17th Symposium on Topological Quantum Information, Garching, 17.04.2015

Topological phases in spin chains with long-range interactions and artificial magnetic fields

Tobias Grass (ICFO - Barcelona)

In collaboration with:

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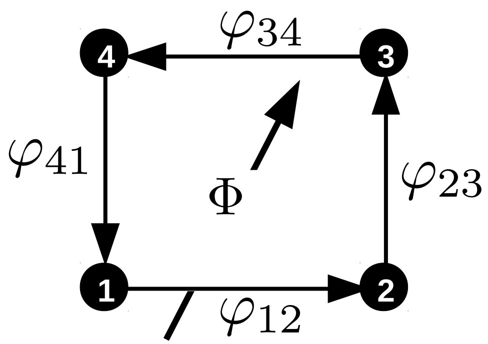
Ravindra Chhajlany (ICFO)
Maciej Lewenstein (ICFO)
Christine Muschik (IQOQI)

Can a magnetic field in 1D be interesting?

In 2 or more dimensions:

non-trivial loops

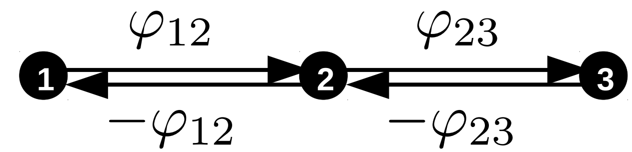


$$\varphi_{ij} = \int_{\mathbf{r}_i}^{\mathbf{r}_j} \mathrm{d}\mathbf{r} \cdot \mathbf{A}(\mathbf{r})$$

$$\Phi = \varphi_{12} + \varphi_{23} + \varphi_{34} + \varphi_{41}$$

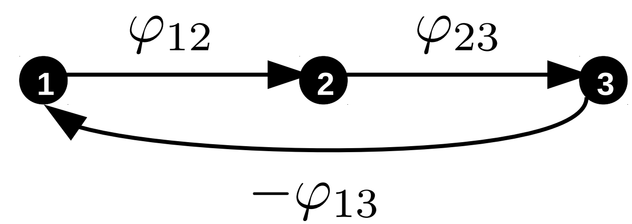
In 1 dimension:

no loops with flux



$$\Phi = \varphi_{12} + \varphi_{23} - \varphi_{23} - \varphi_{14} = 0$$

unless we consider long-range hopping with generic Peierls phases:



$$\Phi = \varphi_{12} + \varphi_{23} - \varphi_{13}$$

Possible platforms

1. Cold atoms in optical lattices

- typically only nearest-neighbor hopping
- artificial gauge fields already exist in 2d lattices

2. Trapped ions

- linear arrangement
- long-range spin-spin interactions (mediated by phonons)

3. Cold atoms coupled to a nanophotonic fiber

- Similar properties as for the ions:
  linear, long-range interaction (mediated by photons)
- Less developed than trapped ions,
  but with the prospect of better scalability (>1000 atoms)

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Outline

1. Mapping: spin-flip interactions ↔ hopping

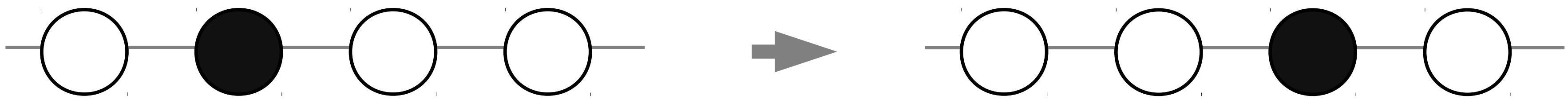
2. Model: XY chain with nearest and
   next-to-nearest neighbor interactions
   - Mapping onto triangular ladder
   - Magnetic flux via complex interaction strength

   Results:
   - Fractal energy spectrum
   - Topological bands
   - Topological many-body states

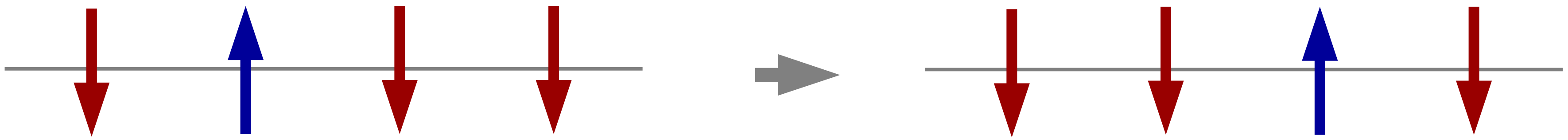
3. Realization of the model with ions or atoms
   - Engineering interactions via periodic driving

Mapping: Hopping ↔ XY model

Hopping: $H = -J \sum_{ij} a_i^\dagger a_j$



Spin flip: $H = -J \sum_{ij} \sigma_i^+ \sigma_j^-$



For XY chain with nearest-neighbor interaction:

- Jordan-Wigner transformation: equivalence of spin flip model and free fermion model

In the presence of interactions beyond nearest neighbors:

- Jordan-Wigner does not work
- Spin flips operators σ are bosonic
- Hard-core constraint: strong interactions

Mapping: Hopping ↔ XY model

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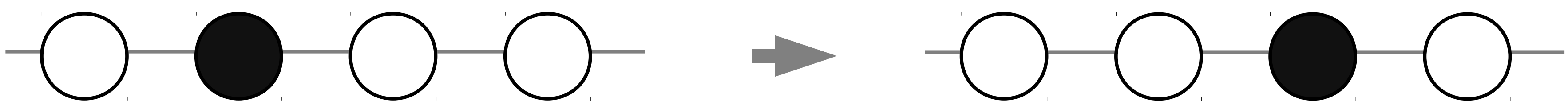
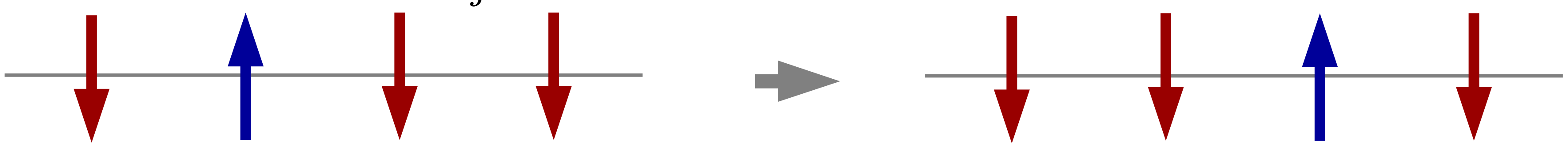


Spin flip: $H = -J \sum_{ij} \sigma_i^+ \sigma_j^-$



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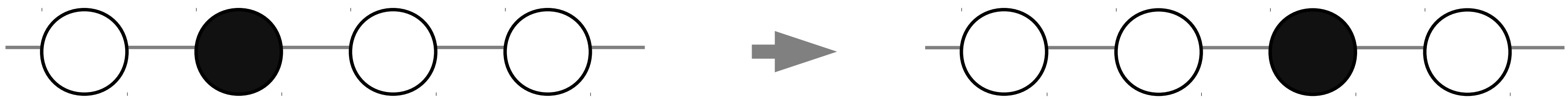

