Synthetic Quantum Magnetism, MPI PKS Dresden, 01.09.2015

# Chains with Loops - Synthetic Magnetic Fluxes in 1D

Tobias Grass (ICFO - Barcelona)

In collaboration with:

Alessio Celi (ICFO)

Ravindra Chhajlany (U Poznań)

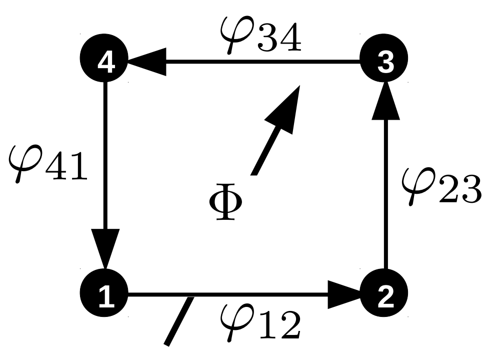
Maciej Lewenstein (ICFO)

Christine Muschik (IQOQI)

# Can a magnetic field in 1D be interesting?

*In 2 or more dimensions:*

non-trivial loops

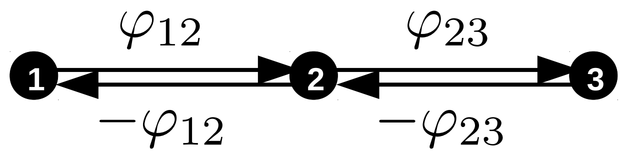



$$\varphi_{ij} = \int_{\mathbf{r}_i}^{\mathbf{r}_j} \mathrm{d}\mathbf{r} \cdot \mathbf{A}(\mathbf{r})$$

$$\Phi = \varphi_{12} + \varphi_{23} + \varphi_{34} + \varphi_{41}$$

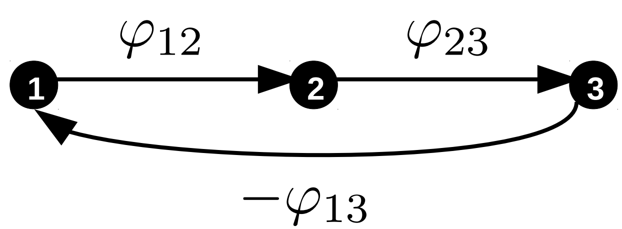
*In 1 dimension:*

no loops with flux




$$\Phi = \varphi_{12} + \varphi_{23} - \varphi_{23} - \varphi_{14} = 0$$

unless we consider long-range hopping with generic Peierls phases:




$$\Phi = \varphi_{12} + \varphi_{23} - \varphi_{13}$$

# Possible platforms

## 1. Cold atoms in optical lattices

- typically only nearest-neighbor hopping
- artificial gauge fields already exist in 2d lattices

## 2. Trapped ions

- linear arrangement
- long-range spin-spin interactions (mediated by phonons)

LETTER

doi:10.1038/nature13461

**Quasiparticle engineering and entanglement propagation in a quantum many-body system**

P. Jurcevic[1,2]*, B. P. Lanyon[1,2]*, P. Hauke[1,3], C. Hempel[1,2], P. Zoller[1,3], R. Blatt[1,2] & C. F. Roos[1,2]

LETTER

doi:10.1038/nature13450

**Non-local propagation of correlations in quantum systems with long-range interactions**

Philip Richerme[1], Zhe-Xuan Gong[1], Aaron Lee[1], Crystal Senko[1], Jacob Smith[1], Michael Foss-Feig[1], Spyridon Michalakis[2], Alexey V. Gorshkov[1] & Christopher Monroe[1]

## 3. Cold atoms coupled to a nanophotonic fiber

- Long-range spin-spin interaction (mediated by photons)
- Not yet mature technology, but with the prospect of good scalability (>1000 atoms)

# Possible platforms

1. Cold atoms in optical lattices
   - typically only nearest-neighbor hopping
   - artificial gauge fields already exist in 2d lattices

2. Trapped ions
   - linear arrangement
   - long-range spin-spin interactions (mediated by phonons)

LETTER

doi:10.1038/nature13461

**Quasiparticle engineering and entanglement propagation in a quantum many-body system**

P. Jurcevic[1,2]*, B. P. Lanyon[1,2]*, P. Hauke[1,3], C. Hempel[1,2], P. Zoller[1,3], R. Blatt[1,2] & C. F. Roos[1,2]

LETTER

doi:10.1038/nature13450

**Non-local propagation of correlations in quantum systems with long-range interactions**

Philip Richerme[1], Zhe-Xuan Gong[1], Aaron Lee[1], Crystal Senko[1], Jacob Smith[1], Michael Foss-Feig[1], Spyridon Michalakis[2], Alexey V. Gorshkov[1] & Christopher Monroe[1]

3. Cold atoms coupled to a nanophotonic fiber
   - Long-range spin-spin interaction (mediated by photons)
   - Not yet mature technology,
     but with the prospect of good scalability (>1000 atoms)

# Outline

1. Mapping: spin-flip interactions ↔ hopping

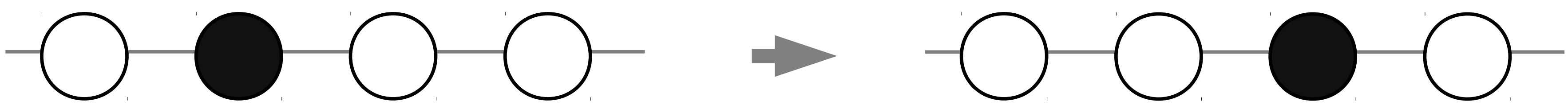
2. Model: XY chain with nearest and
   next-to-nearest neighbor interactions
   - Mapping onto triangular ladder
   - Magnetic flux via complex interaction strength

   Results:
   - Fractal energy spectrum
   - Topological bands
   - Topological many-body states

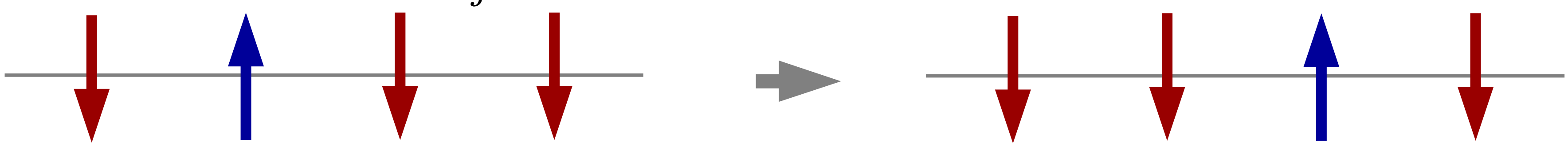
3. Realization of the model with ions or atoms
   - Engineering interactions via periodic driving

# Mapping: Hopping ↔ XY model

**Hopping:** $H = -J \sum_{ij} a_i^\dagger a_j$



**Spin flip:** $H = -J \sum_{ij} \sigma_i^+ \sigma_j^-$



For XY chain with nearest-neighbor interaction:

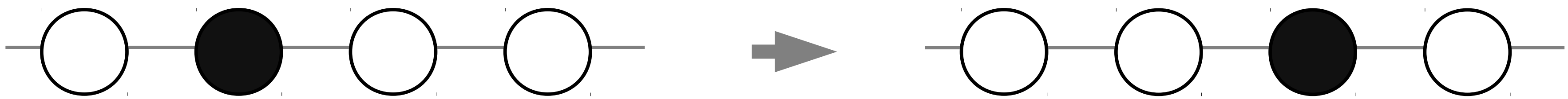
- Jordan-Wigner transformation: equivalence of spin flip model and free fermion model

In the presence of interactions beyond nearest neighbors:

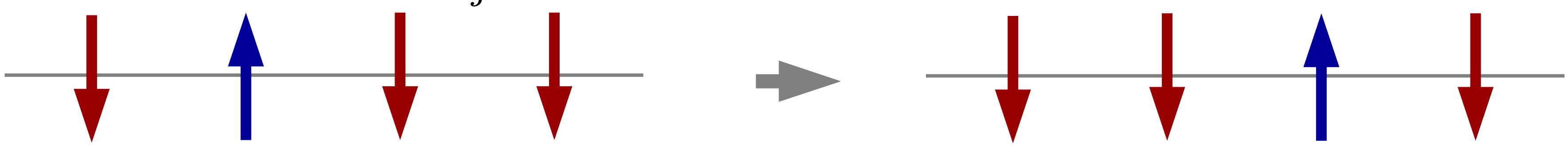
- Jordan-Wigner does not work
- Spin flips operators $\sigma$ are bosonic
- Hard-core constraint: strong interactions

# Mapping: Hopping ↔ XY model

**Hopping:** $H = -J \sum_{ij} a_i^\dagger a_j$



**Spin flip:** $H = -J \sum_{ij} \sigma_i^+ \sigma_j^-$



For XY chain with nearest-neighbor interaction:

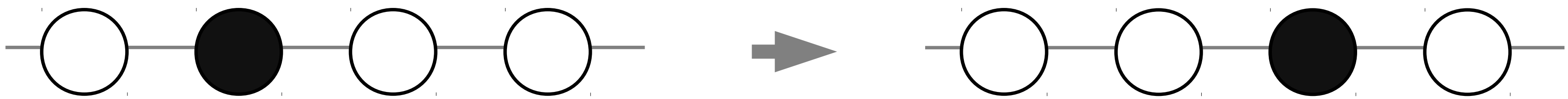
- Jordan-Wigner transformation: equivalence of spin flip model and free fermion model

In the presence of interactions beyond nearest neighbors:

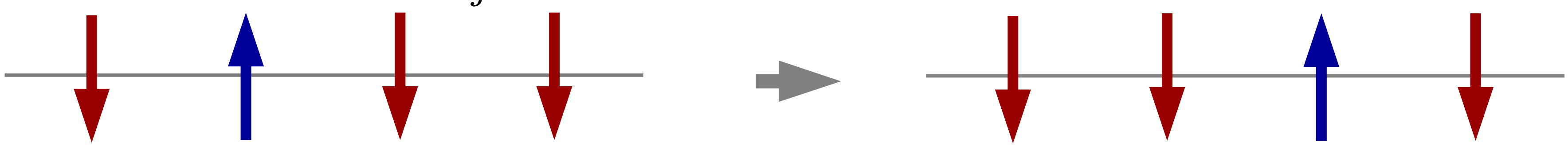
- Jordan-Wigner does not work
- Spin flips operators $\sigma$ are bosonic
- Hard-core constraint: strong interactions

