Uni Köln 27/01/2016

Make ions count: solving computational problems via quantum simulation

Tobias Grass (ICFO - Barcelona)

In collaboration with:

Christian Gogolin (ICFO+MPQ)

Bruno Julía-Díaz (U Barcelona)

Maciej Lewenstein (ICFO)

David Raventós (ICFO)

Modern history of trapped ions

Modern history of trapped ions

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*
Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
(Received 30 November 1994)

VOLUME 75, NUMBER 25 PHYSICAL REVIEW LETTERS 18 DECEMBER 1995

Demonstration of a Fundamental Quantum Logic Gate

C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland
National Institute of Standards and Technology, Boulder, Colorado 80303
(Received 14 July 1995)

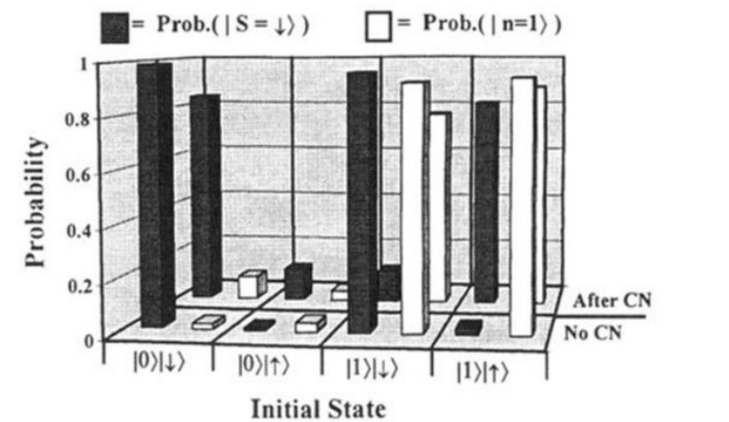


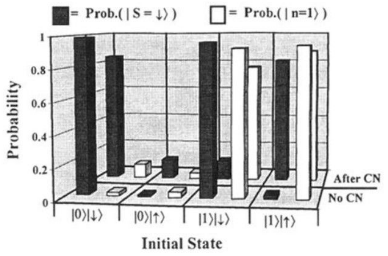
FIG. 2. Controlled-NOT (CN) truth table measurements for eigenstates. The two horizontal rows give measured final

1995 2004 08 10 12 14

Modern history of trapped ions

1995 2004 08 10 12 14

Quantum logic gates



Modern history of trapped ions

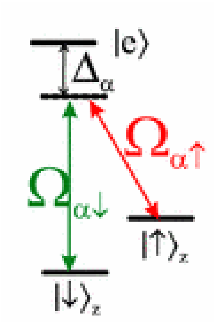
VOLUME 92, NUMBER 20 PHYSICAL REVIEW LETTERS week ending 21 MAY 2004

Effective Quantum Spin Systems with Trapped Ions

D. Porras* and J. I. Cirac†

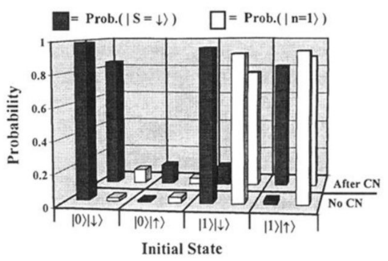
Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, Garching, D-85748, Germany

(Received 16 January 2004; published 20 May 2004)



1995 2004 08 10 12 14

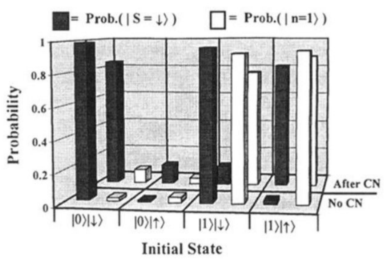
Quantum logic gates



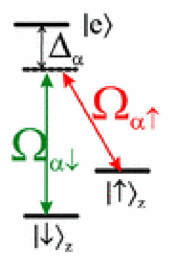
Modern history of trapped ions

1995 2004 08 10 12 14

Quantum logic gates



Spin chains



Modern history of trapped ions

LETTERS

Simulating a quantum magnet with trapped ions

A. FRIEDENAUER*, H. SCHMITZ*, J. T. GLUECKERT, D. PORRAS AND T. SCHAETZ†
Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, D-85748 Garching, Germany

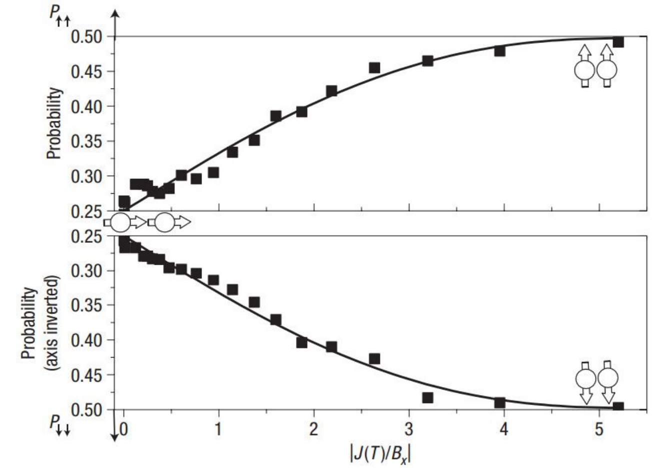
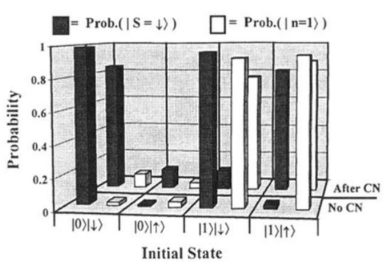


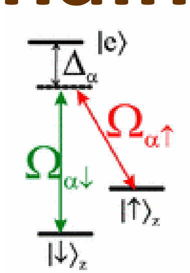
Figure 3 Quantum magnetization of the spin system. We initialize the spins in the

1995 2004 08 10 12 14

Quantum logic gates



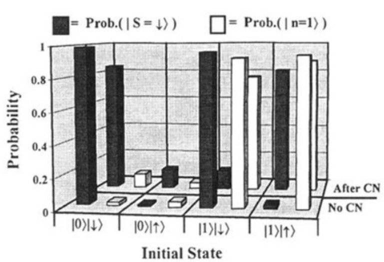
Spin chains



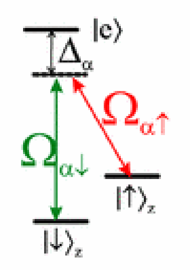
Modern history of trapped ions

1995 2004 08 **10** 12 14

Quantum logic gates



Spin chains



Ising spins

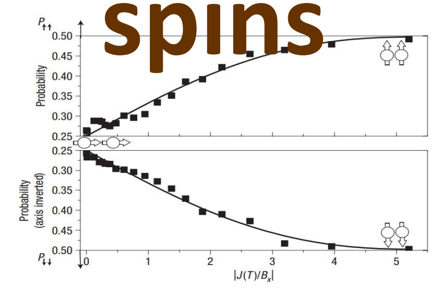


Figure 3 Quantum magnetization of the spin system. We initialize the spins in the