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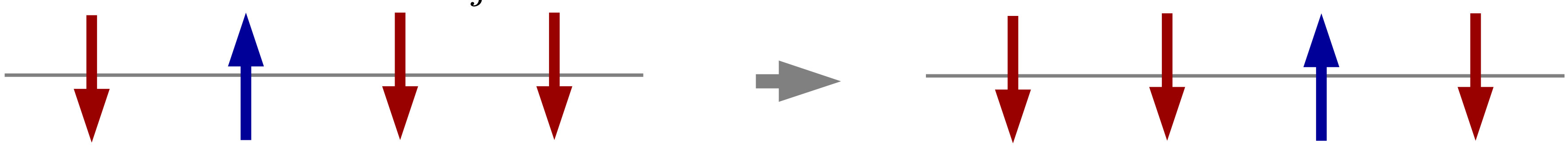

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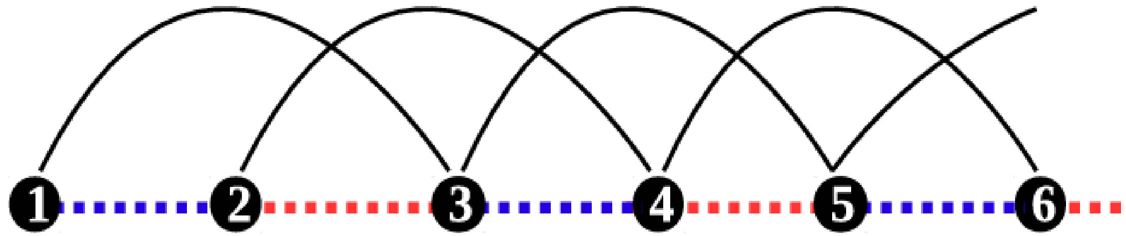
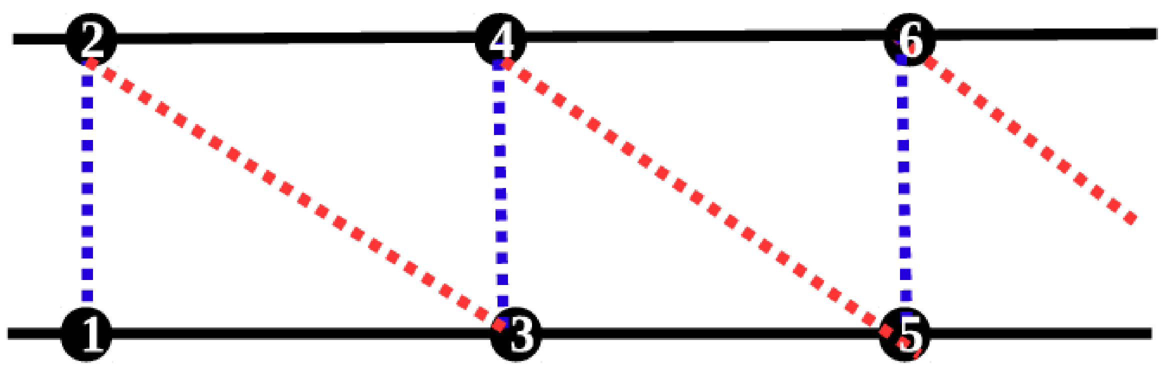
XY model with magnetic fluxes

XY chain with NN and NNN interactions:



Mapping onto triangular ladder:



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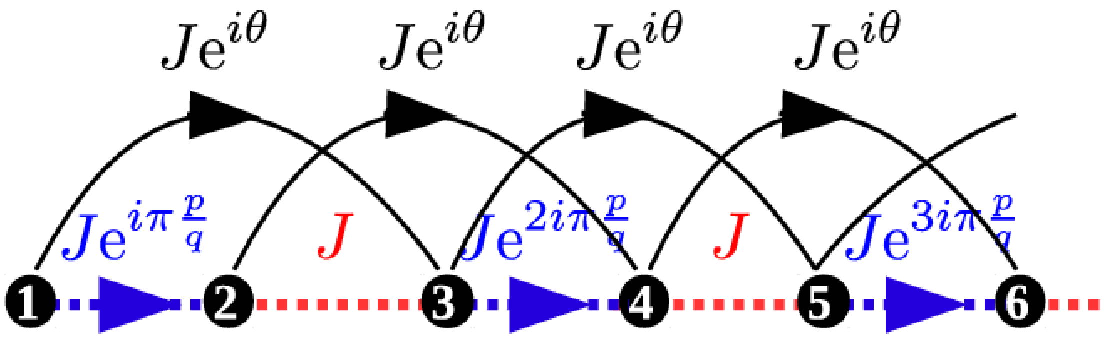

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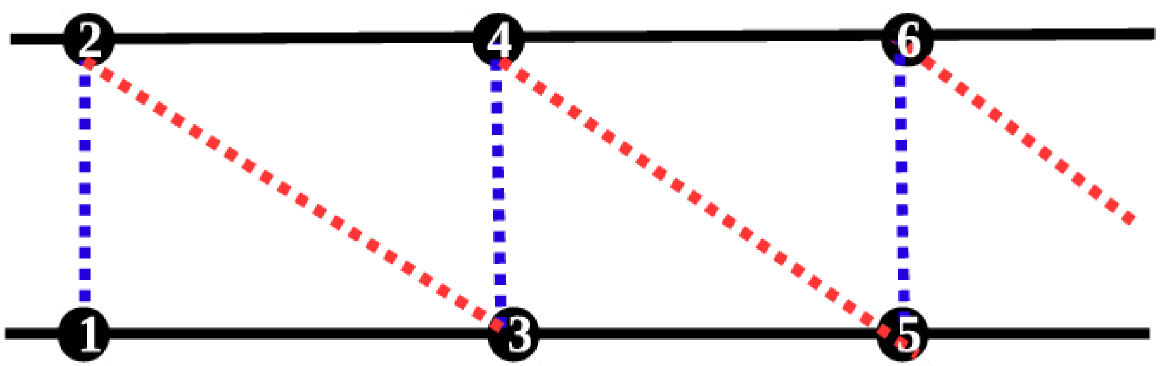