# Mapping: Hopping $\leftrightarrow$ XY model

**Hopping:** $$H = -J \sum_{ij} a_i^\dagger a_j$$



**Spin flip:** $$H = -J \sum_{ij} \sigma_i^+ \sigma_j^-$$



For XY chain with nearest-neighbor interaction:

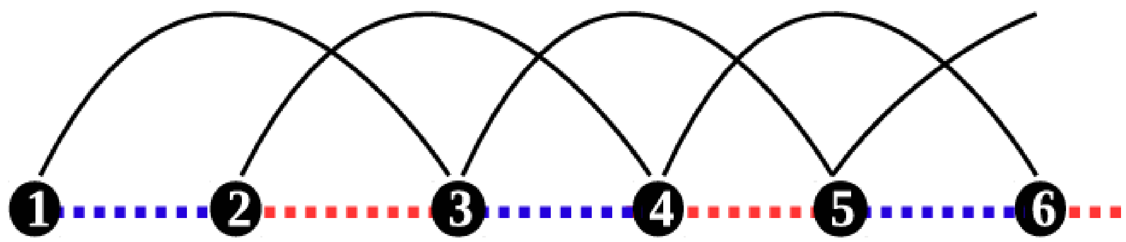
- Jordan-Wigner transformation: equivalence of spin flip model and free fermion model

In the presence of interactions beyond nearest neighbors:

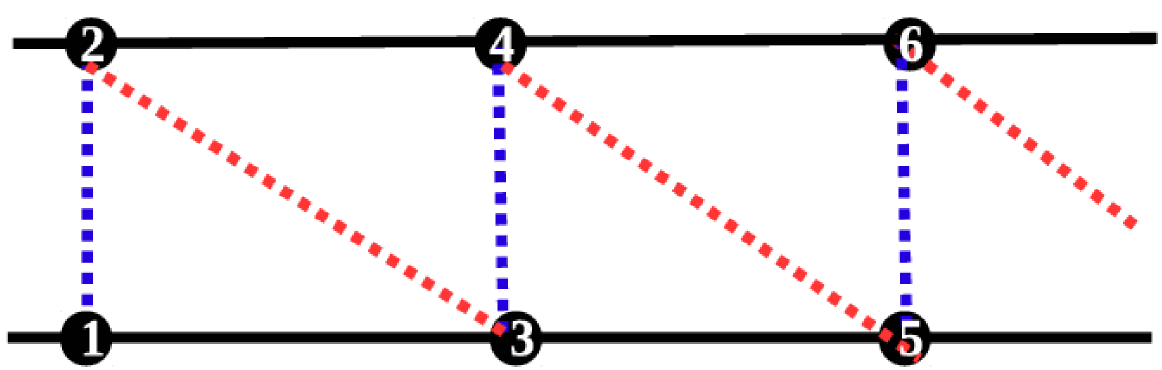
- Jordan-Wigner does not work
- Spin flips operators $\sigma$ are bosonic
- Hard-core constraint: strong interactions

# XY model with magnetic fluxes

XY chain with NN and NNN interactions:

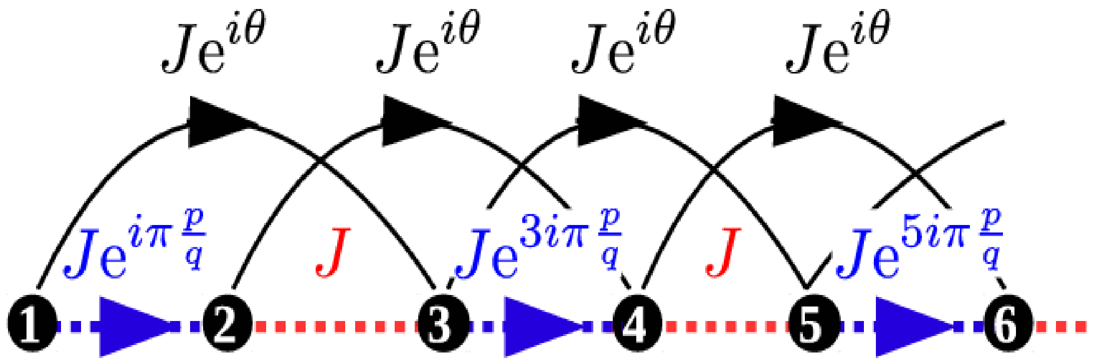



Mapping onto triangular ladder:

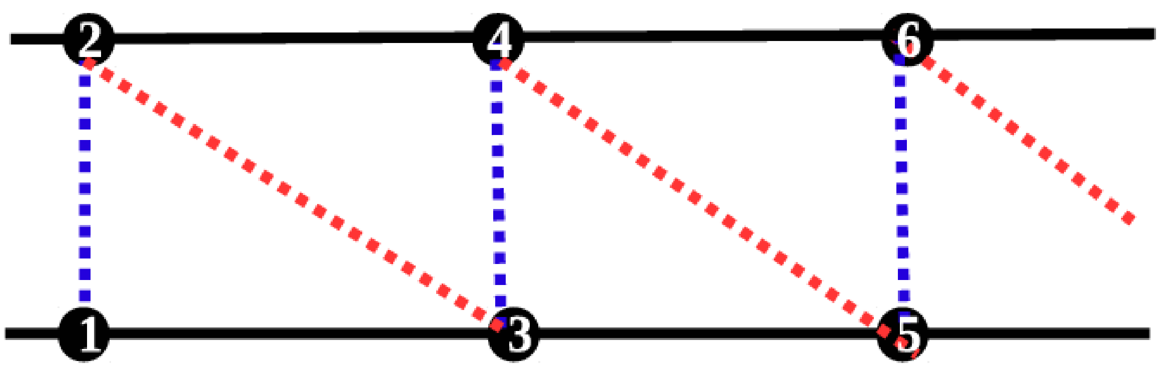


# XY model with magnetic fluxes

XY chain with NN and NNN interactions:

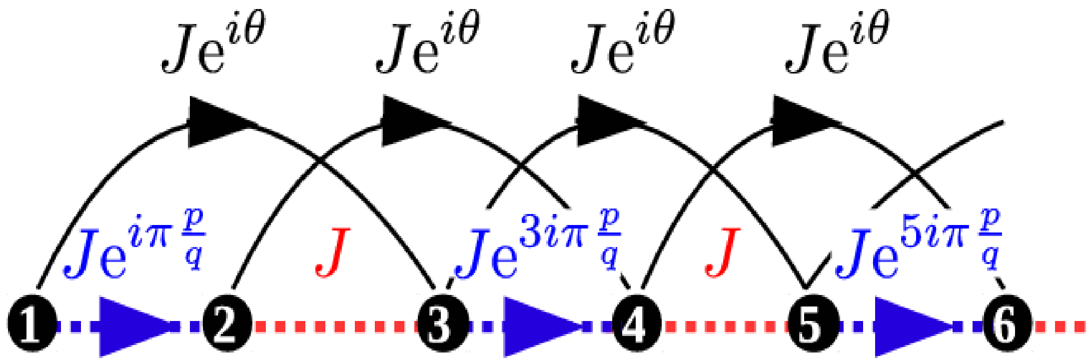



Mapping onto triangular ladder:

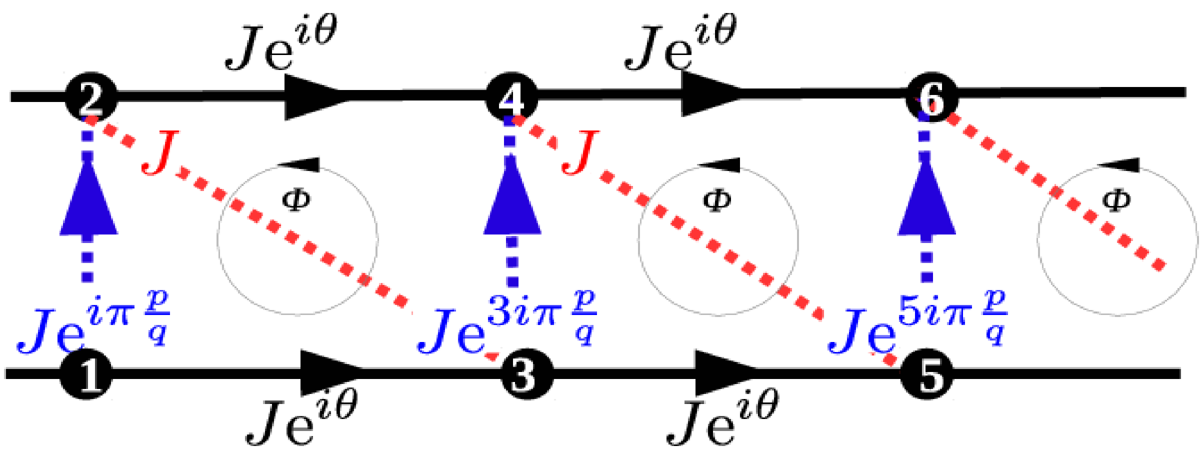


# XY model with magnetic fluxes

XY chain with NN and NNN interactions:




Mapping onto triangular ladder:

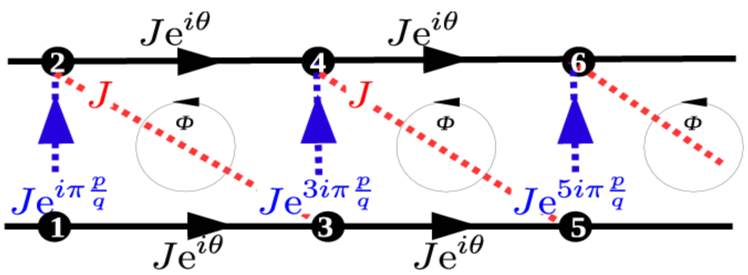



$$\Phi = 2\pi\frac{p}{q}$$

# Butterfly spectrum

## System

$$H = -\sum_{i \neq j} J_{ij} \sigma_i^+ \sigma_j^- + h \sum_i \sigma_i^z$$

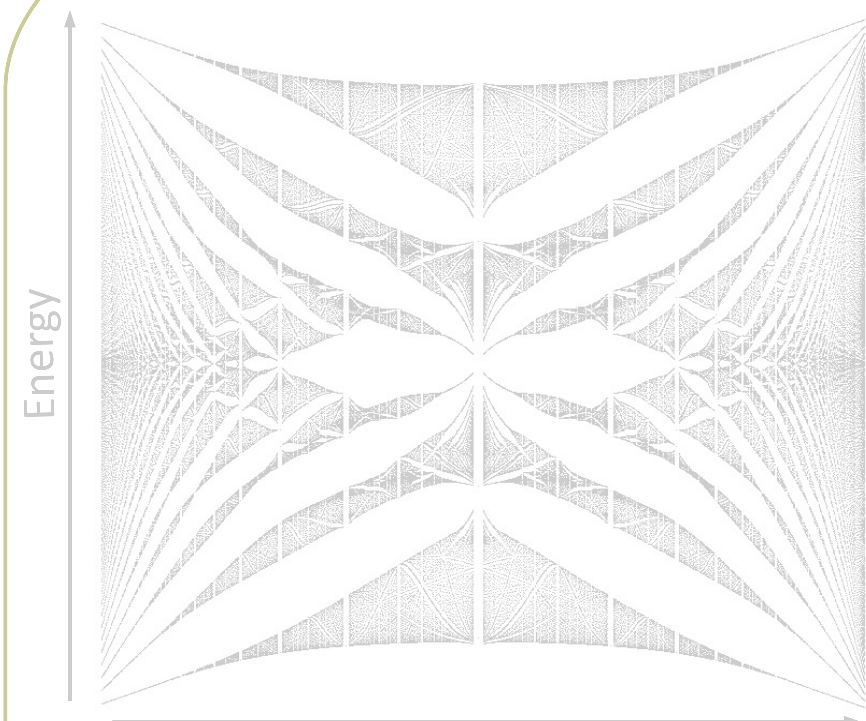



Difference to Hofstadter model:

- Interactions: irrelevant for a single spin-flip
  $$S_z = N - 2$$
- Ladder instead of infinite square lattice
- Diagonal link

Fractal energy spectrum?

## Result

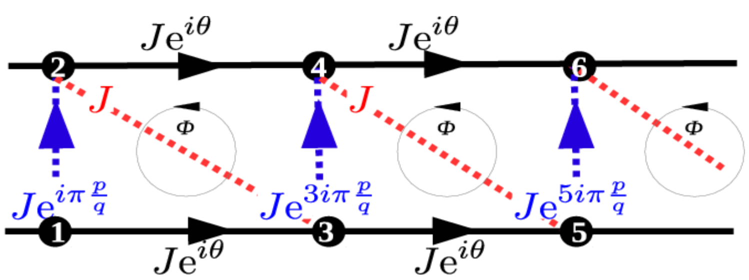



Fractal structure disappears for a square ladder structure.

# Butterfly spectrum

## System

$$H = -\sum_{i \neq j} J_{ij} \sigma_i^+ \sigma_j^- + h \sum_i \sigma_i^z$$

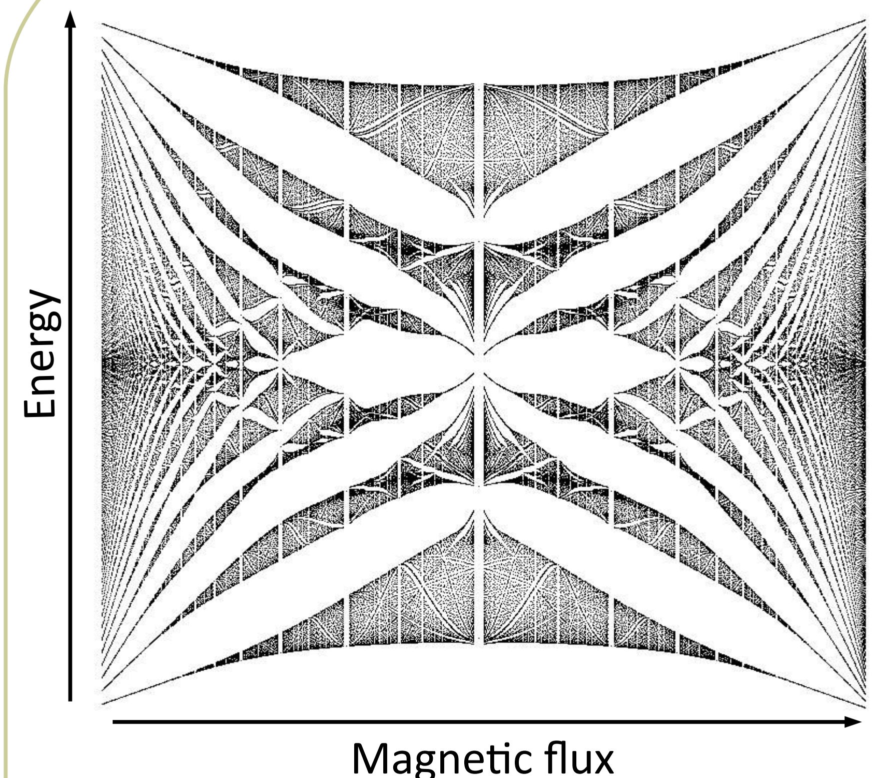



Difference to Hofstadter model:

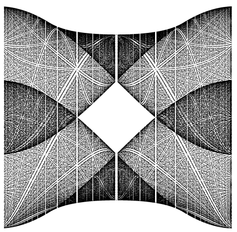
- Interactions: irrelevant for a single spin-flip
  $$S_z = N - 2$$
- Ladder instead of infinite square lattice
- Diagonal link

Fractal energy spectrum?

## Result



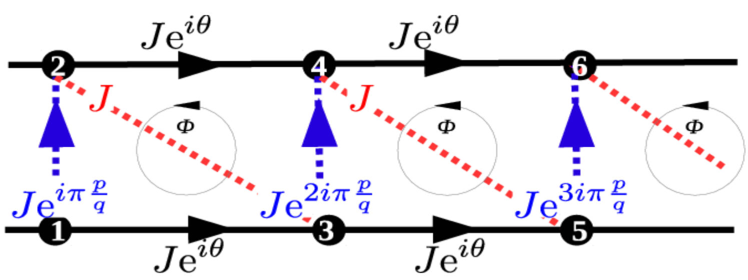

Fractal structure disappears for a square ladder structure.



# Butterfly spectrum

System

$$H = -\sum_{i\neq j} J_{ij}\sigma_i^+\sigma_j^- + h\sum_i \sigma_i^z$$

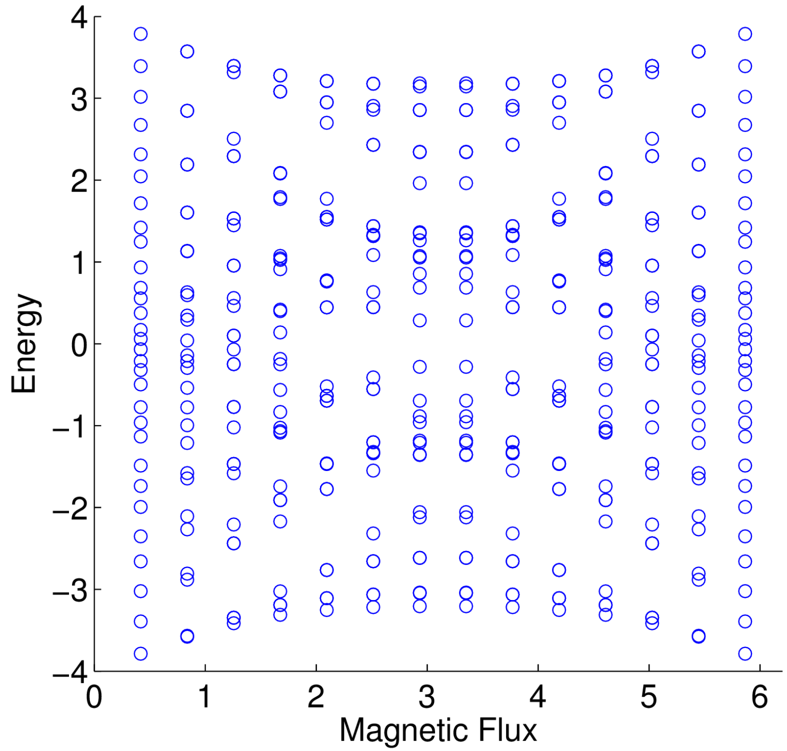



Difference to Hofstadter model:

- Interactions: irrelevant for a single spin-flip
$$S_z = N-2$$
- Ladder instead of infinite square lattice
- Diagonal link

Fractal energy spectrum?

Result

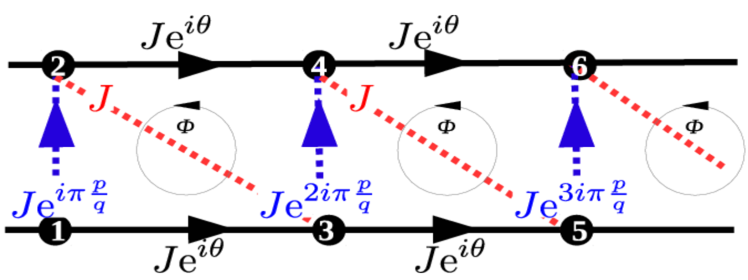



Finite system: N=30

# Butterfly spectrum

## System

$$H = -\sum_{i \neq j} J_{ij} \sigma_i^+ \sigma_j^- + h \sum_i \sigma_i^z$$

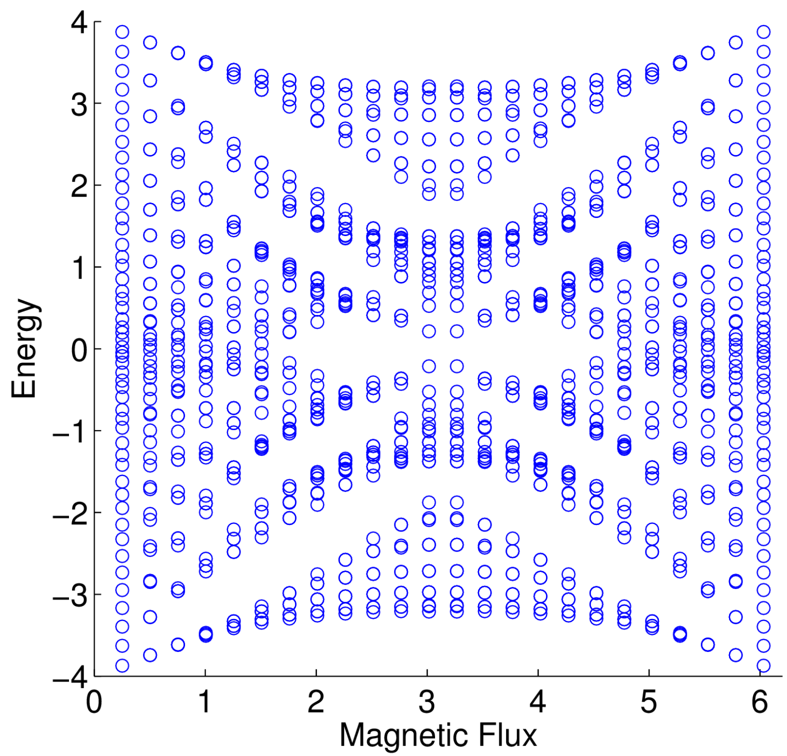



Difference to Hofstadter model:

- Interactions: irrelevant for a single spin-flip
  $S_z = N - 2$
- Ladder instead of infinite square lattice
- Diagonal link

Fractal energy spectrum?