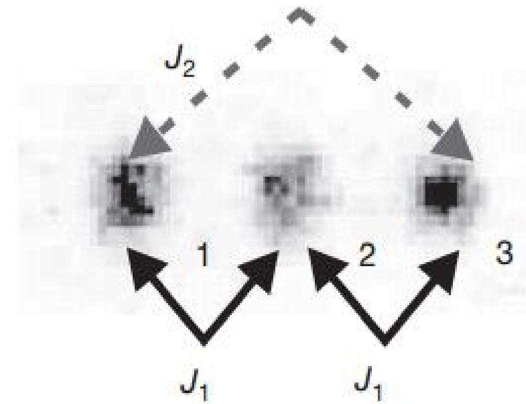
Modern history of trapped ions

nature Vol 465|3 June 2010|doi:10.1038/nature09071

LETTERS

Quantum simulation of frustrated Ising spins with trapped ions

K. Kim[1], M.-S. Chang[1], S. Korenblit[1], R. Islam[1], E. E. Edwards[1], J. K. Freericks[2], G.-D. Lin[3], L.-M. Duan[3] & C. Monroe[1]

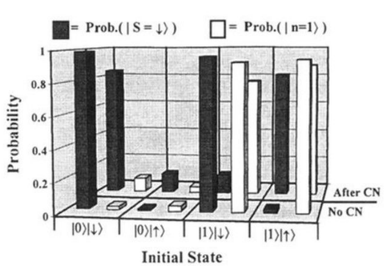


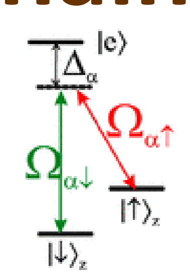
1995 2004 08 **10** 12 14

Quantum logic gates

Spin chains

Ising spins





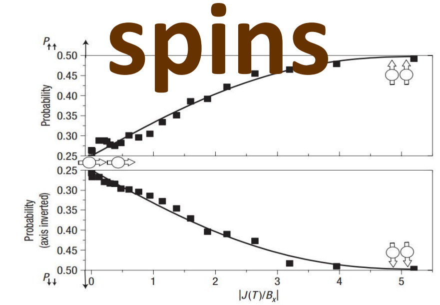
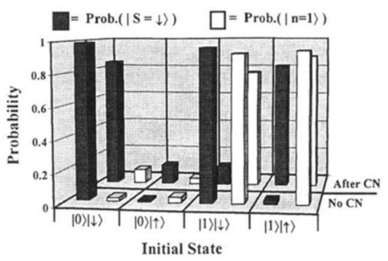


Figure 3 **Quantum magnetization of the spin system.** We initialize the spins in the

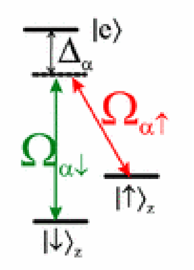
Modern history of trapped ions

1995 2004 08 10 **12** 14

Quantum logic gates



Spin chains



Ising spins

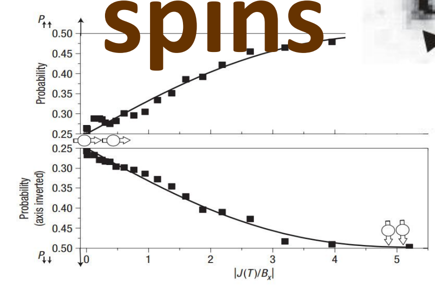
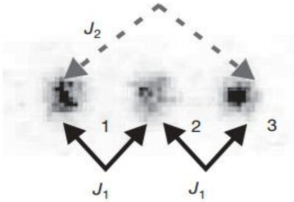


Figure 3 Quantum magnetization of the spin system. We initialize the spins in the



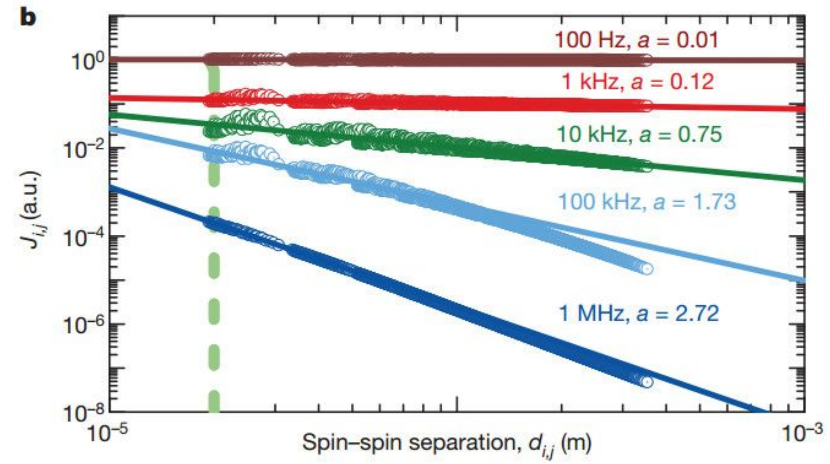
Modern history of trapped ions

LETTER

doi:10.1038/nature10981

Engineered two-dimensional Ising interactions in a trapped-ion quantum simulator with hundreds of spins

Joseph W. Britton[1], Brian C. Sawyer[1], Adam C. Keith[2,3], C.-C. Joseph Wang[2], James K. Freericks[2], Hermann Uys[4], Michael J. Biercuk[5] & John J. Bollinger[1]

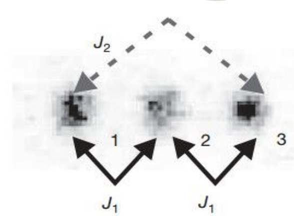


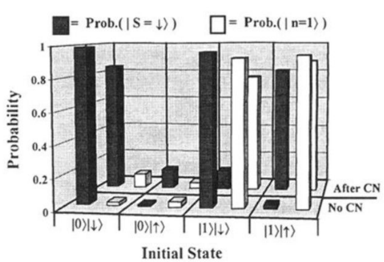
1995 2004 08 10 **12** 14

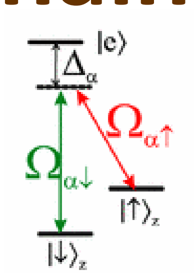
Quantum logic gates

Spin chains

Ising spins







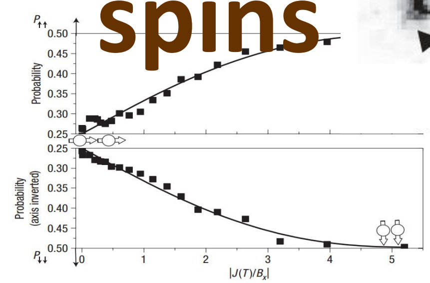
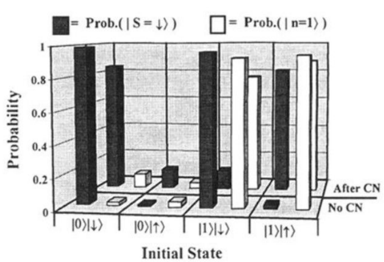