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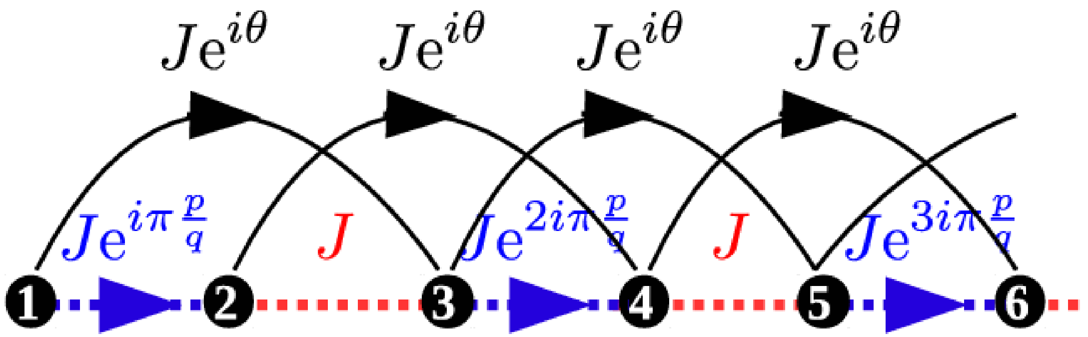

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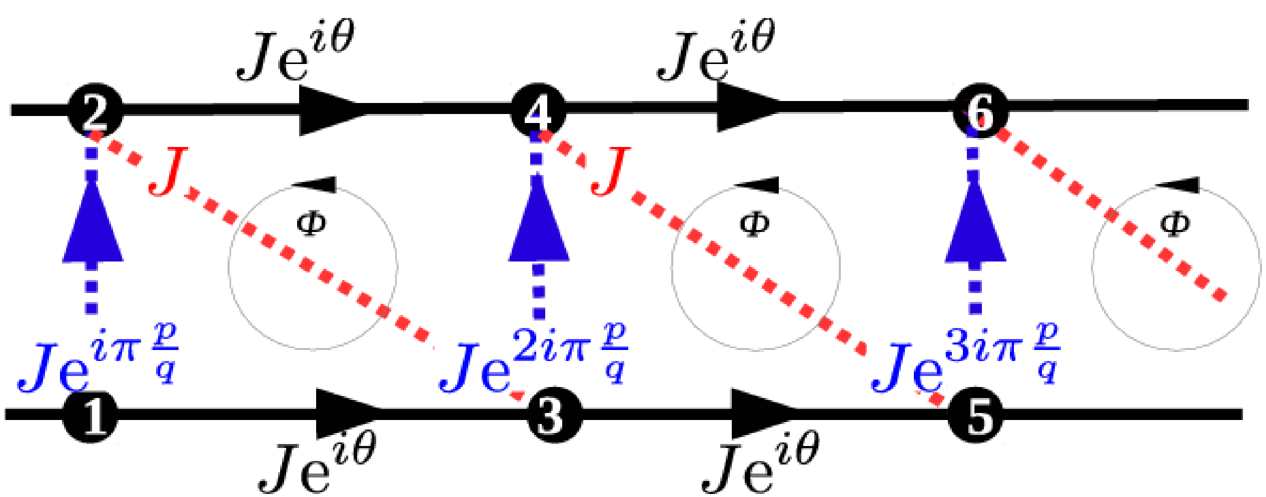
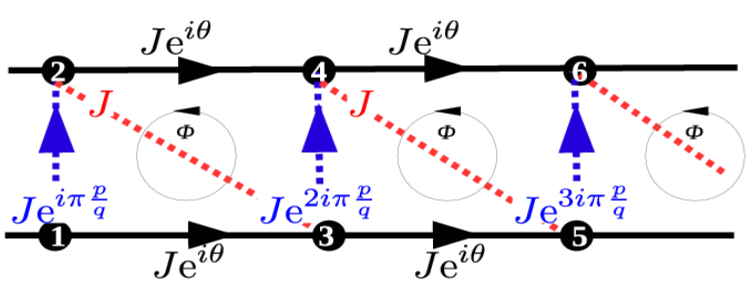


$$\Phi = \pi \frac{p}{q}$$

Butterfly spectrum

System

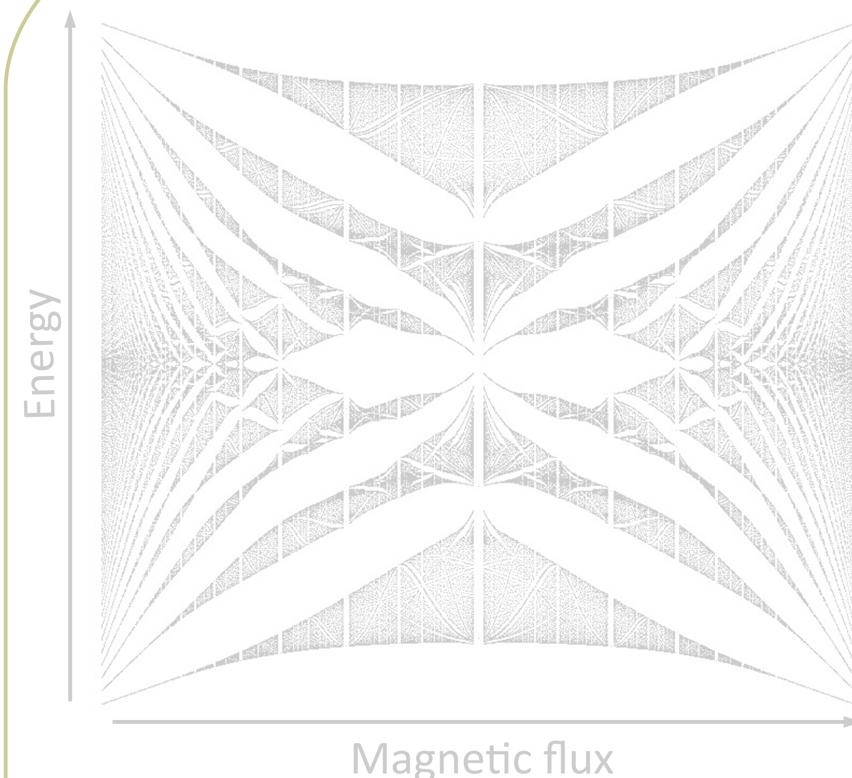
$$H = -\sum_{i \neq j} J_{ij} \left(\sigma_i^+ \sigma_j^- + \text{H.c.} \right) + h \sum_i \sigma_i^z$$



For a single spin-flip ($S_z = N - 2$), the spin chain realizes the Hofstadter model on a triangular ladder.

Fractal energy spectrum?

Result



Fractal structure disappears for a square ladder structure.

