseudopotentials

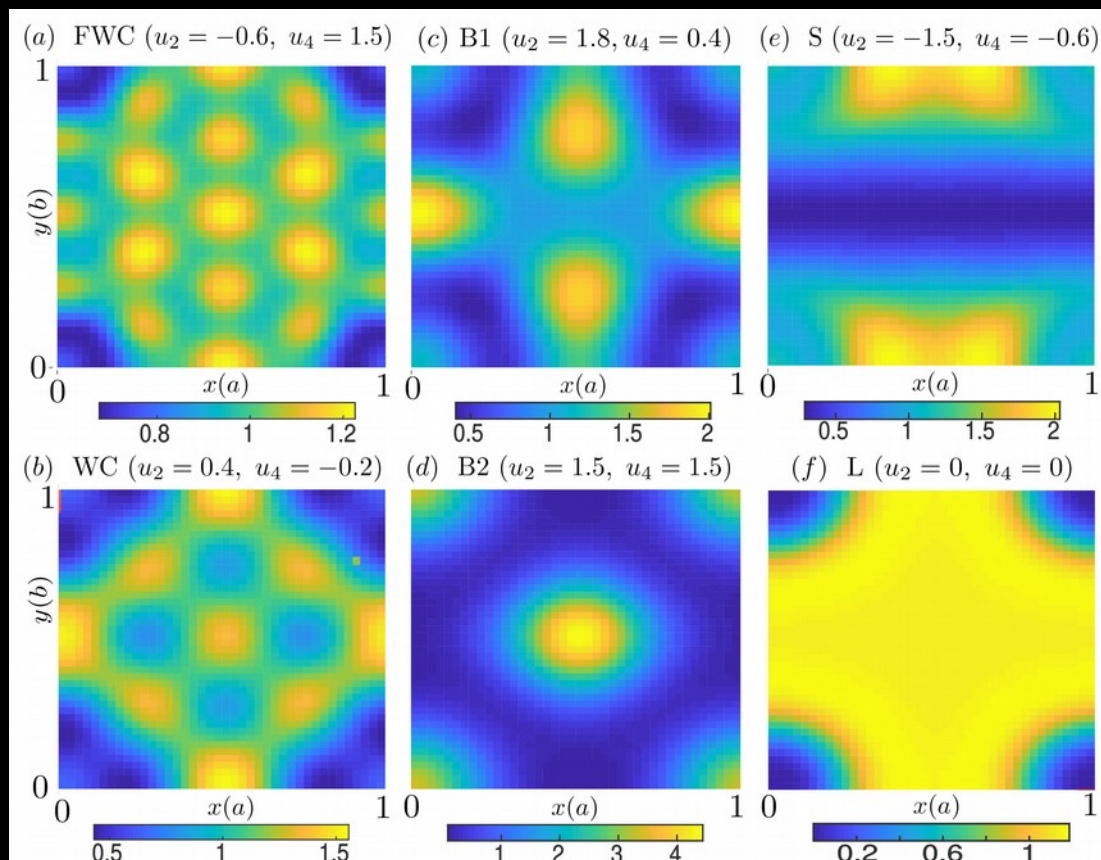
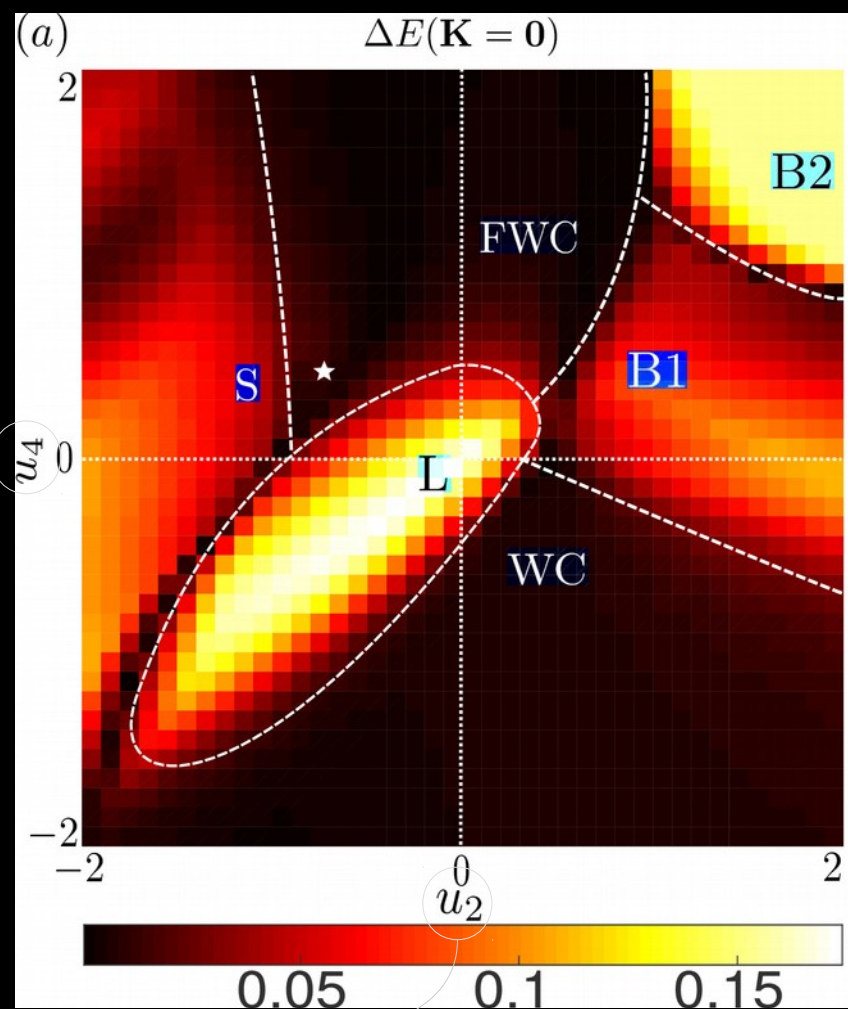