Figure 3 **Quantum magnetization of the spin system.** We initialize the spins in the

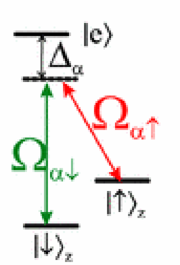
Modern history of trapped ions

1995 2004 08 10 12 14

Quantum logic gates



Spin chains



Ising spins

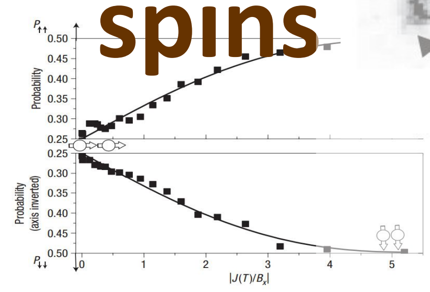
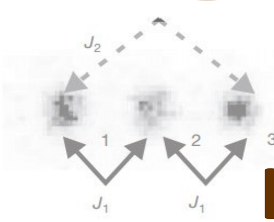
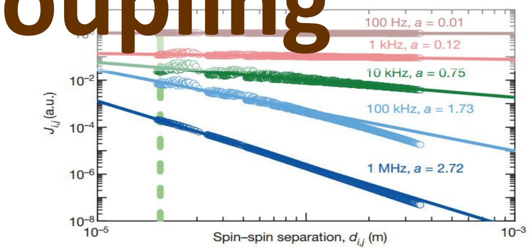


Figure 3 Quantum magnetization of the spin system. We initialize the spins in the



Long range coupling



Modern history of trapped ions

LETTER

doi:10.1038/nature13461

Quasiparticle engineering and entanglement propagation in a quantum many-body system

P. Jurcevic[1,2]*, B. P. Lanyon[1,2]*, P. Hauke[1,3], C. Hempel[1,2], P. Zoller[1,3], R. Blatt[1,2] & C. F. Roos[1,2]

LETTER

doi:10.1038/nature13450

Non-local propagation of correlations in quantum systems with long-range interactions

Philip Richerme[1], Zhe-Xuan Gong[1], Aaron Lee[1], Crystal Senko[1], Jacob Smith[1], Michael Foss-Feig[1], Spyridon Michalakis[2], Alexey V. Gorshkov[1] & Christopher Monroe[1]

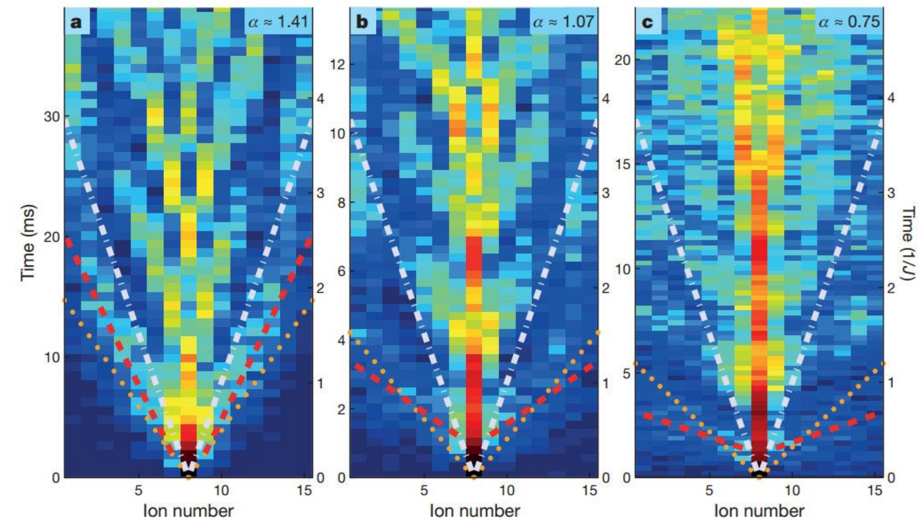
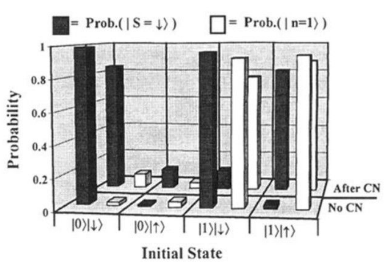


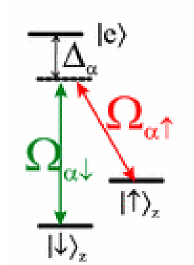
Figure 4 | Measured quantum dynamics for increasing spin–spin interaction ranges. a–c, Measured magnetization $\langle\sigma_i^z(t)\rangle$ (colour coded) … lines, Gaussian fits to measured mag… neighbour Lieb–Robinson bound ca…

1995 2004 08 10 12 14

Quantum logic gates



Spin chains



Ising spins

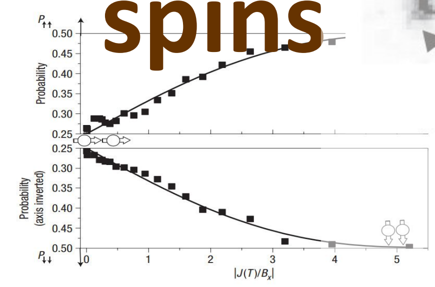
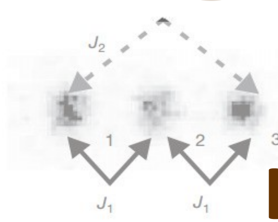
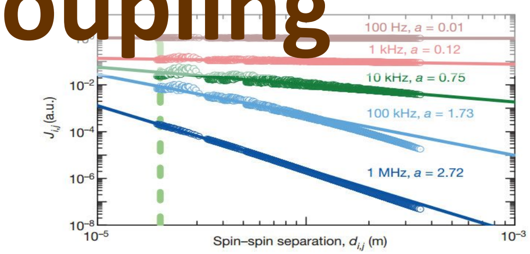


Figure 3 Quantum magnetization of the spin system. We initialize the spins in the