## Result

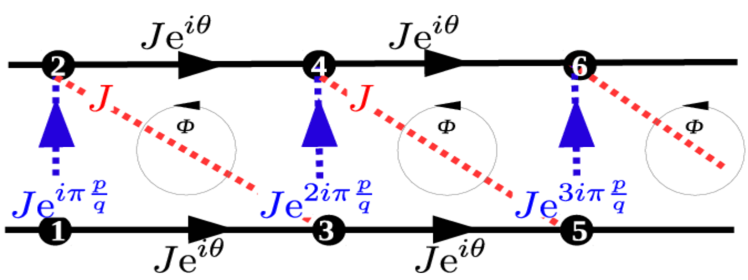



Finite system: N=50

# Butterfly spectrum

## System

$$H = -\sum_{i \neq j} J_{ij}\sigma_i^+\sigma_j^- + h\sum_i \sigma_i^z$$

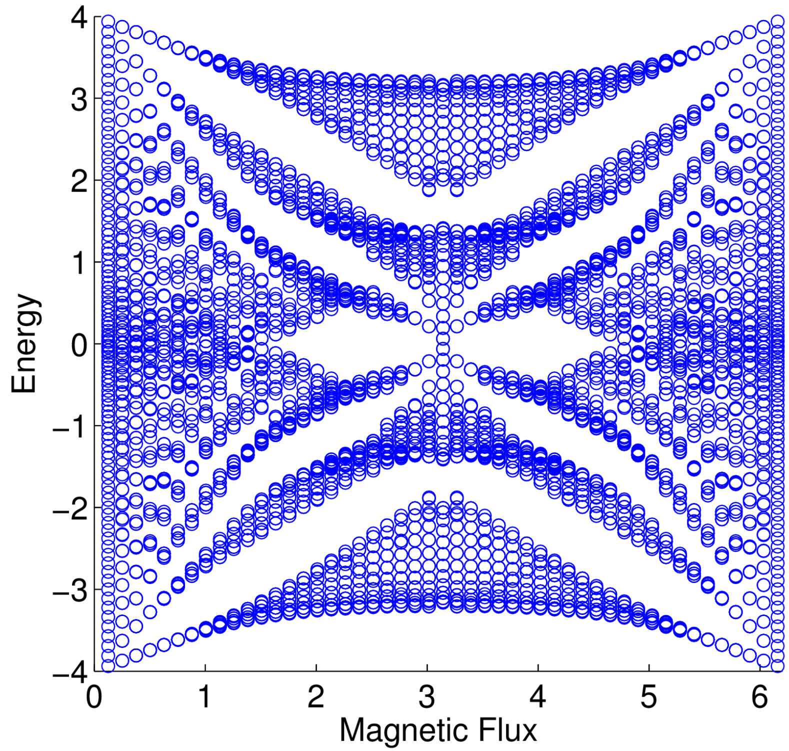



Difference to Hofstadter model:

- Interactions: irrelevant for a single spin-flip
$$S_z = N - 2$$
- Ladder instead of infinite square lattice
- Diagonal link

Fractal energy spectrum?

## Result

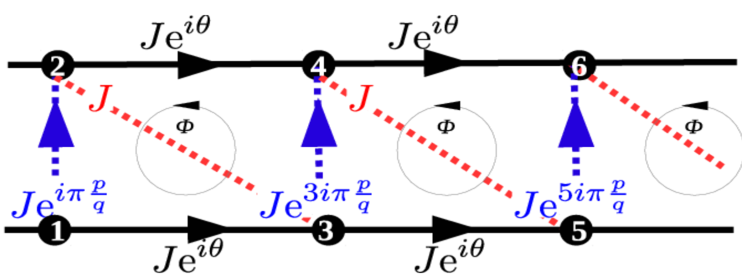



Finite system: N=100

# Topological bands

## System

$$H = -\sum_{i \neq j} J_{ij} \sigma_i^+ \sigma_j^- + h \sum_i \sigma_i^z$$



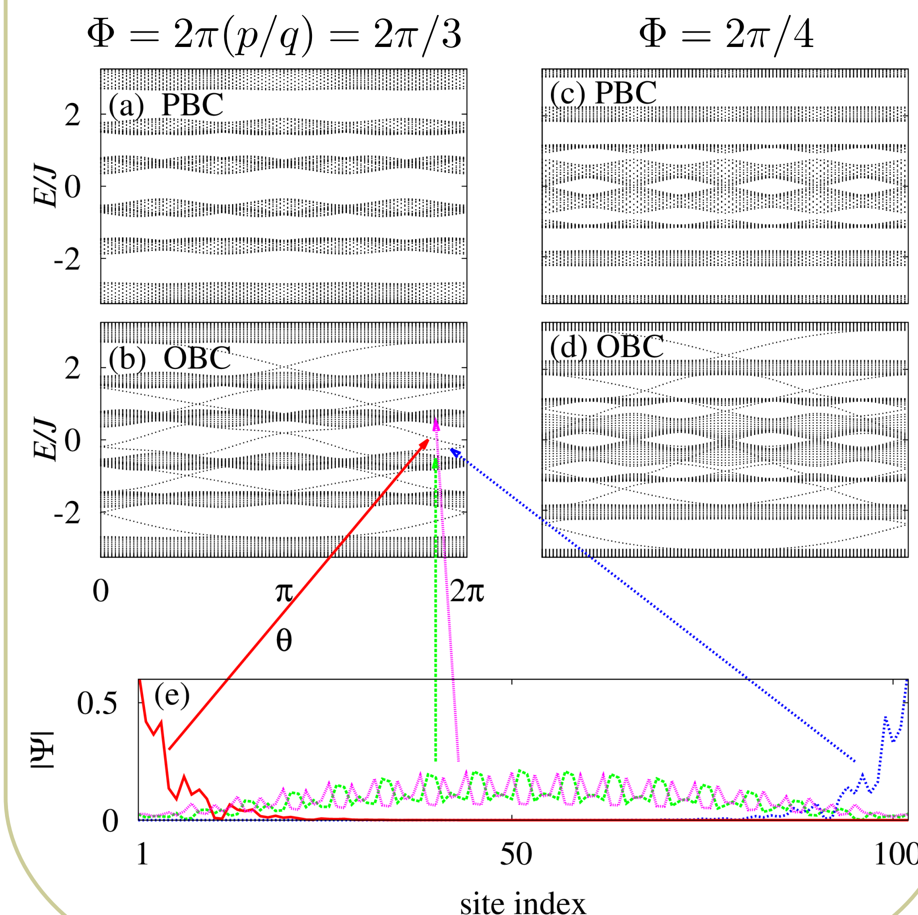

## Chern numbers (single-particle bands)

for bands parametrized by $k$ and $\theta$ at $\Phi = \frac{2\pi}{q}$

| $q$ | Chern numbers |
|---|---|
| 3 | -1, -1, 2, 2, -1, -1 |
| 4 | -1, -1, -1, 6, -1, -1, -1 |
| 5 | -1, -1, -1, -1, 4, 4, -1, -1, -1, -1 |