Combined s- and p- state dressing for enhanced flexibility

Tunable via principal quantum numbers, detuning, Rabi frequencies

[Grass, Bienias, Gullans, Lundgren, Maciejko, Gorshkov, PRL (2018)]

## Landau level filling $\nu=1/2$ : Laughlin liquid and what else?

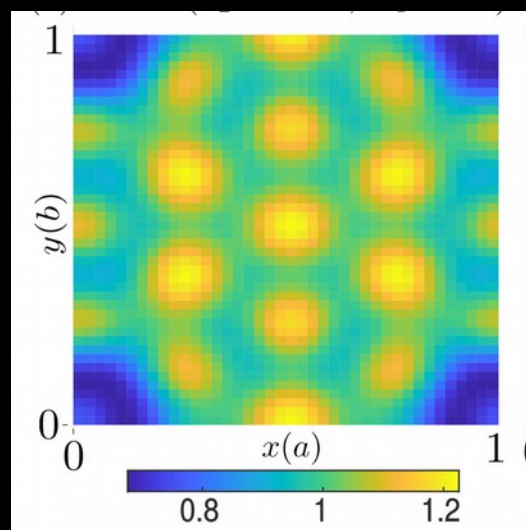


2-body correlation functions (on torus):  
Symmetry-broken phases

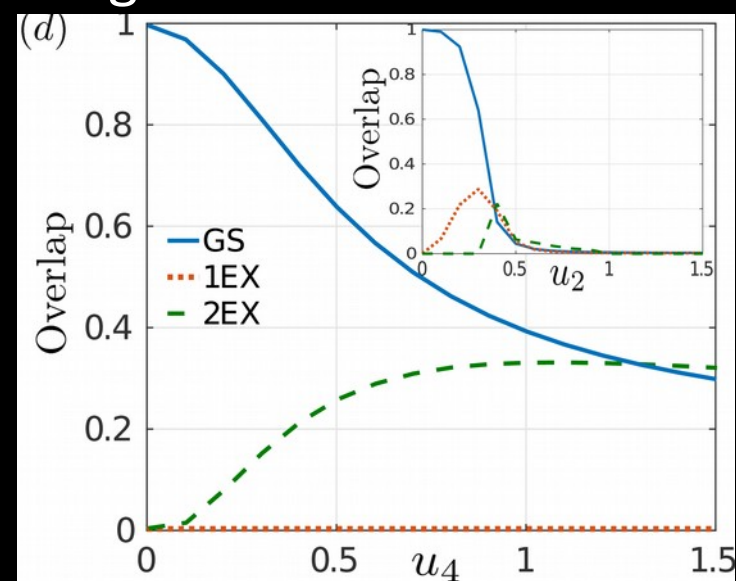
$u_2$  and  $u_4$  are pseudopotentials: interaction strength at fixed relative angular momentum  $2\hbar$  and  $4\hbar$

# Fractional Wigner Crystal

Crystal structure with  $2N$  peaks:  
formed by fractionalized bosons?



Wave-function overlap with  
Laughlin state