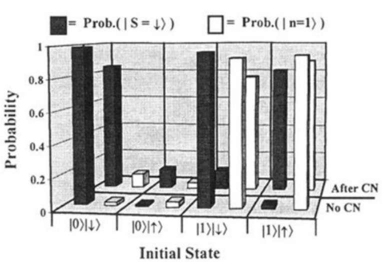


Long range coupling

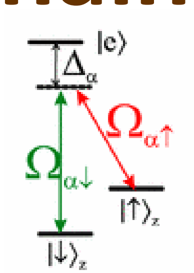


Modern history of trapped ions

1995 — Quantum logic gates



2004 — Spin chains



08 — Ising spins

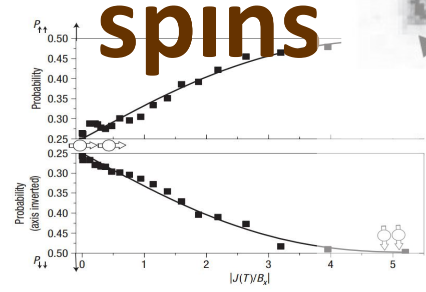
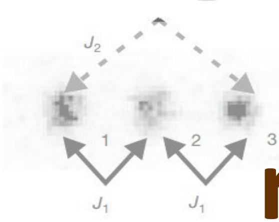


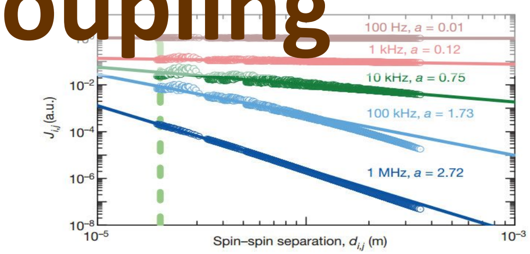
Figure 3 Quantum magnetization of the spin system. We initialize the spins in the

10



12 — Long range coupling

14 — Dyna- mics



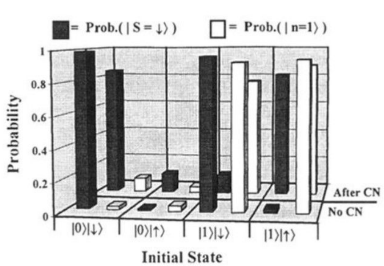
Modern history of trapped ions

Flexible emulator of spin models:

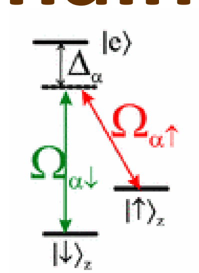
→ tunable interactions
→ good access to many observables
→ microscopic systems

1995 2004 08 10 12 14

Quantum logic gates



Spin chains



Ising spins

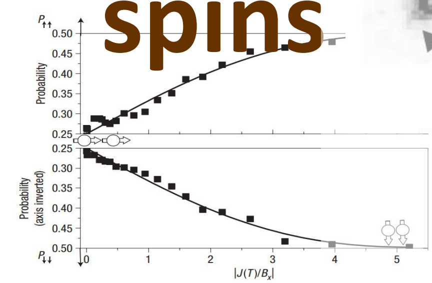
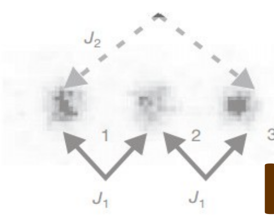
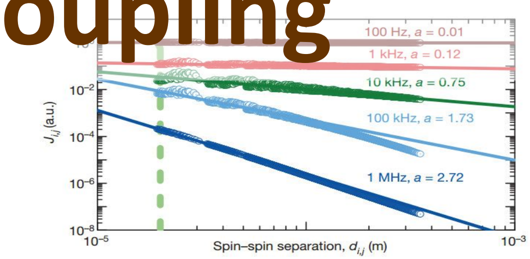


Figure 3 Quantum magnetization of the spin system. We initialize the spins in the



Long range coupling

Dyna- mics



My work on ions

SU(3) models and quantum chaos

PRL **111,** 090404 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Quantum Chaos in SU(3) Models with Trapped Ions

Tobias Graß,[1] Bruno Juliá-Díaz,[1,2] Marek Kuś,[3] and Maciej Lewenstein[1,4]
[1]*ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain*
[2]*Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, 08028 Barcelona, Spain*
[3]*Center for Theoretical Physics, Polish Academy of Sciences, 02-668 Warszawa, Poland*
[4]*ICREA—Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*
(Received 29 May 2013; published 28 August 2013)

Artificial magnetic fluxes

PHYSICAL REVIEW A **91**, 063612 (2015)

Synthetic magnetic fluxes and topological order in one-dimensional spin systems

Tobias Graß,[1] Christine Muschik,[1,2,3] Alessio Celi,[1] Ravindra W. Chhajlany,[1,4] and Maciej Lewenstein[1,5]
[1]*ICFO-Institut de Ciències Fotòniques, Av. Carl Friedrich Gauss 3, 08860 Barcelona, Spain*
[2]*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria*
[3]*Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria*
[4]*Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*
[5]*ICREA-Institució Catalana de Recerca i Estudis Avançats, Lluis Campanys 23, 08010 Barcelona, Spain*
(Received 13 January 2015; revised manuscript received 8 April 2015; published 11 June 2015)

Heisenberg models

Graß and Lewenstein *EPJ Quantum Technology* 2014, 1:8
http://www.epjquantumtechnology.com/content/1/1/8
EPJ .org
EPJ Quantum Technology
a SpringerOpen Journal
RESEARCH Open Access

Trapped-ion quantum simulation of tunable-range Heisenberg chains

Tobias Graß[1*] and Maciej Lewenstein[1,2]

Mattis glass and number partitioning

arXiv.org > cond-mat > arXiv:1507.07863
Condensed Matter > Quantum Gases

Controlled complexity in trapped ions: from quantum Mattis glasses to number partitioning

Tobias Graß, David Raventós, Bruno Juliá-Díaz, Christian Gogolin, Maciej Lewenstein

(Submitted on 28 Jul 2015)

My work on ions

SU(3) models and quantum chaos

PRL 111, 090404 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Quantum Chaos in SU(3) Models with Trapped Ions

Tobias Graß,[1] Bruno Juliá-Díaz,[1,2] Marek Kuś,[3] and Maciej Lewenstein[1,4]
[1]ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain
[2]Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, 08028 Barcelona, Spain
[3]Center for Theoretical Physics, Polish Academy of Sciences, 02-668 Warszawa, Poland
[4]ICREA—Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain
(Received 29 May 2013; published 28 August 2013)

Artificial magnetic fluxes

PHYSICAL REVIEW A 91, 063612 (2015)

Synthetic magnetic fluxes and topological order in one-dimensional spin systems

Tobias Graß,[1] Christine Muschik,[1,2,3] Alessio Celi,[1] Ravindra W. Chhajlany,[1,4] and Maciej Lewenstein[1,5]
[1]ICFO-Institut de Ciències Fotòniques, Av. Carl Friedrich Gauss 3, 08860 Barcelona, Spain
[2]Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria
[3]Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria
[4]Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland
[5]ICREA-Institució Catalana de Recerca i Estudis Avançats, Lluis Campanys 23, 08010 Barcelona, Spain
(Received 13 January 2015; revised manuscript received 8 April 2015; published 11 June 2015)

