Tailoring spin systems in a shaken Dicke model

Tobias Graß[1], Alessio Celi[2], Guido Pagano[1], Maciej Lewenstein[2]

(1) JQI – Joint Quantum Institute, University of Maryland, College Park, USA (2) ICFO – The Institute of Photonic Sciences, Castelldefels (Barcelona), Spain

ABSTRACT:

The Dicke model, describing the coupling of spins and bosons, gives rise to an effective long-range Ising model. In past years, this relation has been exploited for using trapped ions as quantum simulators of spin models [1-4]. Here we suggest to gain additional control by applying a periodically driven potential to the Dicke model. Such 'shaking' can be used to enhance or suppress interactions between selected spins, or even to render spin-spin interactions complex-valued. The latter case allows for mimicking the presence of artificial magnetic fluxes, which can give rise to phenomena known from topological insulators, such as fractal energy spectra or end states located between bulk energy bands [5].

FROM DICKE TO ISING:

In second order Magnus expansion, a Dicke-type spin-phonon coupling, as realized in trapped ion systems, gives rise to long-range Ising model:

$$H_{\rm s-ph} = \Omega \sum_{i,m} \eta_m^{(i)} a_m \sigma_x^{(i)} e^{-i\delta_m t} + \text{H.c.}$$

▼

$$H_{\rm eff} = \sum_{i,j} J^{(i,j)} \sigma_x^{(i)} \sigma_x^{(j)} \quad \text{with} \quad J^{(i,j)} = \Omega^2 \sum_m \frac{\eta_m^{(i)} \eta_m^{(j)}}{\delta_m}$$

Validity of spin model description requires sufficiently large detuning:

$$\delta_m \gg \Omega \eta_m^{(i)} \quad \Rightarrow \quad \delta_m \gg J^{(i,j)}$$

Neglecting higher-order terms, we can add a transverse field term:

$$H'_{\rm eff} = H_{\rm eff} + H_{\rm B} \quad \text{with} \quad H_{\rm B} = \sum_i B_i \sigma_z^{(i)}$$

FLOQUET ENGINEERING:

Floquet theorem:

$$H = H_0 + H_1(t)$$
$$\text{where} \quad H_1(t+T) = H_1(t)$$
$$\Rightarrow U(t,t_0) = U(t+T, t_0+T)$$
$$\Rightarrow U(mT+\epsilon, 0) = U(\epsilon) U(T)^m$$

For small ϵ, this leads to an effective Floquet Hamiltonian:

$$H_{\rm F} = \frac{i}{mT} \ln U(mT)$$

Interaction picture:

$$H_{\rm I}(t) = U_{\rm I}(t)^\dagger H(t) U_{\rm I}(t) - i U_{\rm I}^\dagger(t) \frac{d}{dt} U_{\rm I}(t)$$
$$\text{where} \quad U_{\rm I}(t) = \exp\left[-i \int_0^t dt' H_1(t')\right]$$

Then, the average over one period provides an estimate for the Floquet Hamiltonian:

$$H_{\rm F} \approx \frac{1}{T} \int_0^T dt \, H_{\rm I}(t)$$

Example: XX model with shaken field

$$H_0 = \sum_{i,j} J^{(i,j)} \sigma_+^{(i)} \sigma_-^{(j)} + \text{H.c.}$$
$$H_1(t) = \sum_i B_i(t) \sigma_z^{(i)}$$
$$\Rightarrow H_F \approx \sum_{i,j} \tilde{J}^{(i,j)} \sigma_+^{(i)} \sigma_-^{(j)} + \text{H.c.}$$
$$\tilde{J}^{(i,j)} = \frac{J^{(i,j)}}{T} \int_0^T dt \, e^{2i \int_0^t dt' [B_i(t') - B_j(t')]}$$

SHAKING:

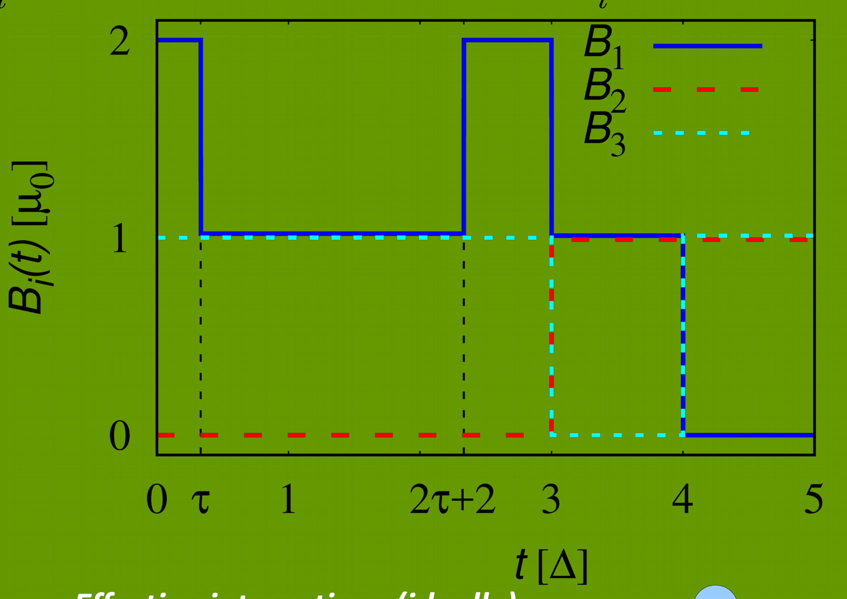
$$H(t) = \sum_m \omega_m a_m^\dagger a_m + \Omega \sum_{i,m} \eta_m^{(i)} (a_m + a_m^\dagger) \sigma_x^{(i)} \sin(\omega_{\rm L} t) + \sum_i B_i(t) \sigma_z^{(i)}$$

- Apply periodic driving on magnetic field strength $B_i(t)$:

  Shaking period T shall be a multiple of a time unit Δ

  Shaking strength shall be a multiple of $\mu_0 = \pi/\Delta$
- Then, only intervals with $B_i = B_j$ contribute to the interaction between ion i and ion j → *adjust interaction strength!*
- The time τ_{ij} at which the detuning of two ions vanishes determines the phase of the interactions:

$$\arg[J^{(i,j)}] = 2\pi \bmod[\tau_{ij}, \Delta]$$



Main requirements for high fidelity:

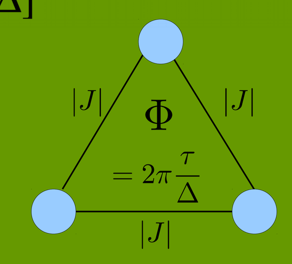
- Separation of time scales: $\delta_m \gg \mu_0 \gg J^{(i,j)}$
- $J^{(i,j)}$ should be fast compared to noise (~ 1 kHz)

Is this feasible? Realistic parameters:

$$\delta_m \approx 2\pi \times 100 \text{ kHz}$$
$$J^{(1,2)} = J^{(2,3)} \approx 2 J^{(1,3)} \approx 2\pi \times 400 \text{ Hz}$$
$$\mu_0 \approx 2\pi \times 5 \text{ kHz}$$

Effective interactions (ideally):

$$\tilde{J}^{(1,2)} = \frac{1}{5} J^{(1,2)} \equiv J$$
$$\tilde{J}^{(2,3)} = \frac{1}{5} J^{(2,3)} \approx J$$
$$\tilde{J}^{(1,3)} = \frac{2}{5} e^{i2\pi\frac{\tau}{\Delta}} J^{(1,3)} \approx J e^{i2\pi\frac{\tau}{\Delta}}$$



ERROR ESTIMATES:

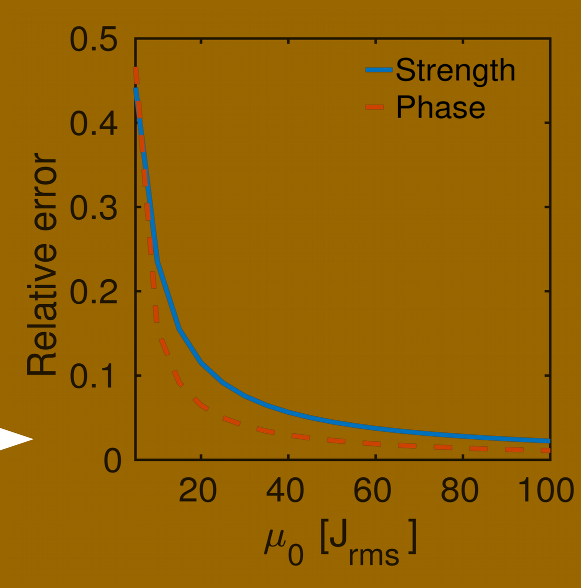
- Problem: *H(t)* involves two time scales.