Coexistence of topological order and symmetry-broken order?

Nematic FQH – Experiments:

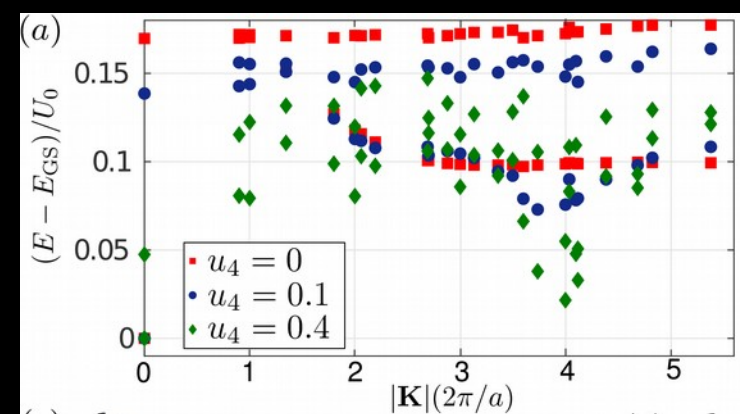
[Xia, Eisenstein, Pfeiffer, West, Nat. Phys. (2011)]

[Samkharadze, Schreiber, Gardner, Manfra, Fradkin, Csáthy, Nat. Phys (2016)]

Explanation through softening of a  
collective mode (magnetoroton)?

[Maciejko, Hsu, Kivelson, Park, Sondhi, PRB (2013)]

[You, Cho, Fradkin, PRX, (2014)]



[Grass, Bienias, Gullans, Lundgren, Maciejko, Gorshkov, PRL (2018)]



# (3)

## QUANTUM SIMULATION OF TOPOLOGICAL MATTER WITH SOLID-STATE SYSTEMS

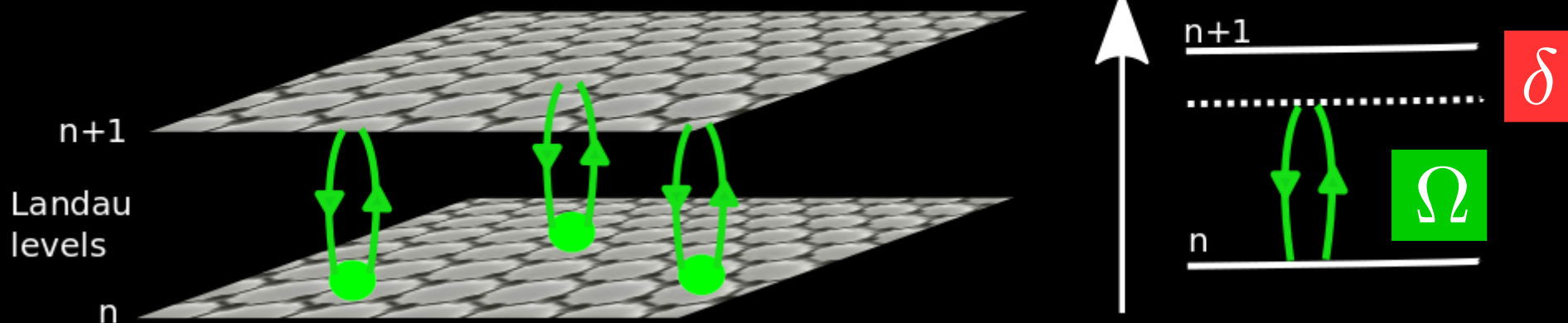
Engineering of non-Abelian phase in synthetic bilayer graphene