Heisenberg models

Graß and Lewenstein *EPJ Quantum Technology* 2014, 1:8
http://www.epjquantumtechnology.com/content/1/1/8

EPJ .org

EPJ Quantum Technology
a SpringerOpen Journal

RESEARCH Open Access

Trapped-ion quantum simulation of tunable-range Heisenberg chains

Tobias Graß[1*] and Maciej Lewenstein[1,2]

Mattis glass and number partitioning

arXiv.org > cond-mat > arXiv:1507.07863

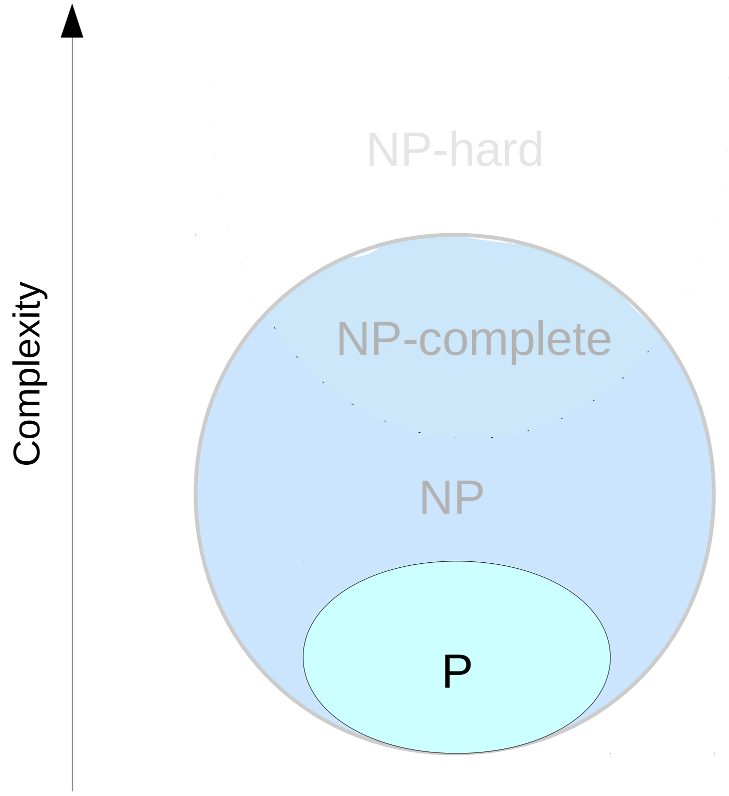
Condensed Matter > Quantum Gases

Controlled complexity in trapped ions: from quantum Mattis glasses to number partitioning

Tobias Graß, David Raventós, Bruno Juliá-Díaz, Christian Gogolin, Maciej Lewenstein

(Submitted on 28 Jul 2015)

Complexity classes



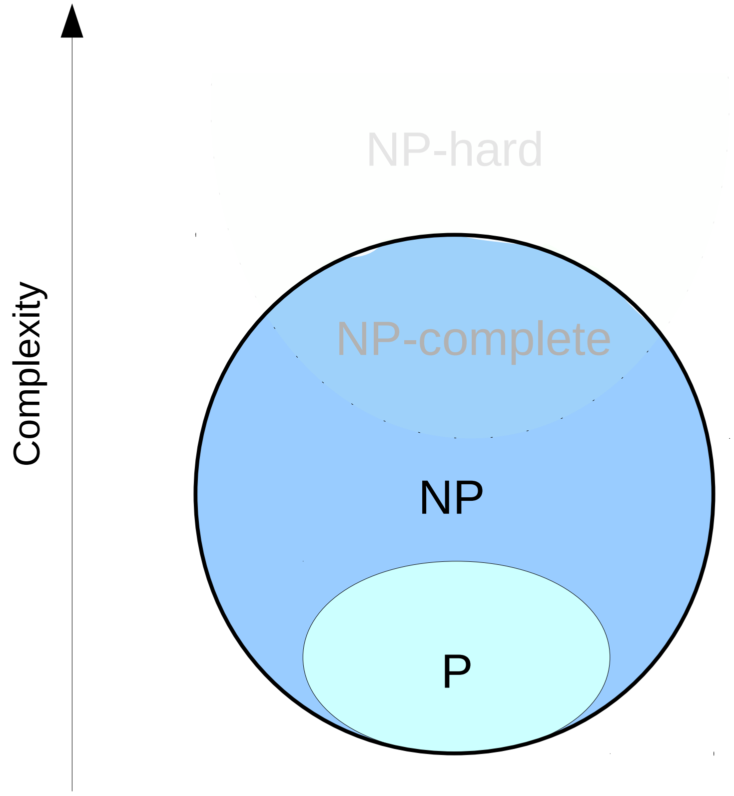
NP-hard: Problems at least as hard as NP-complete problems, but not necessarily in NP

NP-complete: "Hardest" problems in NP (to which any NP problem can be mapped in polynomial time)

NP: Decision problems which can be *solved* on a **non-deterministic** computer (or whose positive answer can be *verified* on a deterministic computer) in polynomial time

P: Decision problems solvable on a deterministic computer in polynomial time

Complexity classes



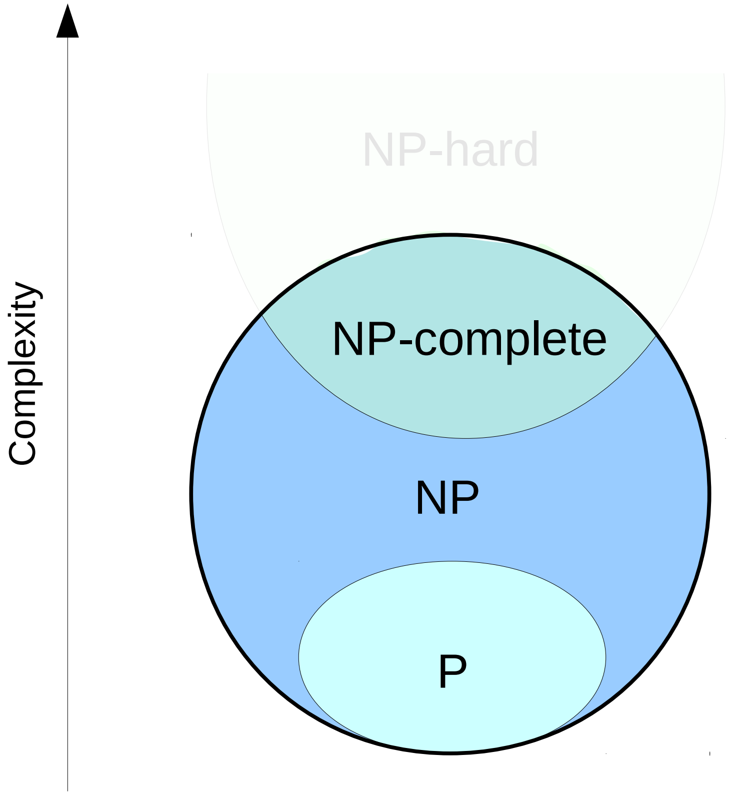
NP-hard: Problems at least as hard as NP-complete problems, but not necessarily in NP