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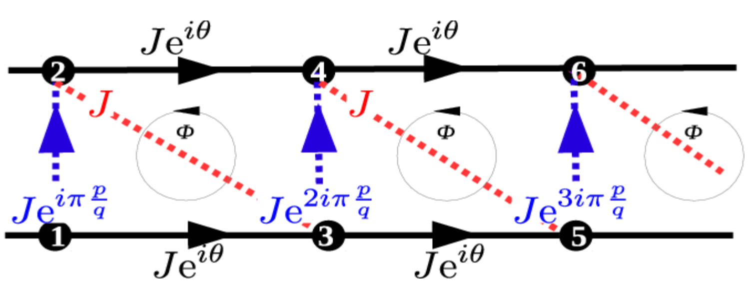

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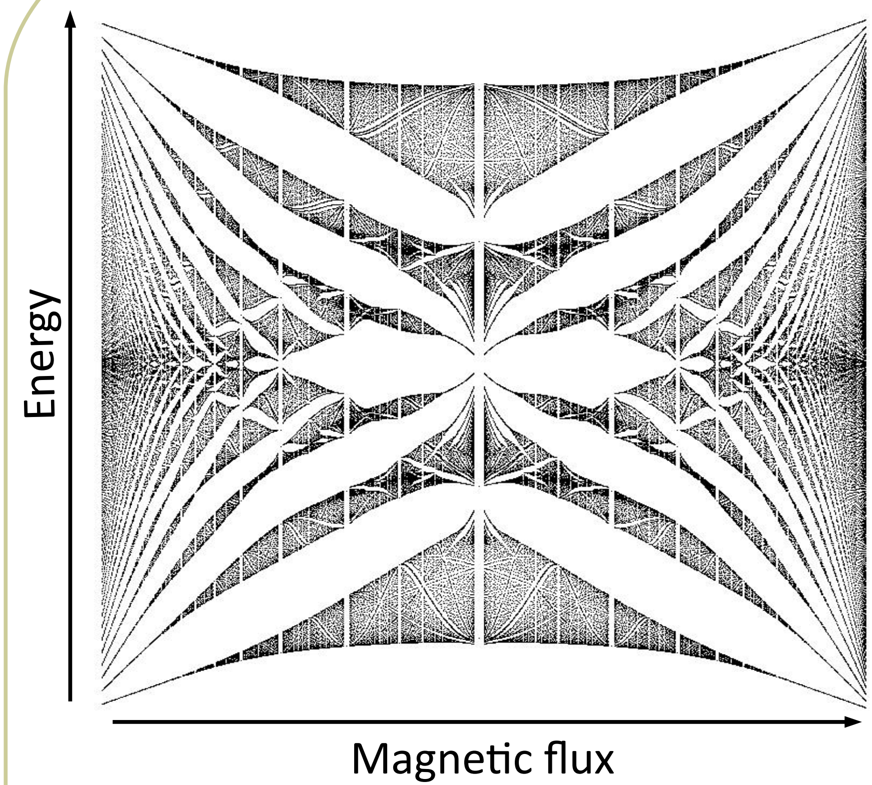

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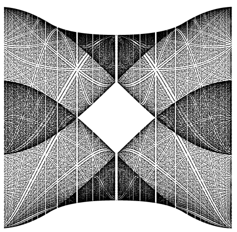

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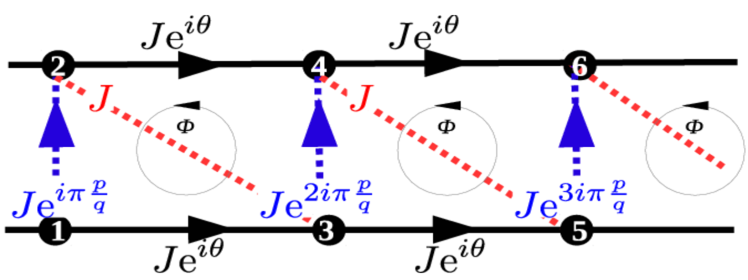

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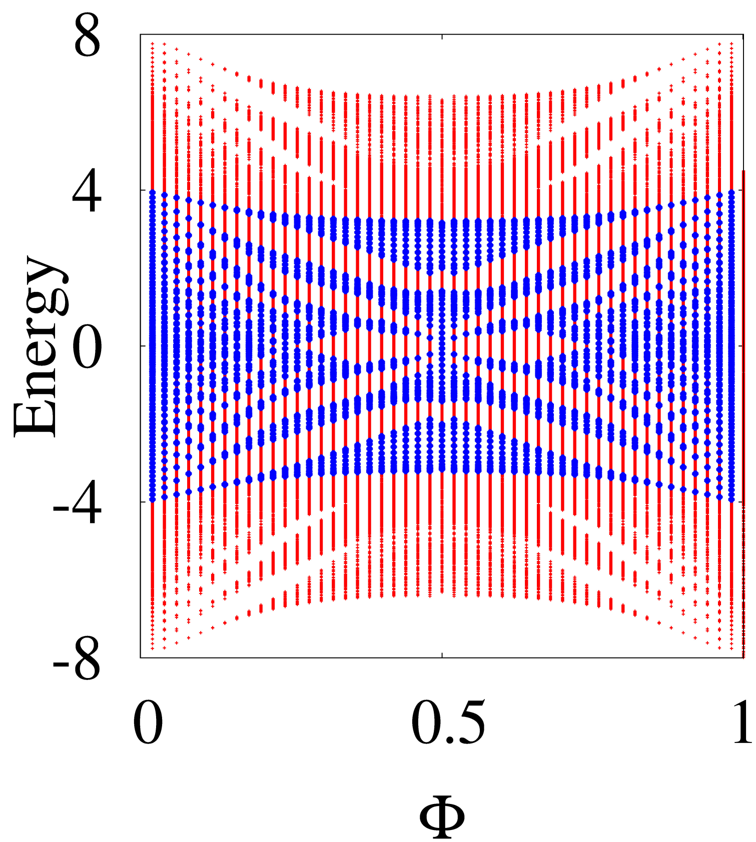
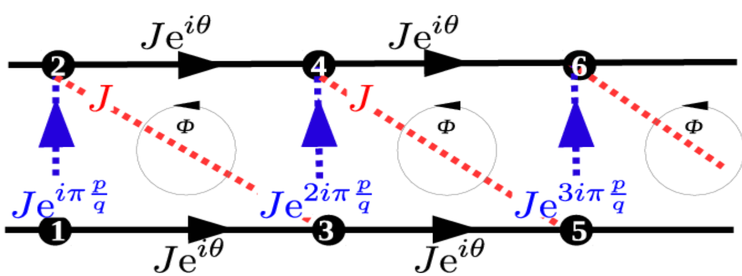


Finite system (N=100 spins), with one (blue) and two (red) spin flips

Topological bands

System

$$H = -\sum_{i \neq j} J_{ij} \left(\sigma_i^+ \sigma_j^- + \text{H.c.} \right) + h \sum_i \sigma_i^z$$