## Edge states

For 100 spins with a single spin flip



# Topological bands

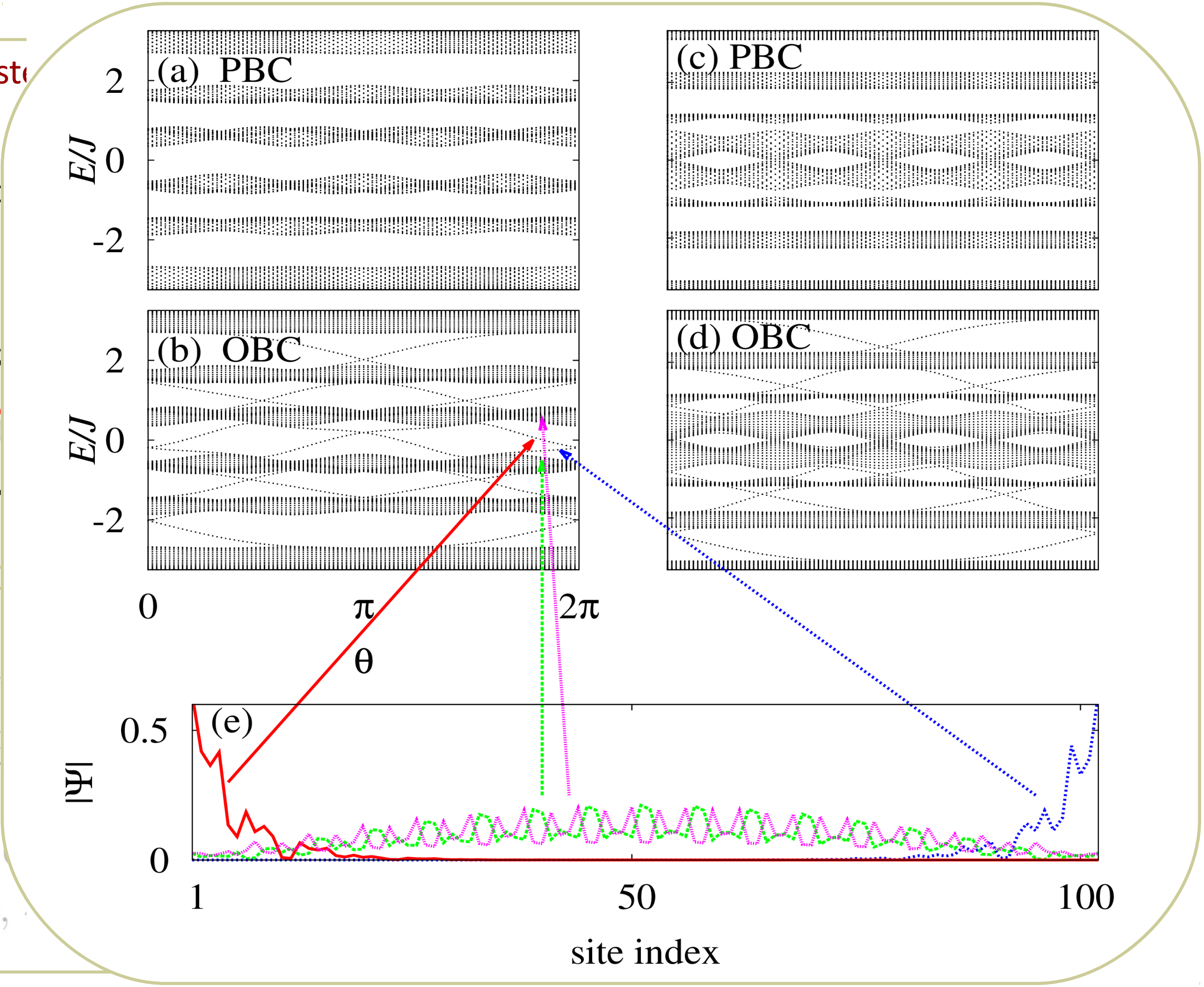
Syst

$$H = -\sum_{i\neq j} J_{ij} c$$

Chern numbers (s

for bands parametrize

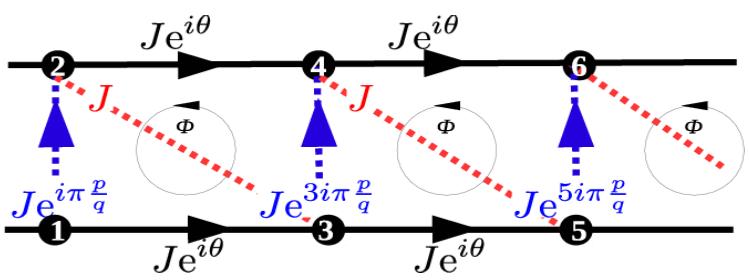
| $q$ | Chern |
|---|---|
| 3 | -1, -1, 2 |
| 4 | -1, -1, -1, |
| 5 | -1, -1, -1, -1, 4, |



# Topological bands

## System

$$H = -\sum_{i \neq j} J_{ij} \sigma_i^+ \sigma_j^- + h \sum_i \sigma_i^z$$



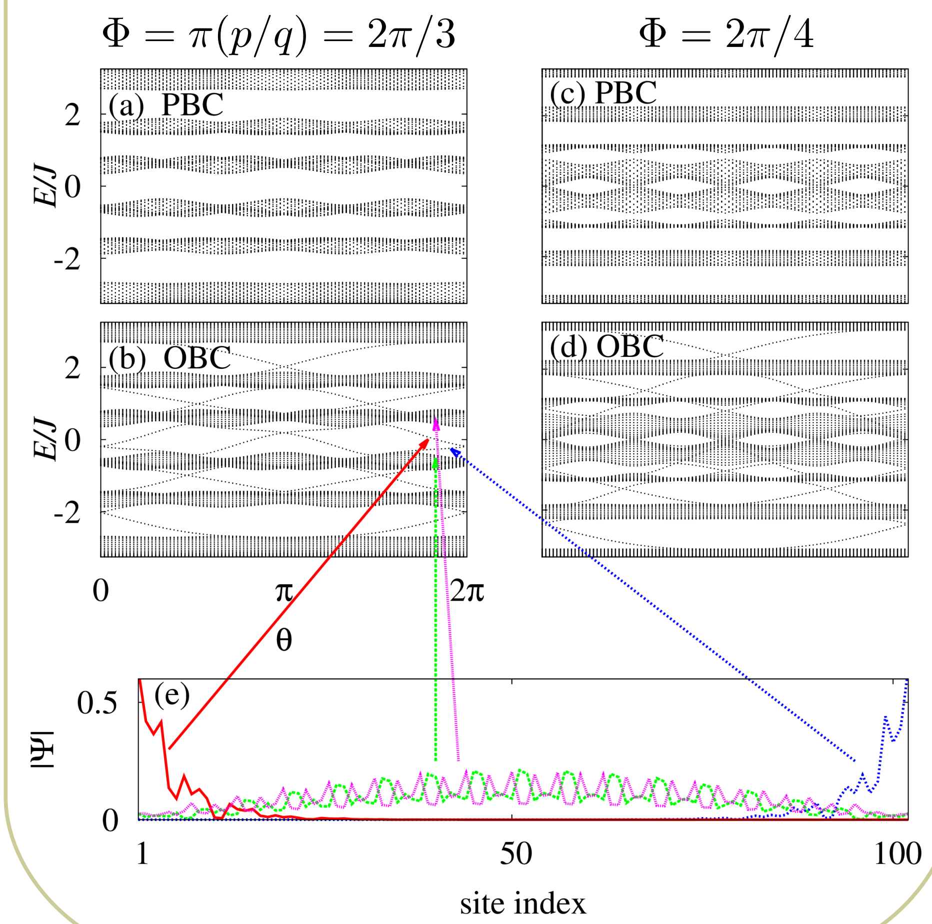

## Chern numbers (single-particle bands)

for bands parametrized by $k$ and $\theta$ at $\Phi = \dfrac{2\pi}{q}$

| $q$ | Chern numbers |
|---|---|
| 3 | -1, -1, 2, 2, -1, -1 |
| 4 | -1, -1, -1, 6, -1, -1, -1 |
| 5 | -1, -1, -1, -1, 4, 4, -1, -1, -1, -1 |

## Edge states

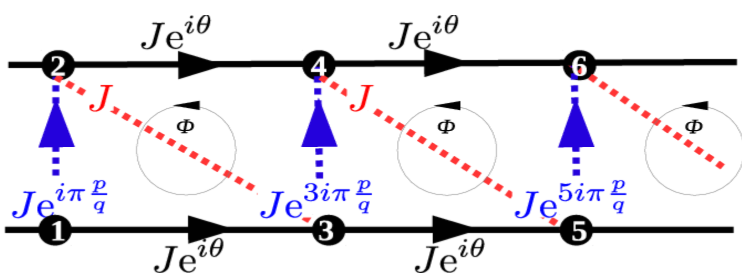
For 100 spins with a single spin flip



# Topological bands

## System

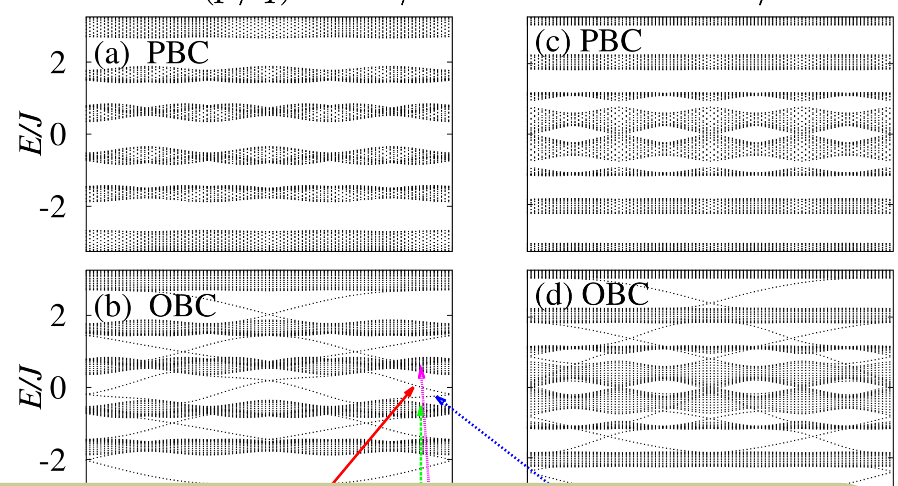
$$H = -\sum_{i\neq j} J_{ij}\sigma_i^+\sigma_j^- + h\sum_i \sigma_i^z$$




## Edge states

For 100 spins with a single spin flip

$\Phi = \pi(p/q) = 2\pi/3$ $\quad\quad$ $\Phi = 2\pi/4$




## Chern numbers (single-particle bands)

$$CN = \frac{i}{2\pi}\int \mathrm{d}\mu_1 \int \mathrm{d}\mu_2 (\langle\partial_1\Psi|\partial_2\Psi\rangle - \langle\partial_2\Psi|\partial_1\Psi\rangle)$$

for bands parametrized by $\mu_1 \equiv k$ and $\mu_2 \equiv \theta$ at $\Phi = \frac{2\pi}{q}$

| $q$ | Chern numbers |
|---|---|
| 3 | -1, -1, 2, 2, -1, -1 |
| 4 | -1, -1, -1, 6, -1, -1, -1 |
| 5 | -1, -1, -1, -1, 4, 4, -1, -1, -1, -1 |

# Classification of topology

PHYSICAL REVIEW B **78**, 195125 (2008)

## Classification of topological insulators and superconductors in three spatial dimensions

Andreas P. Schnyder,[1] Shinsei Ryu,[1] Akira Furusaki,[2] and Andreas W. W. Ludwig[3]

TABLE I. Ten symmetry classes of single-particle Hamiltonians classified in terms of the presence or absence of time-reversal symmetry (TRS) and particle-hole symmetry (PHS), as well as "sublattice" (or "chiral") symmetry (SLS) (Refs. 37 and 38). In the table, the absence of symmetries is denoted by "0." The presence of these symmetries is denoted by either "+1" or "−1," depending on whether the (antiunitary) operator implementing the symmetry at the level of the single-particle Hamiltonian squares to "+1" or "−1" (see text). [The index $\pm 1$ equals $\eta_c$ in Eq. (1b); here $\epsilon_c = +1$ and $-1$ for TRS and PHS, respectively.] For the first six entries of the table (which can be realized in nonsuperconducting systems), TRS$=+1$ when the SU(2) spin is an integer [called TRS (even) in the text] and TRS$=-1$ when it is a half-integer [called TRS (odd) in the text]. For the last four entries, the superconductor "Bogoliubov–de Gennes" (BdG) symmetry classes D, C, DIII, and CI, the Hamiltonian preserves SU(2) spin-1/2 rotation symmetry when PHS$=-1$ [called PHS (singlet) in the text], while it does not preserve SU(2) when PHS$=+1$ [called PHS (triplet) in the text]. The last three columns list all topologically non-trivial quantum ground states as a function of symmetry class and spatial dimension. The symbols $\mathbb{Z}$ and $\mathbb{Z}_2$ indicate whether the space of quantum ground states is partitioned into topological sectors labeled by an integer or a $\mathbb{Z}_2$ quantity, respectively.