NP-complete: "Hardest" problems in NP (to which any NP problem can be mapped in polynomial time)

NP: Decision problems which can be *solved* on a **non-deterministic** computer (or whose positive answer can be *verified* on a deterministic computer) in polynomial time

P: Decision problems solvable on a deterministic computer in polynomial time

Complexity classes



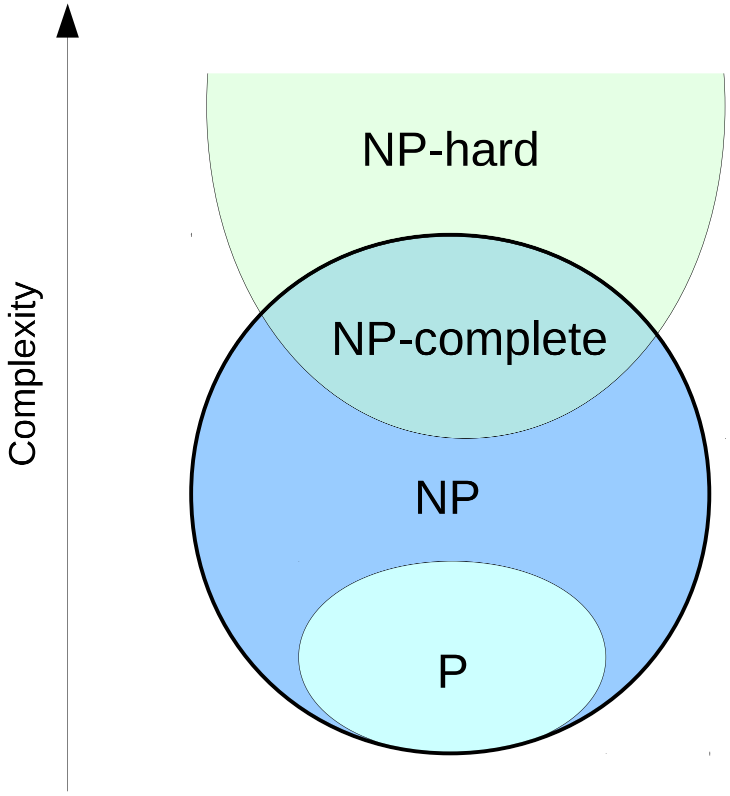
NP-hard: Problems at least as hard as NP-complete problems, but not necessarily in NP

NP-complete: "Hardest" problems in NP (to which any NP problem can be mapped in polynomial time)

NP: Decision problems which can be *solved* on a **non-deterministic** computer (or whose positive answer can be *verified* on a deterministic computer) in polynomial time

P: Decision problems solvable on a deterministic computer in polynomial time

Complexity classes



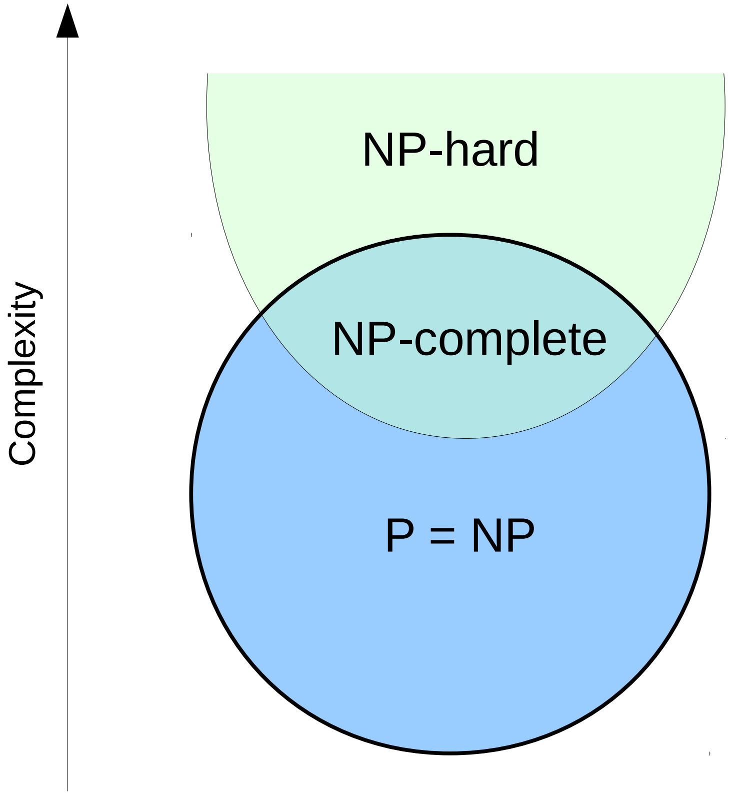
NP-hard: Problems at least as hard as NP-complete problems, but not necessarily in NP

NP-complete: "Hardest" problems in NP (to which any NP problem can be mapped in polynomial time)

NP: Decision problems which can be *solved* on a **non-deterministic** computer (or whose positive answer can be *verified* on a deterministic computer) in polynomial time

P: Decision problems solvable on a deterministic computer in polynomial time

Complexity classes



NP-hard: Problems at least as hard as NP-complete problems, but not necessarily in NP

NP-complete: “Hardest” problems in NP (to which any NP problem can be mapped in polynomial time)

NP: Decision problems which can be *solved* on a **non-deterministic** computer (or whose positive answer can be *verified* on a deterministic computer) in polynomial time