[Ghazaryan, Grass, Gullans, Ghaemi, Hafezi, PRL (2017)]

Creation of anyons by light-matter interactions:

[Grass, Gullans, Bienias, Zhu, Ghazaryan, Ghaemi, Hafezi, PRB (2018)]

## Optically coupled Landau levels:

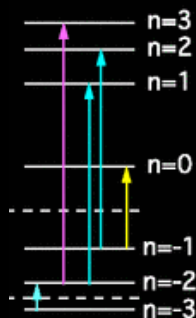


$$H_0 = \sum_m \left[ \hbar\delta c_{n+1,m}^\dagger c_{n+1,m} + \hbar\Omega c_{n+1,m}^\dagger c_{n,m} \right] + \text{h.c.}$$

"chemical potential"

"interlayer tunneling"

[Ghazaryan, Grass, Gullans, Ghaemi, Hafezi, PRL (2017)]

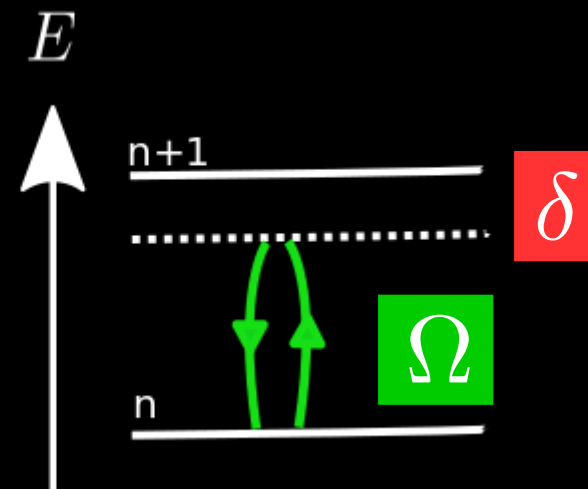
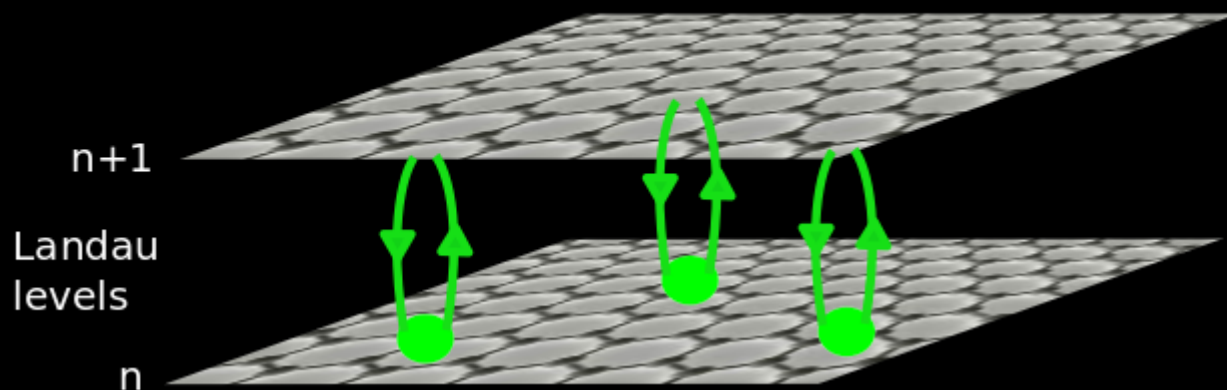


## Selection rule for Landau level coupling in graphene:

$$n \leftrightarrow \pm n \pm 1 \quad (\text{B}=10 \text{ Tesla} \rightarrow \text{near-IR transitions})$$