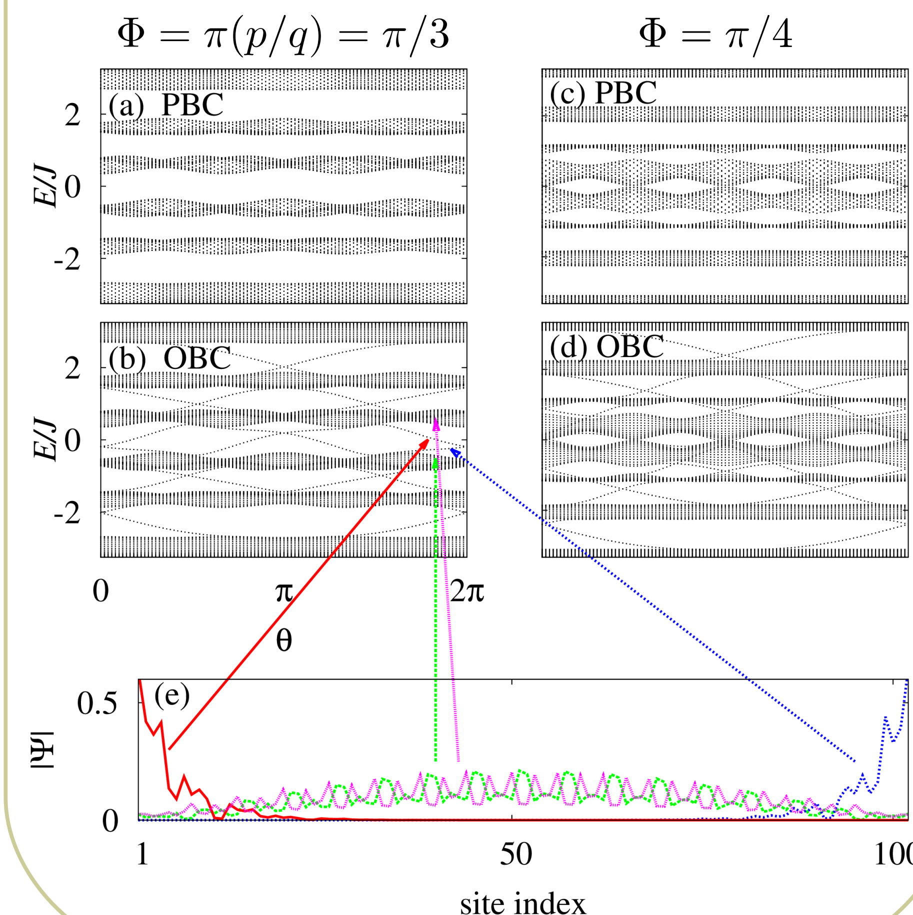
Chern numbers (single-particle bands)

for bands parametrized by k and θ at $\Phi = \frac{\pi}{q}$

q	Chern numbers
3	-1, -1, 2, 2, -1, -1
4	-1, -1, -1, 6, -1, -1, -1
5	-1, -1, -1, -1, 4, 4, -1, -1, -1, -1

Edge states

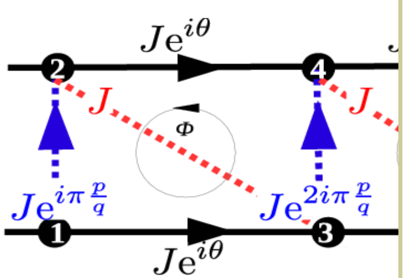
For 100 spins with a single spin flip



Topological bands

Syst

$$H = -\sum_{i\neq j} J_{ij} \left(\sigma_i^+\right.$$



Chern numbers (s

for bands parametrize

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3	-1, -1, 2
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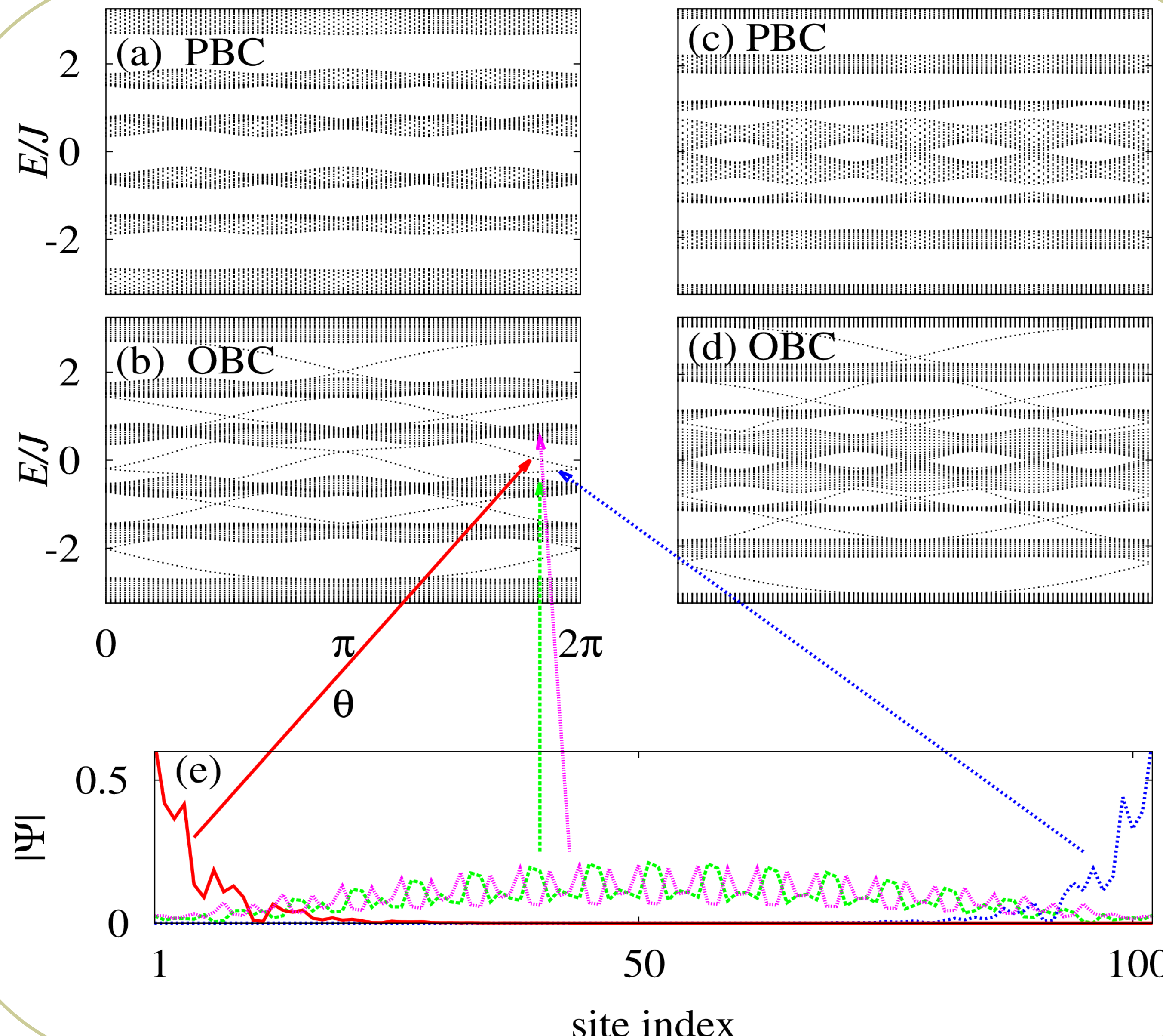