  If shaking is **too slow/weak**:

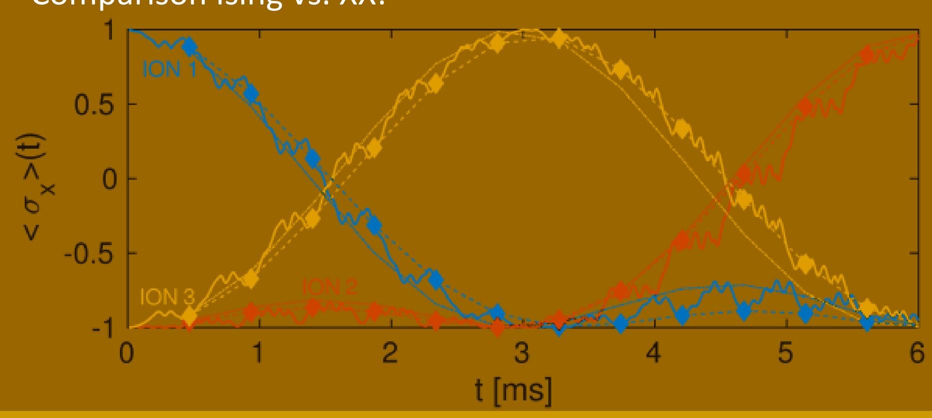
  → effective spin Hamiltonian not the desired one

  If shaking is **too fast/strong**:

  → phonon effects invalidate effective spin model
- To estimate weak shaking errors we calculate the Floquet spin Hamiltonian *exactly* (shaking is just a series of quenches). Assuming an XX model, deviation between exact and desired Floquet Hamiltonian are plotted on the left: **errors < 10% for strengths > 20 J_{rms}**



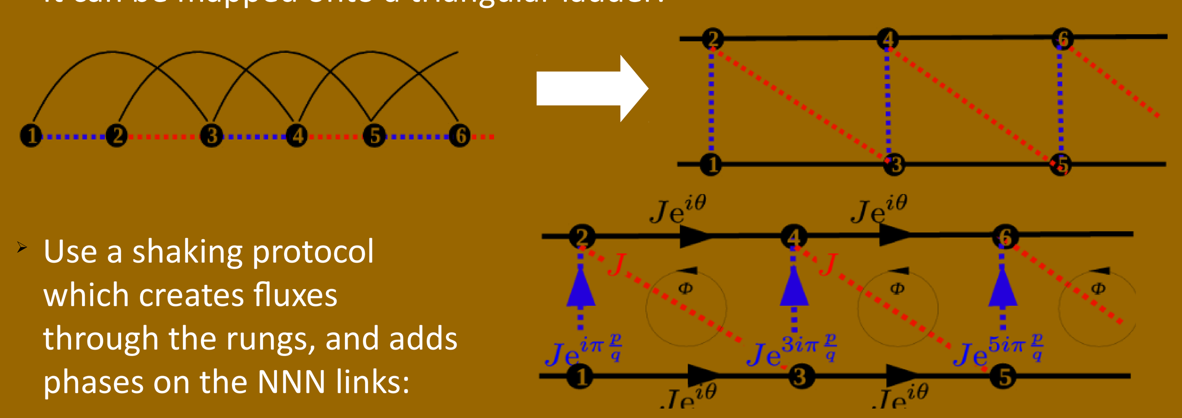
- Comparison Ising vs. XX:



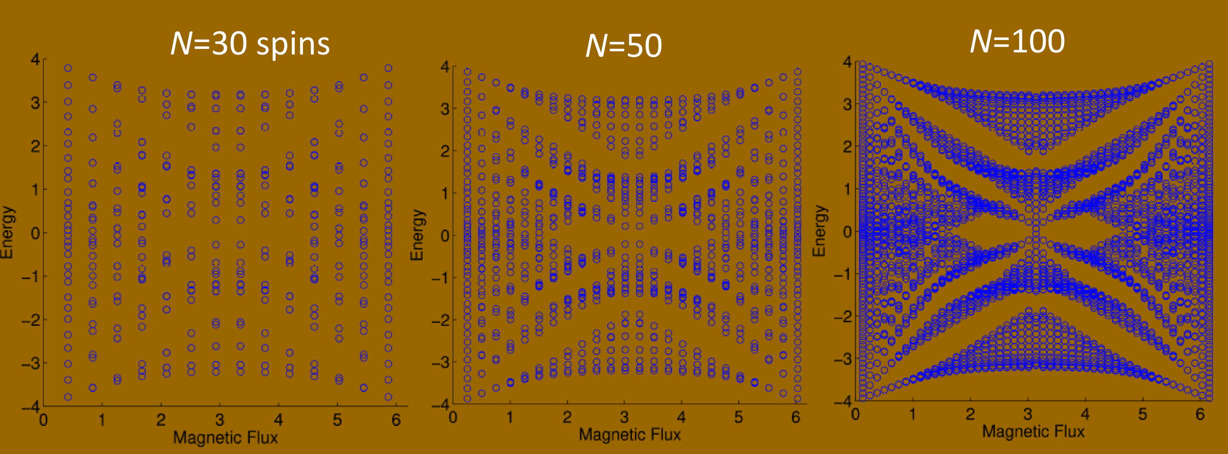
Discrepancies are apparent only on fast (non-stroboscopic) time scales.

SPIN CHAIN WITH FLUXES:

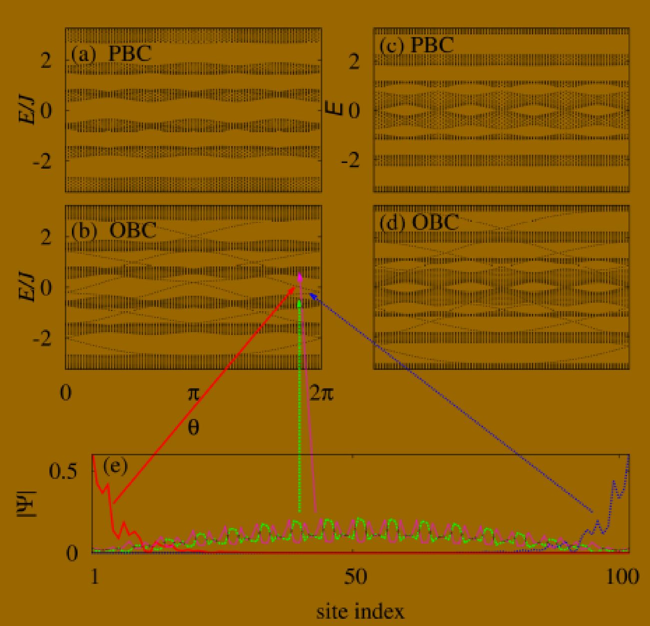
- Assume XX spin chain with nearest- and next-nearest-neighbor interactions. It can be mapped onto a triangular ladder:
- Use a shaking protocol which creates fluxes through the rungs, and adds phases on the NNN links:



- This model features phenomena known from topological quantum systems. In the presence of a single spin flip, it is related to the Hofstadter model (hopping of a free particle in magnetic field). For many spin flips, the model describes bosonic particles with hard-core interactions.
- FRACTAL ENERGY SPECTRA of a single spin flip:



- EDGE STATES and CHERN NUMBERS:

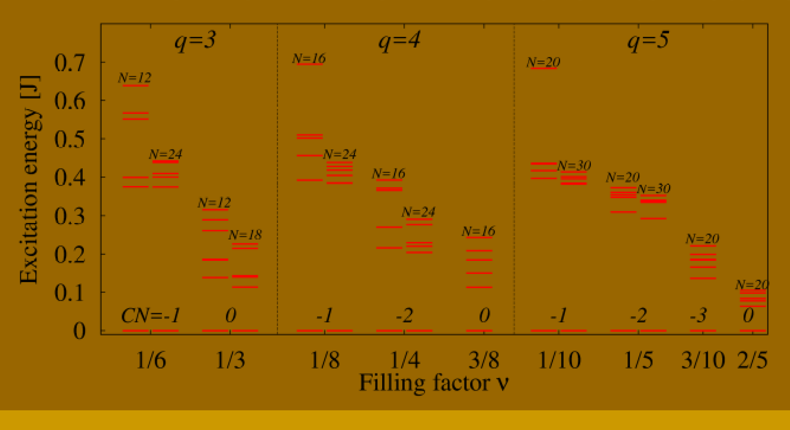


- Energy bands in system with periodic boundary are gapped. Upon opening the boundary, edge states appear.
- Winding of bands can be defined as:

$$CN = \frac{i}{2\pi} \int d\mu_1 d\mu_2 (\langle \partial_1 \Psi | \partial_2 \Psi \rangle - \langle \partial_2 \Psi | \partial_1 \Psi \rangle)$$

for $\mu_1 = k$ and $\mu_2 = \vartheta$.

- Winding numbers are found to be non-zero for all gapped manifolds.
- MANY-BODY CHERN NUMBERS: Chern-to-Mott insulator transition



flux $\Phi = \frac{2\pi}{q}$

filling $\nu = \frac{n}{2q}, \; n \in \mathbb{N}$

polarization $S_z = N(1-2\nu)$

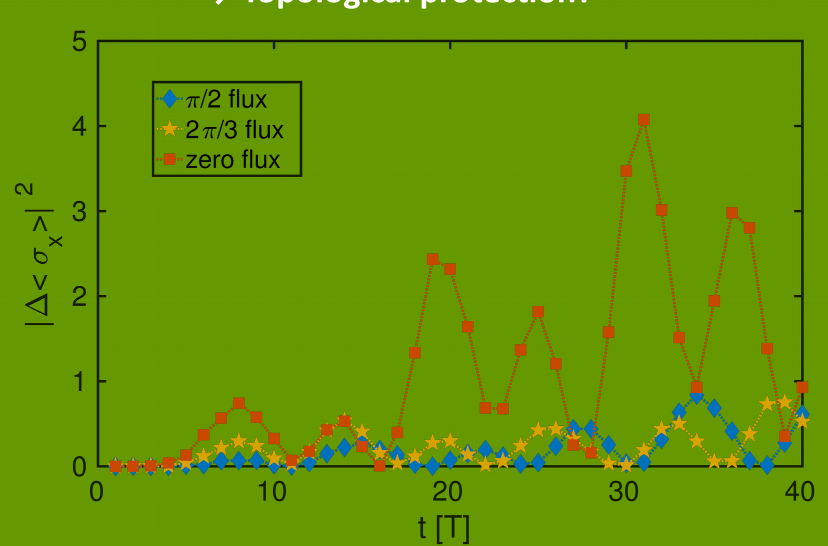
COMPUTER SIMULATION OF DYNAMICS:

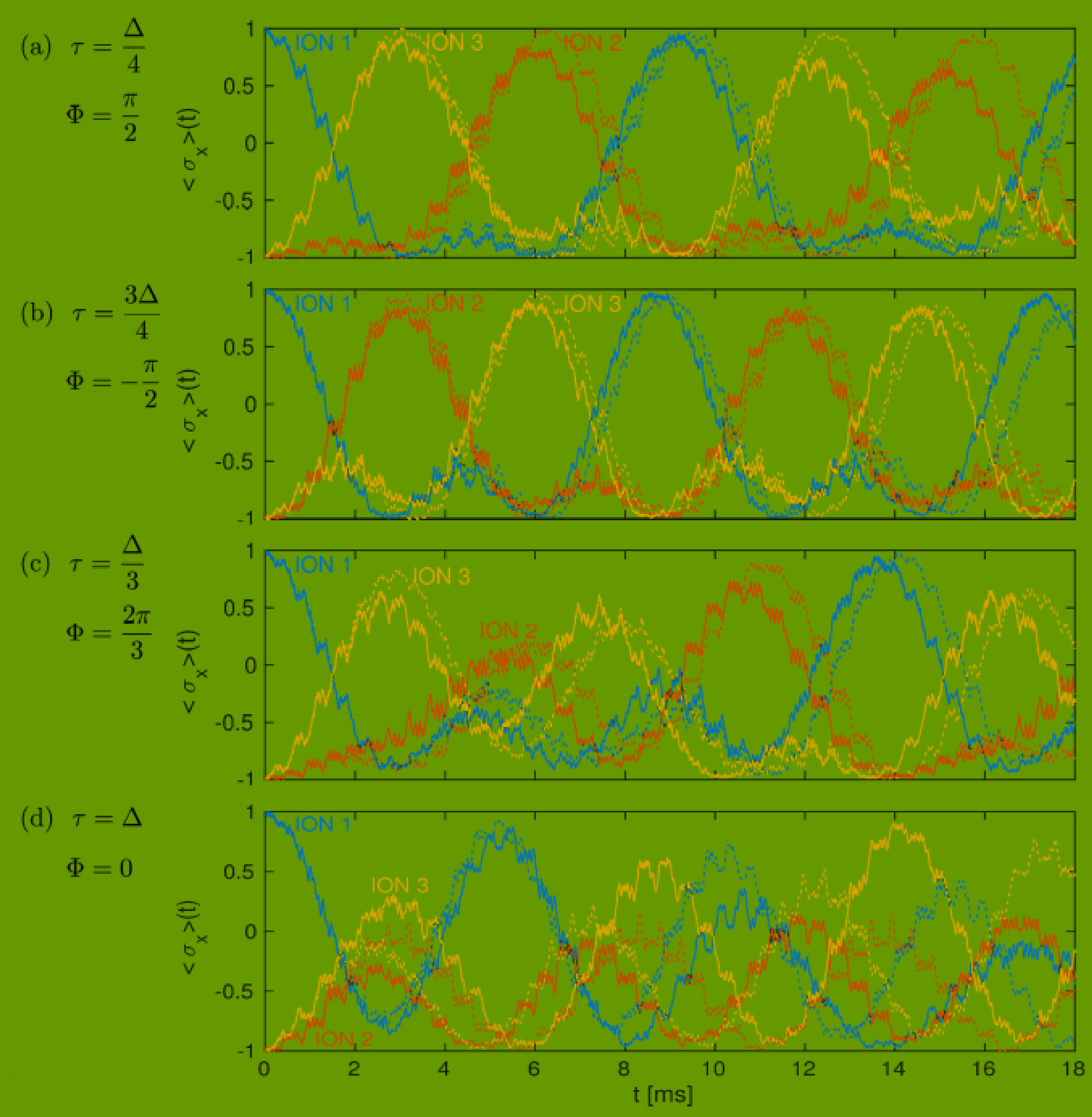
- Initial state: One spin up, other spins down → Spin evolution maps onto the dynamics of a free particle.
- Artificial flux breaks time-reversal symmetry, directly seen in the dynamics.
- Triangle with π/2 flux: chiral spin currents (panels a,b).

  Triangle without flux (or π-flux): spin currents in both directions (panel d).

  Triangle with other values for flux: chiral currents, but longer revival periods (panel c)
- Observation: In the presence of a flux, the dynamics in the Dicke model (solid lines) agrees much better with the Ising model dynamics (dashed lines) than in the absence of flux.

→ **Topological protection?**





OUTLOOK:

- ***Thermalization in the Floquet Hamiltonian: Can we get chiral eigenstates?***
- ***Exact simulations beyond 3 ions: What can we expect in larger systems? Will the observed protection help to scale up?***
- ***In larger systems, also interacting models (hard-core bosons) can be studied.***
- ***Can phonon effects be minimized using other shaking schemes?***
- ***Can the Floquet presricption be useful also for quantum simulations without magnetic field or with constant magnetic field, e.g. to determine the effect of phonons?***
- ***Can phonons be used to control the quantum simulation?***

...

REFERENCES:

[1] K. Kim, M.-S. Chang, R. Islam, S. Korenblit, L.-M. Duan, and C. Monroe, Phys. Rev. Lett. **103**, 120502 (2009).

[2] P. Richerme, Z.-X. Gong, A. Lee, C. Senko, J. Smith, M. Foss-Feig, S. Michalakis, A. V. Gorshkov, and C. Monroe, Nature **511**, 198 (2014).

[3] P. Jurcevic, B. P. Lanyon, P. Hauke, C. Hempel, P. Zoller, R. Blatt, and C. F. Roos, Nature **511**, 202 (2014).

[4] T. Graß, D. Raventós, B. Juliá-Díaz, C. Gogolin, and M. Lewenstein, Nat. Commun. **7**, 11524 (2016)

[5] T. Graß, C. Muschik, A. Celi, R. W. Chhajlany, and M. Lewenstein, Phys. Rev. A 91, 063612 (2015)