[Jiang, ..., Kim, Stormer, PRL (2007)]

## Optically coupled Landau levels:



$$H_0 = \sum_m \left[ \hbar\delta c_{n+1,m}^\dagger c_{n+1,m} + \hbar\Omega c_{n+1,m}^\dagger c_{n,m} \right] + \text{h.c.}$$

“chemical potential”

“interlayer tunneling”

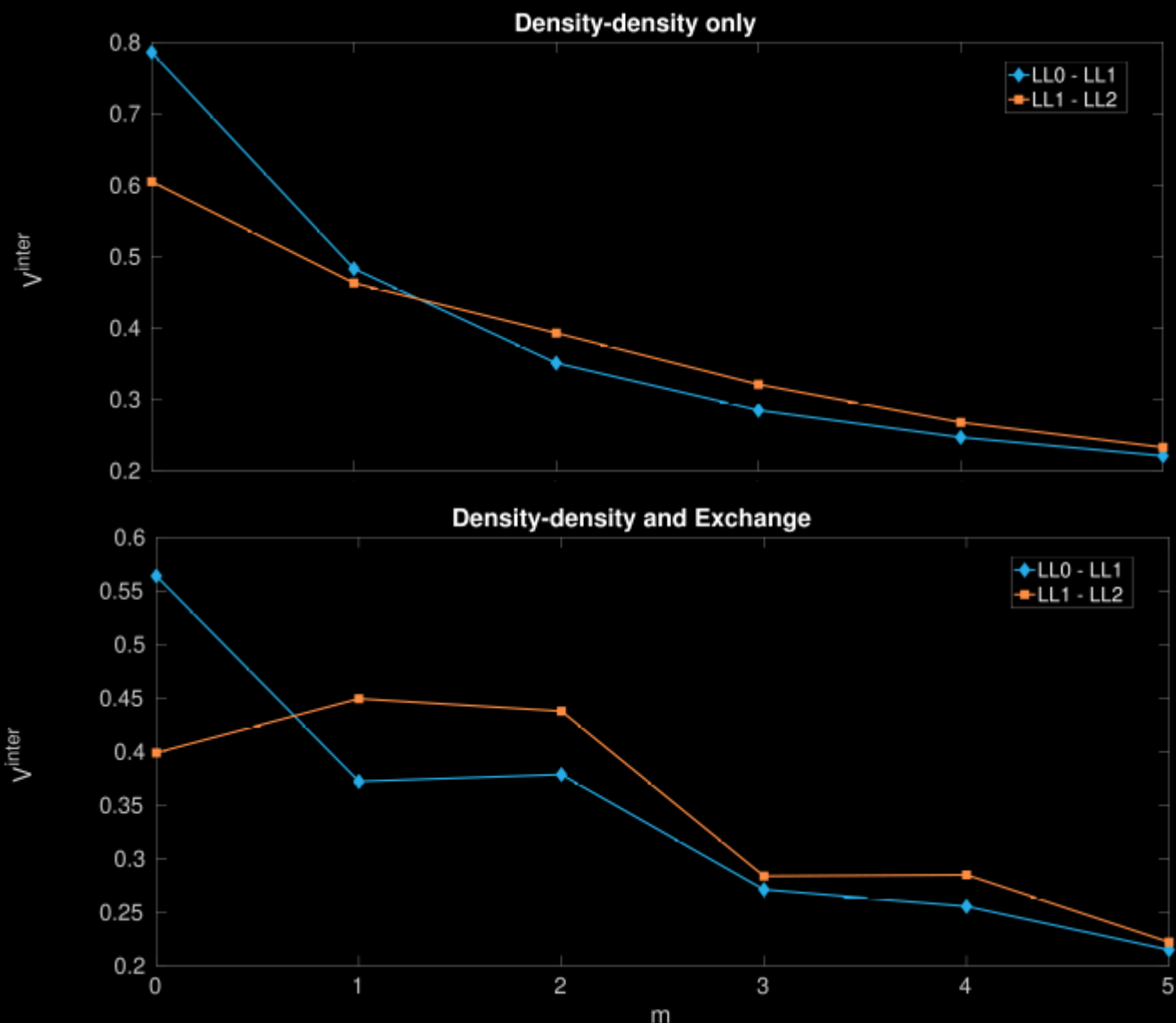
[Ghazaryan, Grass, Gullans, Ghaemi, Hafezi, PRL (2017)]