| | | TRS | PHS | SLS | $d=1$ | $d=2$ | $d=3$ |
|---|---|---|---|---|---|---|---|
| Standard | A (unitary) | 0 | 0 | 0 | - | $\mathbb{Z}$ | - |
| (Wigner-Dyson) | AI (orthogonal) | +1 | 0 | 0 | - | - | - |
| | AII (symplectic) | −1 | 0 | 0 | - | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ |
| Chiral | AIII (chiral unitary) | 0 | 0 | 1 | $\mathbb{Z}$ | - | $\mathbb{Z}$ |
| (sublattice) | BDI (chiral orthogonal) | +1 | +1 | 1 | $\mathbb{Z}$ | - | - |
| | CII (chiral symplectic) | −1 | −1 | 1 | $\mathbb{Z}$ | - | $\mathbb{Z}_2$ |
| BdG | D | 0 | +1 | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}$ | - |
| | C | 0 | −1 | 0 | - | $\mathbb{Z}$ | - |
| | DIII | −1 | +1 | 1 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}$ |
| | CI | +1 | −1 | 1 | - | - | $\mathbb{Z}$ |

see also: A. Kitaev, AIP Conf. Proc. (2009)

# Classification of topology

PHYSICAL REVIEW B **78**, 195125 (2008)

## Classification of topological insulators and superconductors in three spatial dimensions

Andreas P. Schnyder,[1] Shinsei Ryu,[1] Akira Furusaki,[2] and Andreas W. W. Ludwig[3]

TABLE I. Ten symmetry classes of single-particle Hamiltonians classified in terms of the presence or absence of time-reversal symmetry (TRS) and particle-hole symmetry (PHS), as well as "sublattice" (or "chiral") symmetry (SLS) (Refs. 37 and 38). In the table, the absence of symmetries is denoted by "0." The presence of these symmetries is denoted by either "+1" or "−1," depending on whether the (antiunitary) operator implementing the symmetry at the level of the single-particle Hamiltonian squares to "+1" or "−1" (see text). [The index ±1 equals $\eta_c$ in Eq. (1b); here $\epsilon_c=+1$ and −1 for TRS and PHS, respectively.] For the first six entries of the table (which can be realized in nonsuperconducting systems), TRS=+1 when the SU(2) spin is an integer [called TRS (even) in the text] and TRS=−1 when it is a half-integer [called TRS (odd) in the text]. For the last four entries, the superconductor "Bogoliubov–de Gennes" (BdG) symmetry classes D, C, DIII, and CI, the Hamiltonian preserves SU(2) spin-1/2 rotation symmetry when PHS=−1 [called PHS (singlet) in the text], while it does not preserve SU(2) when PHS=+1 [called PHS (triplet) in the text]. The last three columns list all topologically non-trivial quantum ground states as a function of symmetry class and spatial dimension. The symbols $\mathbb{Z}$ and $\mathbb{Z}_2$ indicate whether the space of quantum ground states is partitioned into topological sectors labeled by an integer or a $\mathbb{Z}_2$ quantity, respectively.

| | | TRS | PHS | SLS | $d=1$ | $d=2$ | $d=3$ |
|---|---|---|---|---|---|---|---|
| Standard | A (unitary) | 0 | 0 | 0 | - | $\mathbb{Z}$ | - |
| (Wigner-Dyson) | AI (orthogonal) | +1 | 0 | 0 | - | - | - |
| | AII (symplectic) | −1 | 0 | 0 | - | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ |
| Chiral | AIII (chiral unitary) | 0 | 0 | 1 | $\mathbb{Z}$ | - | $\mathbb{Z}$ |
| (sublattice) | BDI (chiral orthogonal) | +1 | +1 | 1 | $\mathbb{Z}$ | - | - |
| | CII (chiral symplectic) | −1 | −1 | 1 | $\mathbb{Z}$ | - | $\mathbb{Z}_2$ |
| BdG | D | 0 | +1 | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}$ | - |
| | C | 0 | −1 | 0 | - | $\mathbb{Z}$ | - |
| | DIII | −1 | +1 | 1 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}$ |
| | CI | +1 | −1 | 1 | - | - | $\mathbb{Z}$ |

Even dimension:
Quantum Hall systems

Odd dimension:
No topological phases without symmetries

see also: A. Kitaev, AIP Conf. Proc. (2009)

# Classification of topology

PHYSICAL REVIEW B **78**, 195125 (2008)

## Classification of topological insulators and superconductors in three spatial dimensions

Andreas P. Schnyder,[1] Shinsei Ryu,[1] Akira Furusaki,[2] and Andreas W. W. Ludwig[3]

TABLE I. Ten symmetry classes of single-particle Hamiltonians classified in terms of the presence or absence of time-reversal symmetry (TRS) and particle-hole symmetry (PHS), as well as "sublattice" (or "chiral") symmetry (SLS) (Refs. 37 and 38). In the table, the absence of symmetries is denoted by "0." The presence of these symmetries is denoted by either "+1" or "−1," depending on whether the (antiunitary) operator implementing the symmetry at the level of the single-particle Hamiltonian squares to "+1" or "−1" (see text). [The index ±1 equals $\eta_c$ in Eq. (1b); here $\epsilon_c = +1$ and −1 for TRS and PHS, respectively.] For the first six entries of the table (which can be realized in nonsuperconducting systems), TRS = +1 when the SU(2) spin is an integer [called TRS (even) in the text] and TRS = −1 when it is a half-integer [called TRS (odd) in the text]. For the last four entries, the superconductor "Bogoliubov–de Gennes" (BdG) symmetry classes D, C, DIII, and CI, the Hamiltonian preserves SU(2) spin-1/2 rotation symmetry when PHS = −1 [called PHS (singlet) in the text], while it does not preserve SU(2) when PHS = +1 [called PHS (triplet) in the text]. The last three columns list all topologically non-trivial quantum ground states as a function of symmetry class and spatial dimension. The symbols $\mathbb{Z}$ and $\mathbb{Z}_2$ indicate whether the space of quantum ground states is partitioned into topological sectors labeled by an integer or a $\mathbb{Z}_2$ quantity, respectively.

| | | TRS | PHS | SLS | $d=1$ | $d=2$ | $d=3$ |
|---|---|---|---|---|---|---|---|
| Standard | A (unitary) | 0 | 0 | 0 | - | $\mathbb{Z}$ | - |
| (Wigner-Dyson) | AI (orthogonal) | +1 | 0 | 0 | - | - | - |
| | AII (symplectic) | −1 | 0 | 0 | - | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ |
| Chiral | AIII (chiral unitary) | 0 | 0 | 1 | $\mathbb{Z}$ | - | $\mathbb{Z}$ |

**Even dimension:**
Quantum Hall systems

**Odd dimension:**
No topological phases without symmetries

PHYSICAL REVIEW B **88**, 125118 (2013)

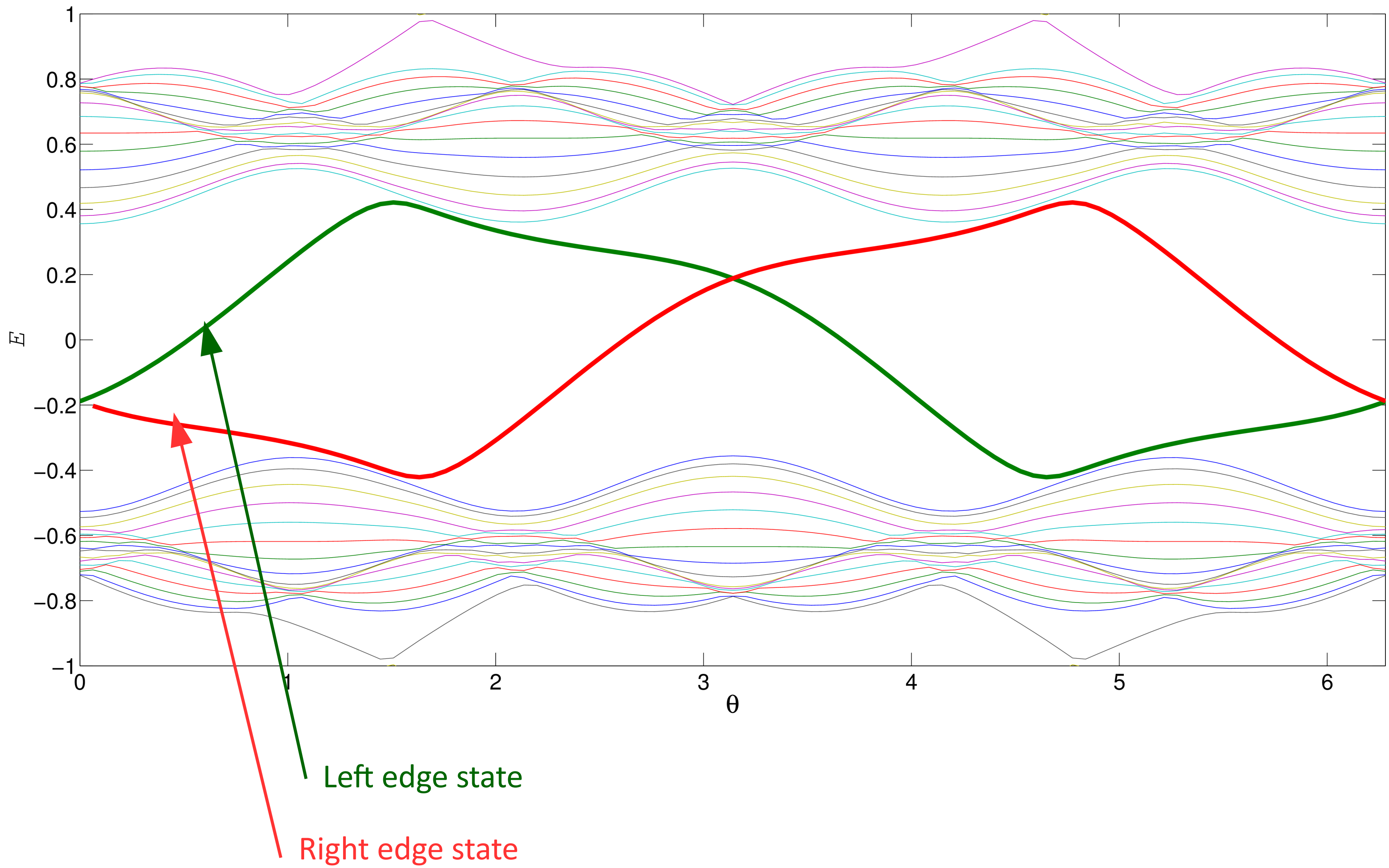
## Topological equivalence of crystal and quasicrystal band structures

Kevin A. Madsen, Emil J. Bergholtz, and Piet W. Brouwer

The familiy of 1D Hamiltonians *H(ϑ)* with periodic parameter *ϑ* (e.g. André-Aubry model) are classified as the corresponding 2D model.