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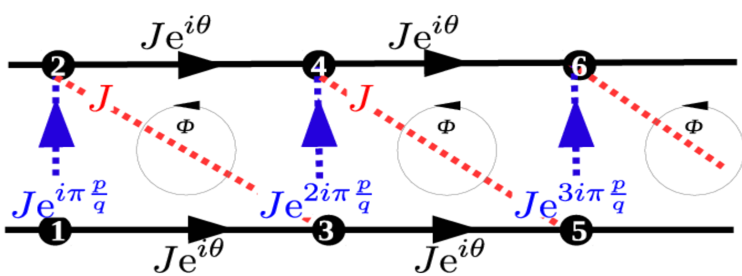

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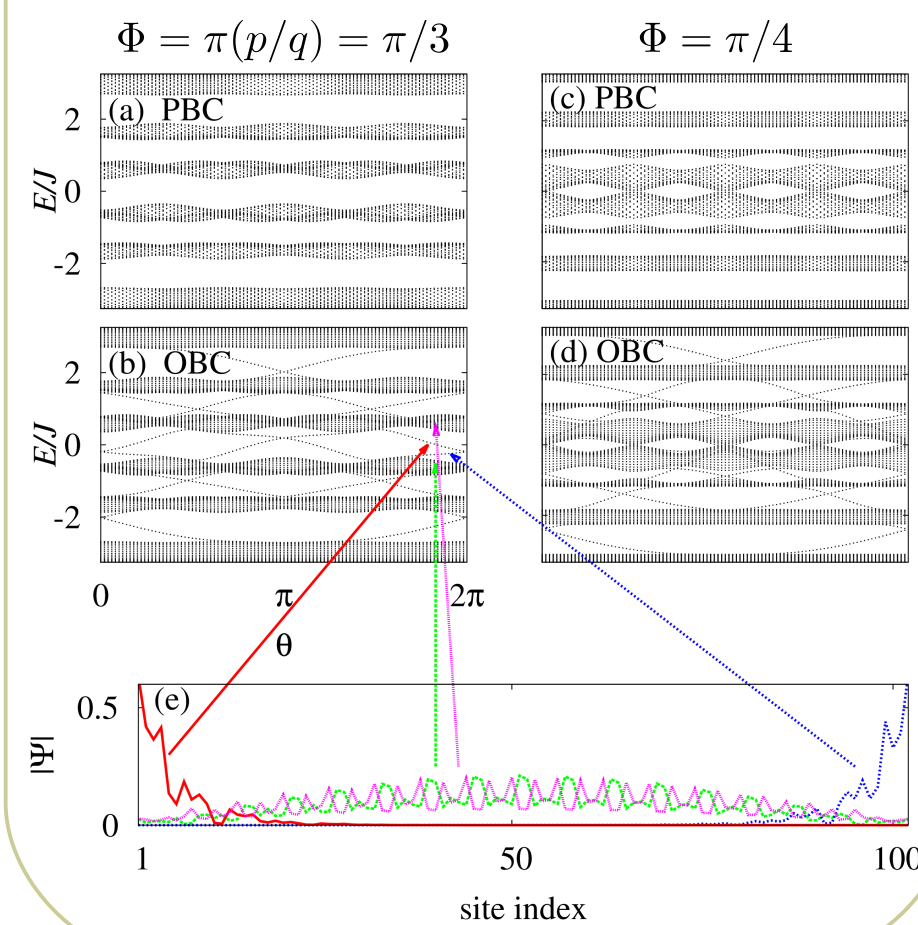
Edge states

For 100 spins with a single spin flip



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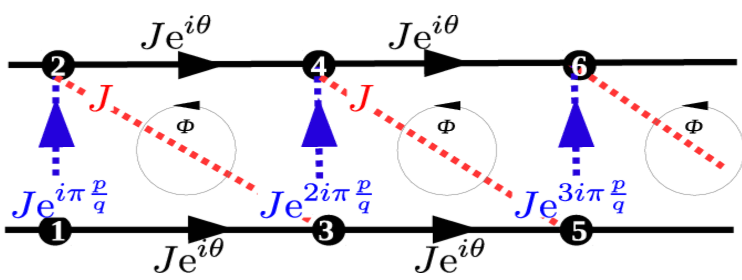

Edge states

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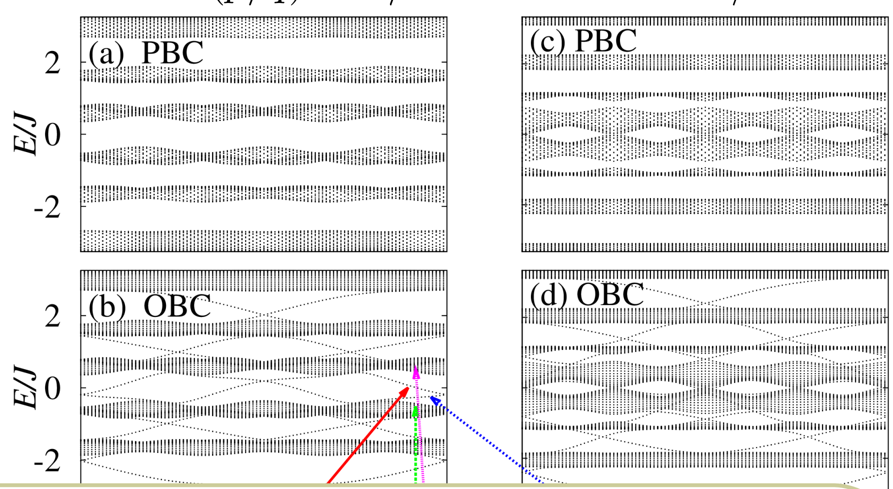

Chern numbers (single-particle bands)

$$CN = \frac{i}{2\pi}\int \mathrm{d}\mu_1 \int \mathrm{d}\mu_2 \left(\langle\partial_1\Psi|\partial_2\Psi\rangle - \langle\partial_2\Psi|\partial_1\Psi\rangle\right)$$

for bands parametrized by $\mu_1 \equiv k$ and $\mu_2 \equiv \theta$ at $\Phi = \frac{\pi}{q}$