P: Decision problems solvable on a deterministic computer in polynomial time

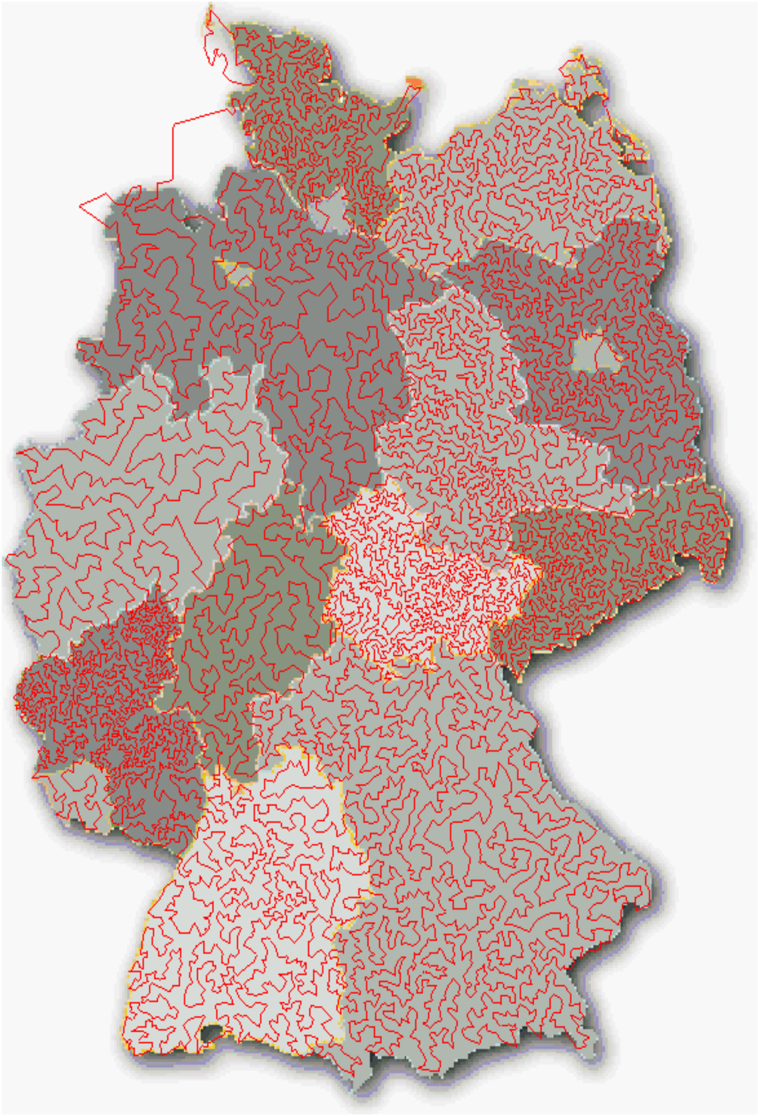
P = NP ?

NP-hard / NP-complete examples

* Traveling salesman problem

15,112 cities in Germany
(2001 world record)

Computation time: 23 CPU yrs.



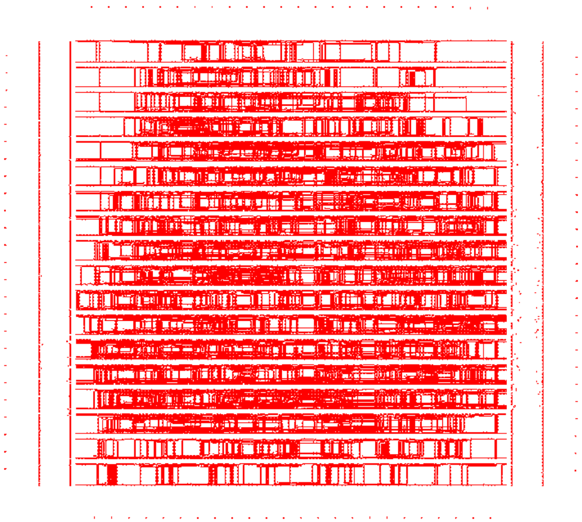
NP-hard / NP-complete examples

* Traveling salesman problem

85,900 connections on a computer chip

(Current world record)

Computation time: 136 CPU yrs.

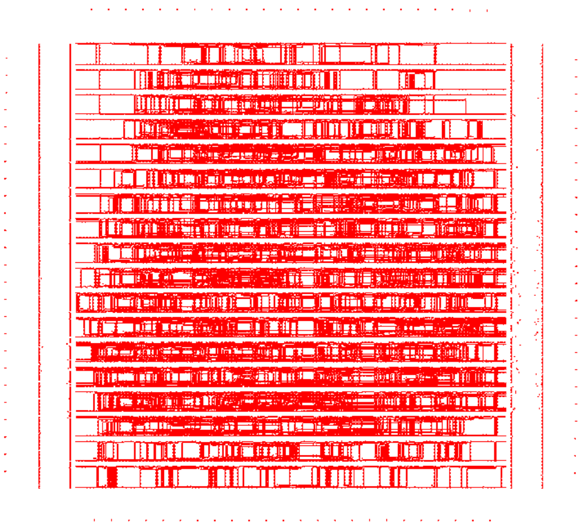


NP-hard / NP-complete examples

* Traveling salesman problem

85,900 connections on a computer chip
(Current world record)

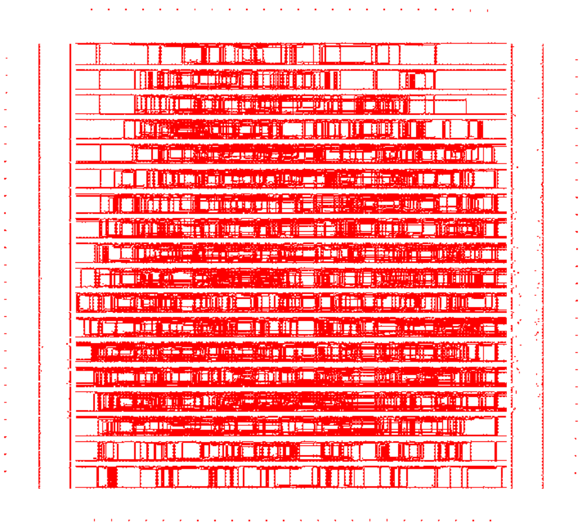
Computation time: 136 CPU yrs.



* Number partitioning

2 6 7 9 12 13 17 20

NP-hard / NP-complete examples



* Traveling salesman problem

  85,900 connections on a computer chip
  (Current world record)

  Computation time: 136 CPU yrs.

* Number partitioning

2 6 7 9 12 13 17 20

2 6 7 9 12 13 17 20

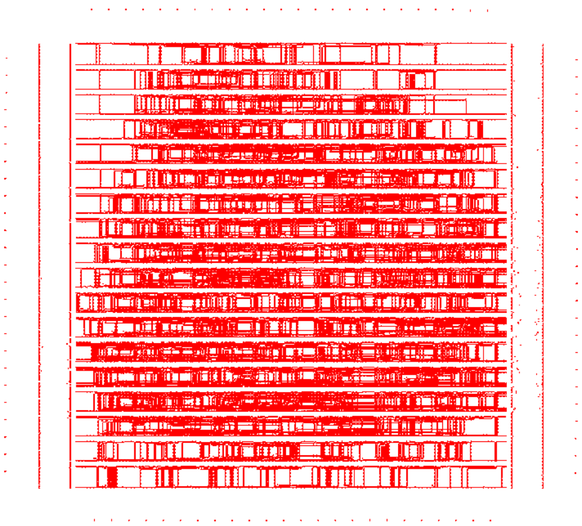
2+9+12+20 -6 -7-13 -17 = 0

NP-hard / NP-complete examples

* Traveling salesman problem

85,900 connections on a computer chip
(Current world record)

Computation time: 136 CPU yrs.