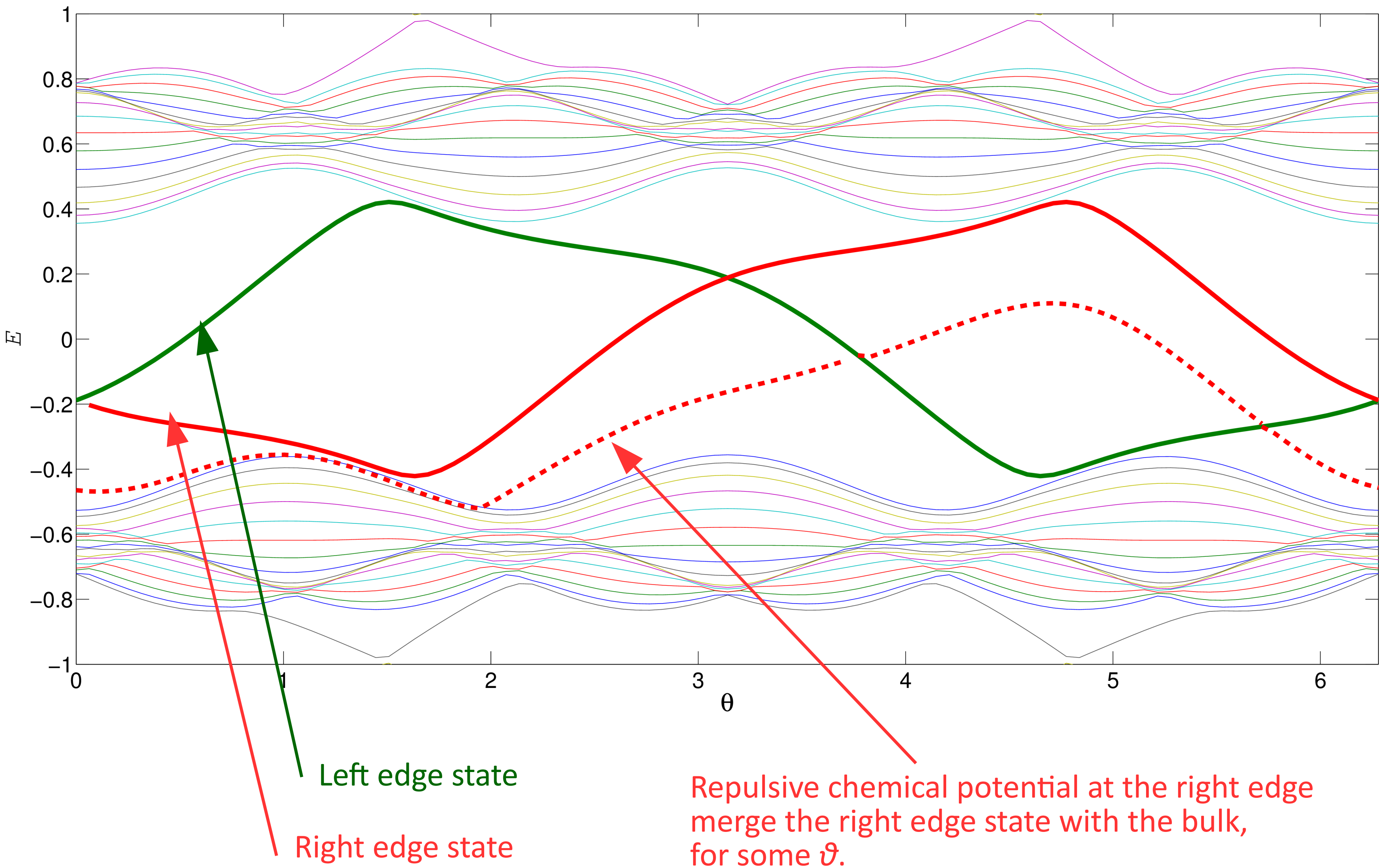
# Robustness of edge states

Edge states in central gap, for 102 spins and $p/q$=1/3



# Robustness of edge states

Edge states in central gap, for 102 spins and *p/q*=1/3



# Bosonic Chern Insulator

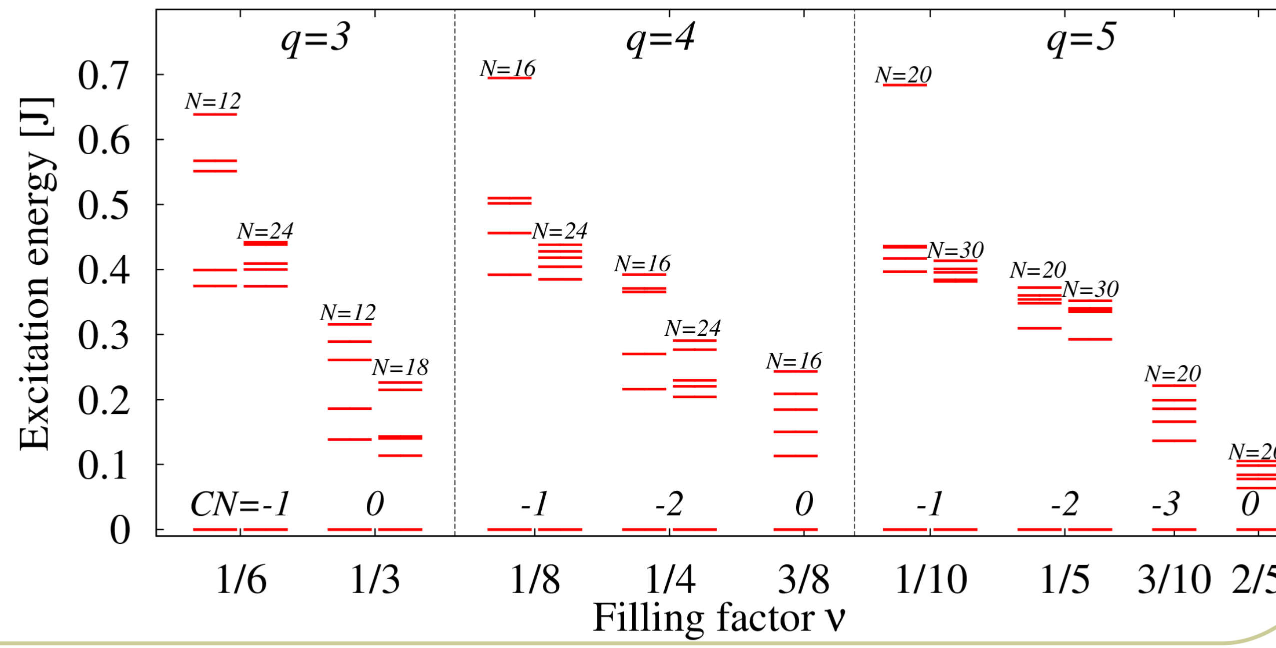
## Many-body Chern numbers

Winding with respect to twisted boundary conditions and phase Θ

flux $\Phi = \frac{2\pi}{q}$

filling $\nu = \frac{n}{2q},\ n \in \mathbb{N}$

polarization $S_z = N(1-2\nu)$




## Chern numbers (single-particle bands)

for bands parametrized by $k$ and $\theta$ at $\Phi = \frac{2\pi}{q}$

| $q$ | Chern numbers |
|---|---|
| 3 | -1, -1, 2, 2, -1, -1 |
| 4 | -1, -1, -1, 6, -1, -1, -1 |
| 5 | -1, -1, -1, -1, 4, 4, -1, -1, -1, -1 |

Small filling: Chern insulating behavior of bosons

Near half-filling: Trivial Mott insulator (1 spin per rung)

# Periodic driving

PRL **95,** 260404 (2005) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2005

**Superfluid-Insulator Transition in a Periodically Driven Optical Lattice**

André Eckardt, Christoph Weiss, and Martin Holthaus
*Institut für Physik, Carl von Ossietzky Universität, D-26111 Oldenburg, Germany*
(Received 16 August 2005; published 21 December 2005)

PRL **99,** 220403 (2007) PHYSICAL REVIEW LETTERS week ending 30 NOVEMBER 2007

**Dynamical Control of Matter-Wave Tunneling in Periodic Potentials**

H. Lignier, C. Sias, D. Ciampini, Y. Singh, A. Zenesini, O. Morsch, and E. Arimondo
*CNR-INFM, Dipartimento di Fisica "E. Fermi," Università di Pisa, Largo Pontecorvo 3, 56127 Pisa, Italy*

Selected for a Viewpoint in *Physics*
PRL **108,** 225304 (2012) PHYSICAL REVIEW LETTERS week ending 1 JUNE 2012

**Tunable Gauge Potential for Neutral and Spinless Particles in Driven Optical Lattices**

J. Struck,[1] C. Ölschläger,[1] M. Weinberg,[1] P. Hauke,[2] J. Simonet,[1] A. Eckardt,[3] M. Lewenstein,[2,4] K. Sengstock,[1,*] and P. Windpassinger[1]
[1]*Institut für Laserphysik, Universität Hamburg, D-22761 Hamburg, Germany*
[2]*Institut de Ciències Fotòniques, Mediterranean Technology Park, Av. Carl Friedrich Gauss 3, E-08860 Castelldefels, Barcelona, Spain*
[3]*Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Str. 38, D-01187 Dresden, Germany*
[4]*ICREA-Instituciò Catalana de Recerca i Estudis Avançats, Lluis Companys 23, E-08010 Barcelona, Spain*
(Received 29 February 2012; published 29 May 2012)

# Periodic driving

Apply the shaking ideas to spin chains in order to modify the interaction parameter:

- Strength of *J*
- Sign of *J*
- Complex phase of *J*

XY model with "shaken" field

$$H(t) = H_{\mathrm{XY}} + \sum_i v_i(t)\sigma_i^z \quad \text{with} \quad H_{\mathrm{XY}} = \sum_{i<j} J_{ij}(\sigma_i^+\sigma_j^- + \mathrm{h.c.})$$

Gauge transform (Floquet basis)