## Real vs. synthetic

- Tunable parameters
- Exotic interactions

	Real bilayer	Synthetic bilayer
Density-density $\Psi_i^\dagger(z_1)\Psi_j^\dagger(z_2)\Psi_j(z_2)\Psi_i(z_1)$	YES	YES
Exchange $\Psi_i^\dagger(z_1)\Psi_j^\dagger(z_2)\Psi_i(z_2)\Psi_j(z_1)$	NO	YES

Haldane pseudopotentials: Expand interaction in terms of their strength for fixed relative angular momentum  $m$



Generic behavior:  
Monotonic decay  
with  $m$

Synthetic bilayer  
interactions:  
Non-monotonic  
behavior  $\rightarrow$   
favoring singlets at  
 $m=0$

[Ghazaryan, Grass, Gullans, Ghaemi, Hafezi, PRL (2017)]

# Identification of the phase: Fibonacci?

Ground state overlaps:  
(at filling 2/3)

“No” overlap with:

- Halperin states (113, 330)
- Composite Fermions
- Intra-layer Pfaffian

“Larger” overlap with:

- Inter-layer Pfaffian
- Fibonacci phase

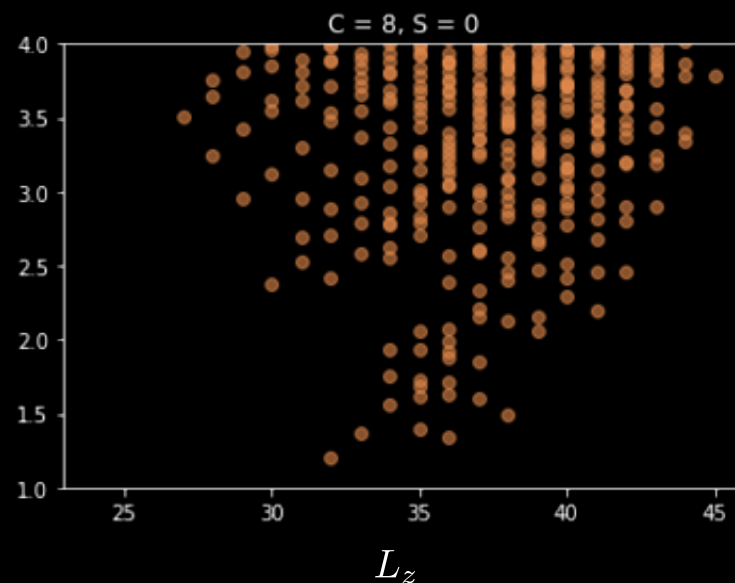


Topological “quantum numbers”:

- Ground state degeneracies on torus
- Edge state counting: 1,1,3,6,...

Conclusion:

Synthetic bilayer likely  
to host Fibonacci anyons



Entanglement spectrum for 16 electrons on sphere  
(DMRG result by Ze-Pei Cian)

## Quantum Simulation:

- Use light-matter interaction to synthesize features of interest
- Matter can be solid (with some intrinsic features) or atomic (featureless)

## Ions: Synthetic Hofstadter ladder

- Floquet-engineered fluxes in 1D geometry
- Microscopic onset of topological protection

## Atoms: Fractional Quantum Hall system

- tunable pseudopotentials through Rydberg dressing
- Fractional Wigner Crystal

## Graphene: Fibonacci anyon phase

- synthetic bilayer through light-matter interactions
- exotic interlayer interactions

# Collaborators

## Theory:

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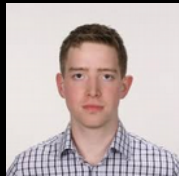
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## Experiments:

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Thank you!