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Many-body states

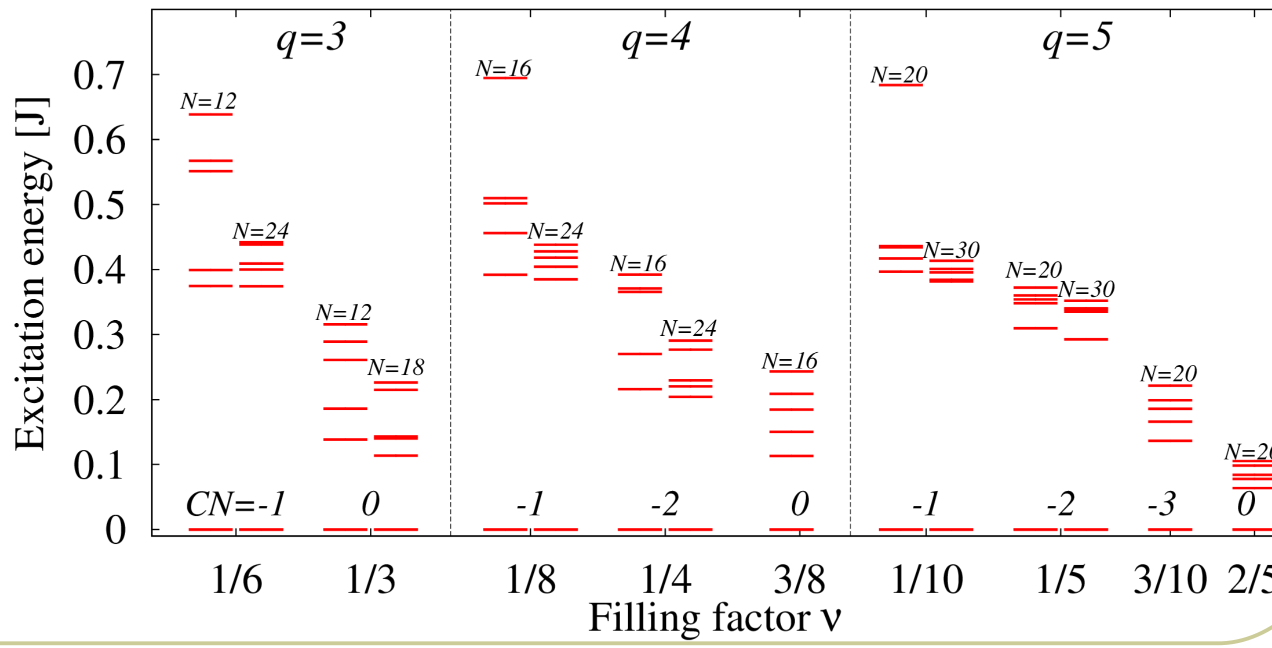
Many-body Chern numbers

Winding with respect to twisted boundary conditions and phase Θ

flux $\Phi = \frac{\pi}{q}$

filling $\nu = \frac{n}{2q}, \quad n \in \mathbb{N}$

polarization $S_z = N(1-2\nu)$



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Sufficiently far from half filling (i.e. S_z=0), the bosonic states are topologically equivalent to fermionic filling of single-particle levels.

Periodic driving

PRL **95,** 260404 (2005) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2005

Superfluid-Insulator Transition in a Periodically Driven Optical Lattice

André Eckardt, Christoph Weiss, and Martin Holthaus
Institut für Physik, Carl von Ossietzky Universität, D-26111 Oldenburg, Germany
(Received 16 August 2005; published 21 December 2005)

PRL **99,** 220403 (2007) PHYSICAL REVIEW LETTERS week ending 30 NOVEMBER 2007

Dynamical Control of Matter-Wave Tunneling in Periodic Potentials

H. Lignier, C. Sias, D. Ciampini, Y. Singh, A. Zenesini, O. Morsch, and E. Arimondo
CNR-INFM, Dipartimento di Fisica "E. Fermi," Università di Pisa, Largo Pontecorvo 3, 56127 Pisa, Italy

Selected for a Viewpoint in *Physics*
PRL **108,** 225304 (2012) PHYSICAL REVIEW LETTERS week ending 1 JUNE 2012

Tunable Gauge Potential for Neutral and Spinless Particles in Driven Optical Lattices

J. Struck,[1] C. Ölschläger,[1] M. Weinberg,[1] P. Hauke,[2] J. Simonet,[1] A. Eckardt,[3] M. Lewenstein,[2,4]
K. Sengstock,[1,*] and P. Windpassinger[1]
[1]*Institut für Laserphysik, Universität Hamburg, D-22761 Hamburg, Germany*
[2]*Institut de Ciències Fotòniques, Mediterranean Technology Park, Av. Carl Friedrich Gauss 3, E-08860 Castelldefels, Barcelona, Spain*
[3]*Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Str. 38, D-01187 Dresden, Germany*
[4]*ICREA-Institució Catalana de Recerca i Estudis Avançats, Lluis Companys 23, E-08010 Barcelona, Spain*
(Received 29 February 2012; published 29 May 2012)

Periodic driving

Apply the shaking ideas to spin chains in order to modify the interaction parameter:

- Strength of *J*
- Sign of *J*
- Complex phase of *J*

XY model with "shaken" field

$$H(t) = H_{\rm XY} + \sum_i v_i(t)\sigma_i^z \quad \text{with} \quad H_{\rm XY} = \sum_{i<j} J_{ij}(\sigma_i^+\sigma_j^- + \text{h.c.})$$

Gauge transform (Floquet basis)

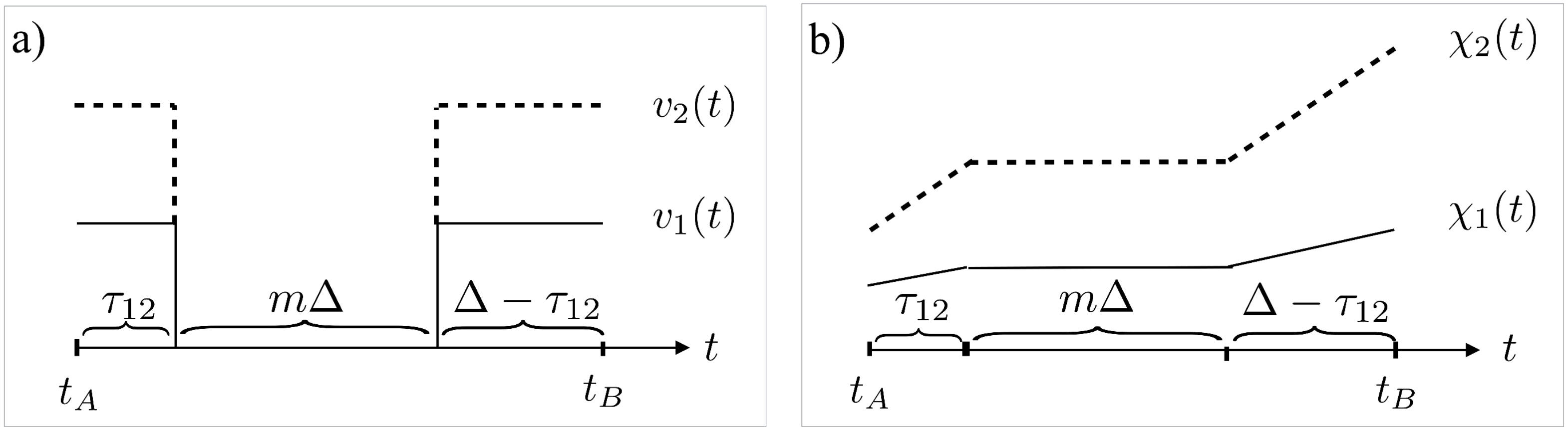
$$U(t) = e^{-i\sum_i \chi_i(t)\sigma_i^z} \quad \text{with} \quad \chi_i(t) = \int_0^t dt' v_i(t')$$