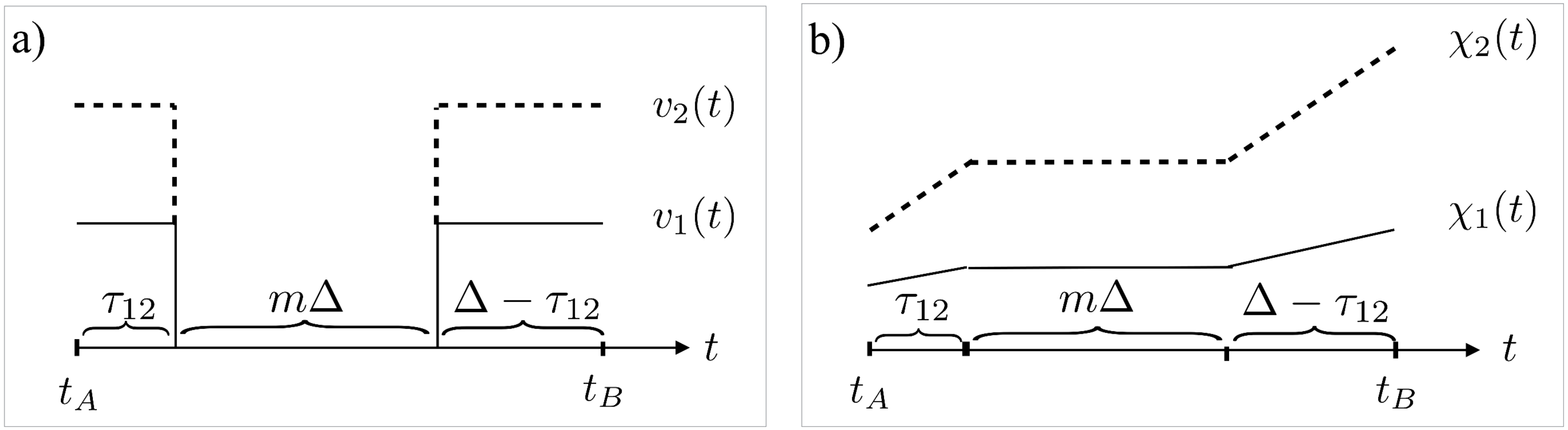
$$U(t) = e^{-i\sum_i \chi_i(t)\sigma_i^z} \quad \text{with} \quad \chi_i(t) = \int_0^t dt' v_i(t')$$

Average over period T

$$H_{\mathrm{eff}} = \sum_{i<j} J_{ij}^{\mathrm{eff}}(\sigma_i^+\sigma_j^- + \mathrm{h.c.}) \quad \text{where} \quad J_{ij}^{\mathrm{eff}} = \frac{\bar{J}_{ij}}{T}\int_0^T \mathrm{d}t e^{2i[\chi_i(t)-\chi_j(t)]}$$

# Periodic driving

Apply the shaking ideas to spin chains in order to modify the interaction parameter:




$$\Delta \cdot v_i(0) = n\pi \Rightarrow \int_{t_A}^{t_B} e^{i2[\chi_1(t')-\chi_2(t')]}dt = e^{i\varphi_{12}} m\Delta \quad \text{with} \quad \varphi_{12} = 2\tau_{12}(v_1(0)-v_2(0))$$

XY model with "shaken" field

$$H(t) = H_{\mathrm{XY}} + \sum_i v_i(t)\sigma_i^z \quad \text{with} \quad H_{\mathrm{XY}} = \sum_{i<j} J_{ij}(\sigma_i^+\sigma_j^- + \mathrm{h.c.})$$

Gauge transform (Floquet basis)

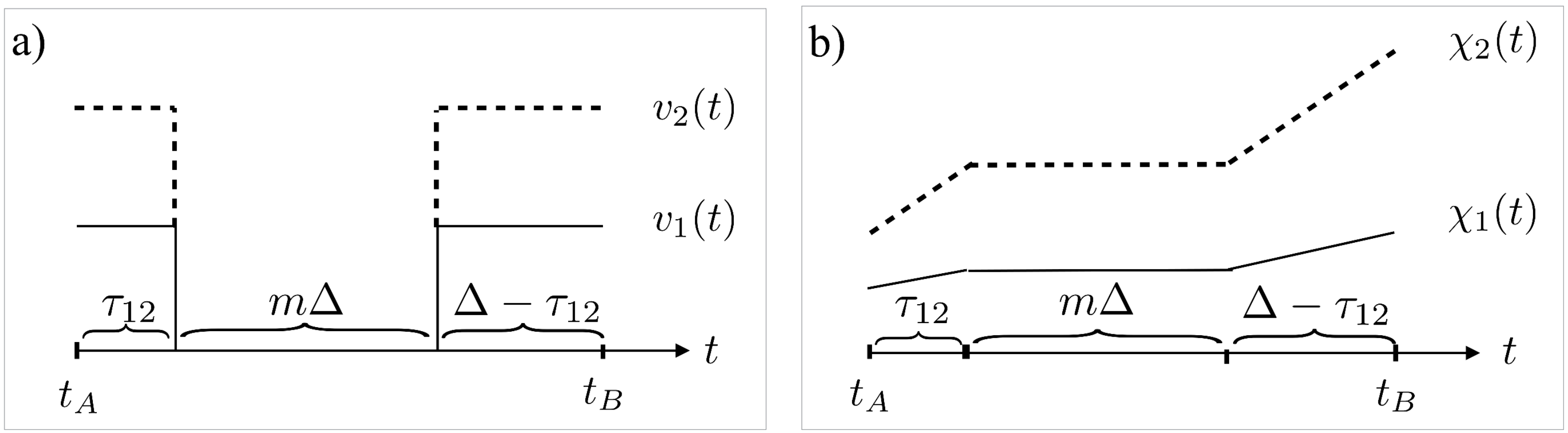
$$U(t) = e^{-i\sum_i \chi_i(t)\sigma_i^z} \quad \text{with} \quad \chi_i(t) = \int_0^t dt' v_i(t')$$

Average over period T

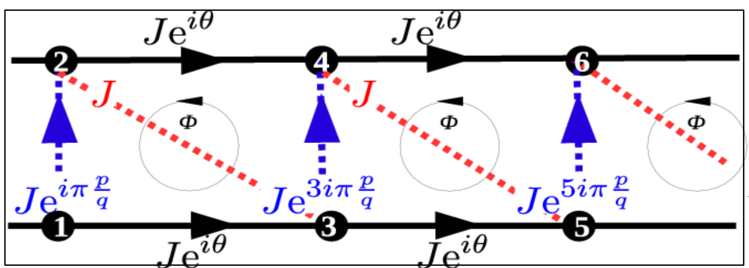
$$H_{\mathrm{eff}} = \sum_{i<j} J_{ij}^{\mathrm{eff}}(\sigma_i^+\sigma_j^- + \mathrm{h.c.}) \quad \text{where} \quad J_{ij}^{\mathrm{eff}} = \frac{\bar{J}_{ij}}{T}\int_0^T \mathrm{d}t e^{2i[\chi_i(t)-\chi_j(t)]}$$

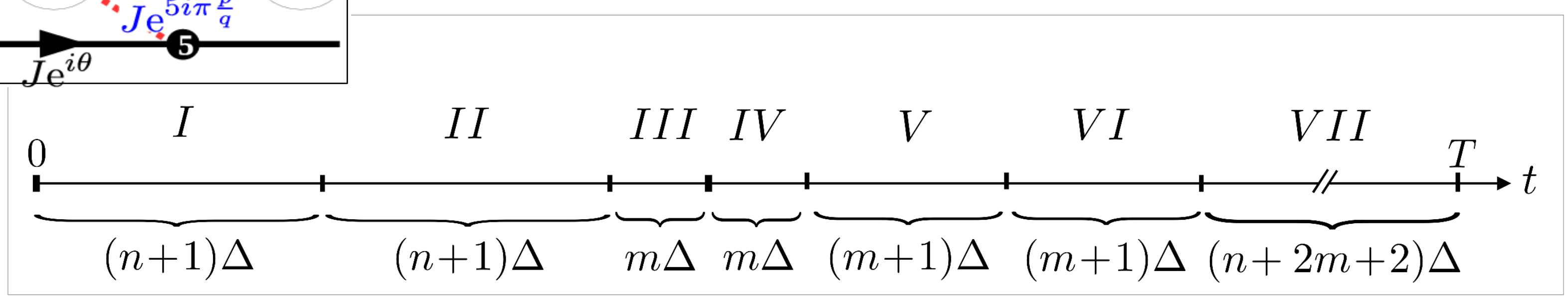
# Periodic driving

Apply the shaking ideas to spin chains in order to modify the interaction parameter:




$$\Delta \cdot v_i(0) = n\pi \Rightarrow \int_{t_A}^{t_B} e^{i2[\chi_1(t') - \chi_2(t')]} dt = e^{i\varphi_{12}} m\Delta \quad \text{with} \quad \varphi_{12} = 2\tau_{12}(v_1(0) - v_2(0))$$

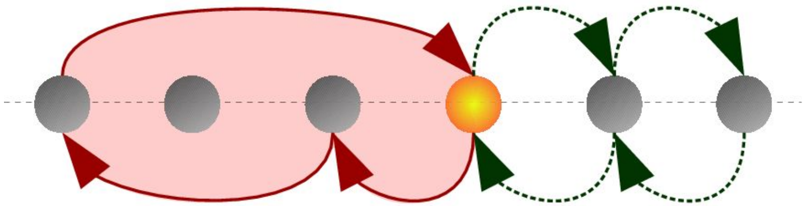


# Summary

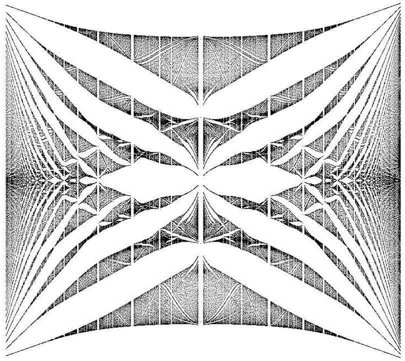
Idee:

- No loops with magnetic flux in short-ranged chains
- Long-range connections allow for loops with flux



Realization:

- Long-range spin chains, e.g. trapped ions or atoms coupled to nanophotonic devices
- Design of complex-valued interactions parameters via shaking



Results:

- Fractal energy spectrum
- Topological band structure
- Bosonic Chern insulator

Phys. Rev. A **91**, 063612 (2015)
Tobias Grass, Christine Muschik, Alessio Celi, Ravindra Chhajlany, Maciej Lewenstein