Average over period T

$$H_{\rm eff} = \sum_{i<j} J_{ij}^{\rm eff}(\sigma_i^+\sigma_j^- + \text{h.c.}) \quad \text{where} \quad J_{ij}^{\rm eff} = \frac{\bar{J}_{ij}}{T}\int_0^T \text{d}t e^{2i[\chi_i(t)-\chi_j(t)]}$$

Periodic driving

Apply the shaking ideas to spin chains in order to modify the interaction parameter:



$$\Delta \cdot v_i(0) = n\pi \Rightarrow \int_{t_A}^{t_B} e^{i2[\chi_1(t')-\chi_2(t')]} dt = e^{i\varphi_{12}} m\Delta \quad \text{with} \quad \varphi_{12} = 2\tau_{12}(v_1(0) - v_2(0))$$

XY model with "shaken" field

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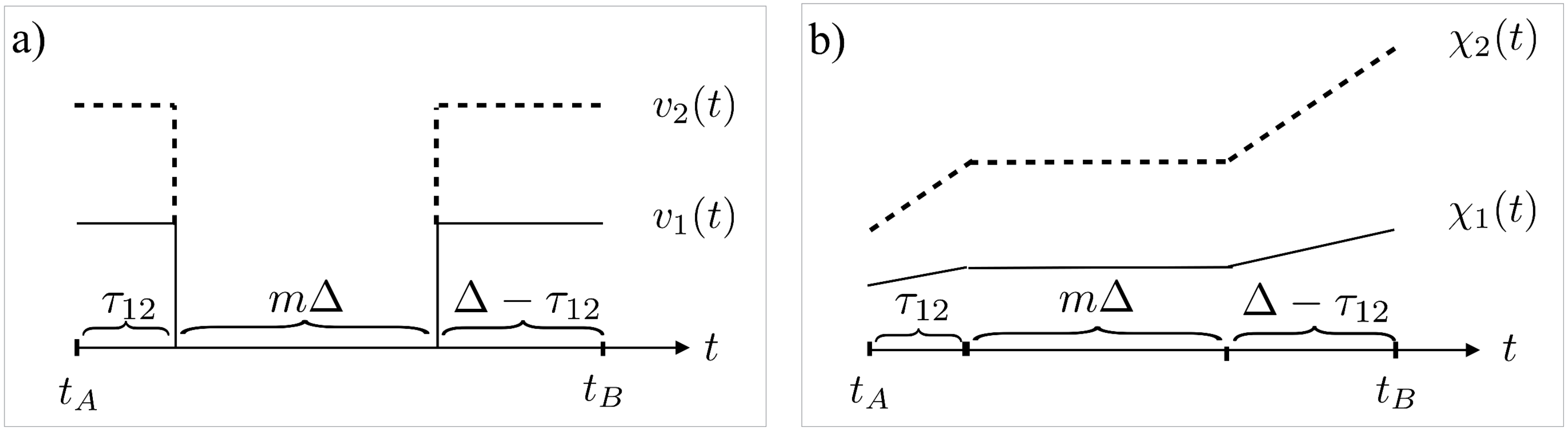

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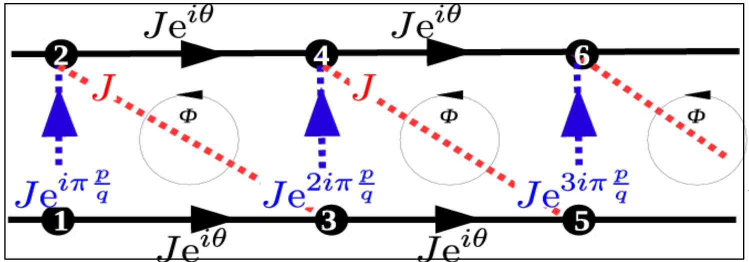
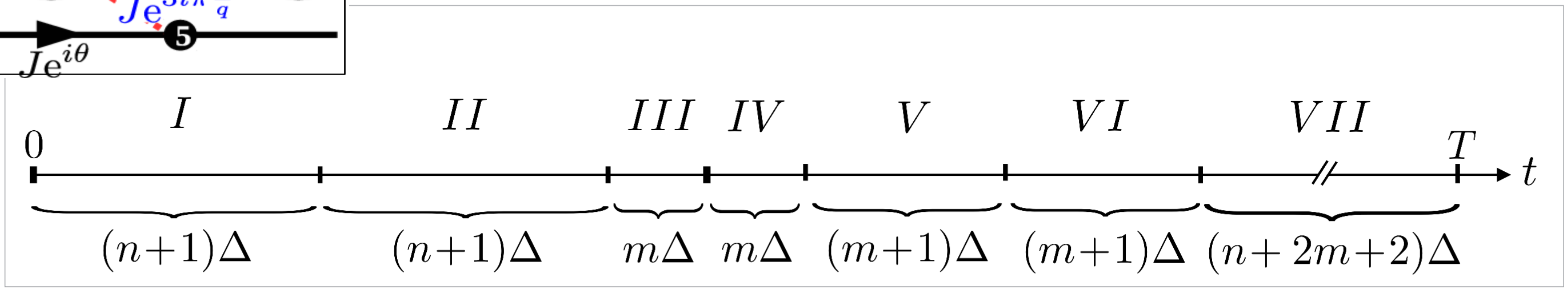
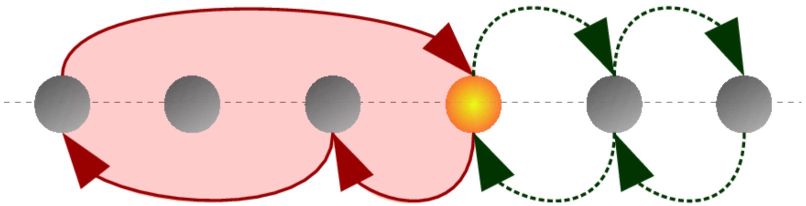




Summary

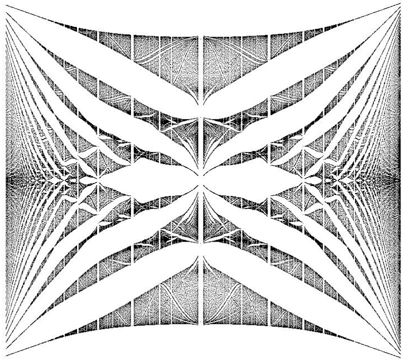
Idee:

- No loops with magnetic flux in short-ranged chains
- Long-range connections allow for loops with flux



Realization:

- Long-range spin chains, e.g. trapped ions or atoms coupled to nanophotonic devices
- Design of complex-valued interactions parameters via shaking



Results:

- Fractal energy spectrum
- Topological band structure
- Bosonic Chern insulator

arXiv:1412.6059
Tobias Grass, Christine Muschik, Alessio Celi, Ravindra Chhajlany, Maciej Lewenstein