* Number partitioning

2 6 7 9 12 13 17 20

2 6 7 9 12 13 17 20
2 6 7 9 12 13 17 20

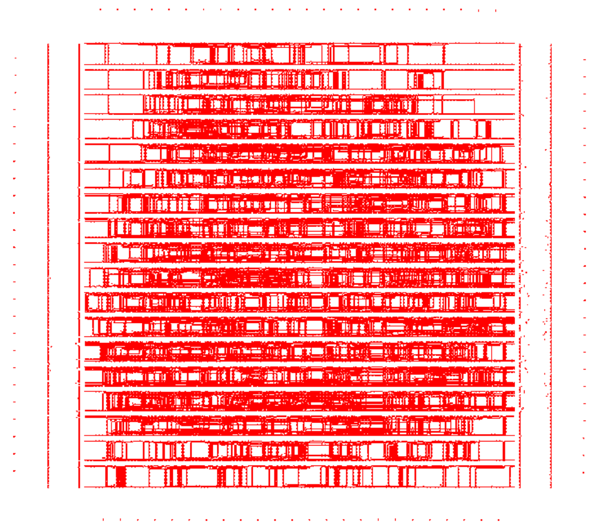
2+9+12+20 -6 -7-13 -17 = 0
6+17+20 -2 -7 -9 -12-13 = 0

NP-hard / NP-complete examples

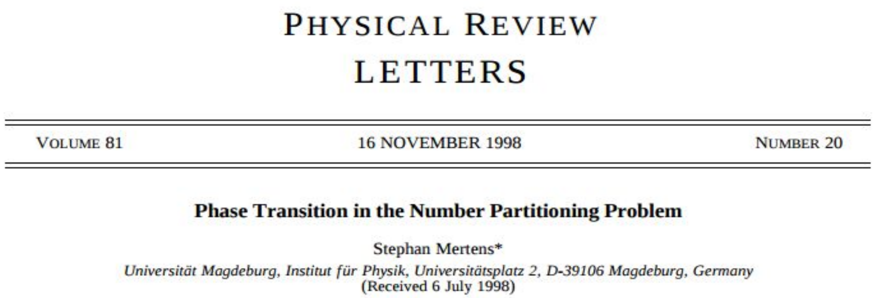
* Traveling salesman problem

85,900 connections on a computer chip
(Current world record)

Computation time: 136 CPU yrs.



* Number partitioning



PHYSICAL REVIEW LETTERS

VOLUME 81 16 NOVEMBER 1998 NUMBER 20

Phase Transition in the Number Partitioning Problem

Stephan Mertens*
Universität Magdeburg, Institut für Physik, Universitätsplatz 2, D-39106 Magdeburg, Germany
(Received 6 July 1998)

2 6 7 9 12 13 17 20

2 6 7 9 12 13 17 20
2 6 7 9 12 13 17 20

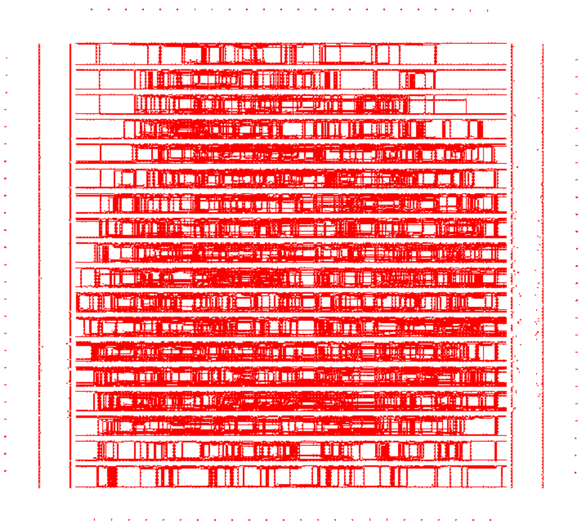
2+9+12+20 -6 -7-13 -17 = 0
6+17+20 -2 -7 -9 -12-13 = 0

NP-hard / NP-complete examples

* Traveling salesman problem

  85,900 connections on a computer chip
  (Current world record)

  Computation time: 136 CPU yrs.



* Number partitioning

PHYSICAL REVIEW LETTERS

VOLUME 81 16 NOVEMBER 1998 NUMBER 20

Phase Transition in the Number Partitioning Problem

Stephan Mertens*
Universität Magdeburg, Institut für Physik, Universitätsplatz 2, D-39106 Magdeburg, Germany
(Received 6 July 1998)

2 6 7 9 12 13 17 20

2 6 7 9 12 13 17 20

2 6 7 9 12 13 17 20

2+9+12+20 -6 -7-13 -17 = 0

6+17+20 -2 -7 -9 -12-13 = 0

* Spin models with random couplings (aka spin glasses)

Journal of Physics A: Mathematical and General

Journal of Physics A: Mathematical and General > Volume 15 > Number 10

On the computational complexity of Ising spin glass models

F Barahona
Show affiliations

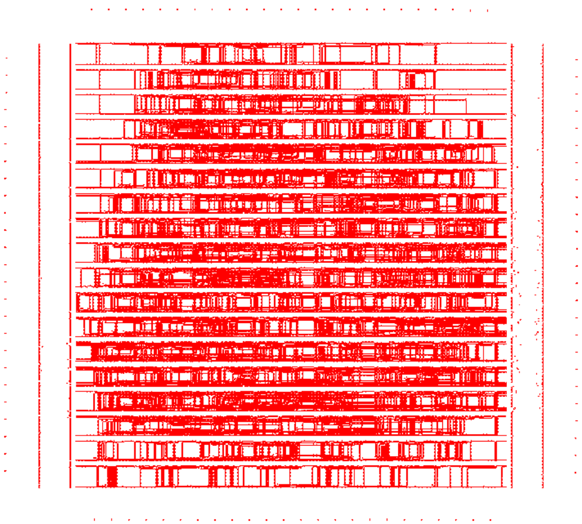
F Barahona 1982 *J. Phys. A: Math. Gen.* **15** 3241. doi:10.1088/0305-4470/15/10/028

NP-hard / NP-complete examples

* Traveling salesman problem

85,900 connections on a computer chip
(Current world record)

Computation time: 136 CPU yrs.



* Number partitioning

PHYSICAL REVIEW
LETTERS

VOLUME 81 16 NOVEMBER 1998 NUMBER 20

Phase Transition in the Number Partitioning Problem

Stephan Mertens*
Universität Magdeburg, Institut für Physik, Universitätsplatz 2, D-39106 Magdeburg, Germany
(Received 6 July 1998)

2 6 7 9 12 13 17 20

2 6 7 9 12 13 17 20
2 6 7 9 12 13 17 20

2+9+12+20 -6 -7-13 -17 = 0
6+17+20 -2 -7 -9 -12-13 = 0

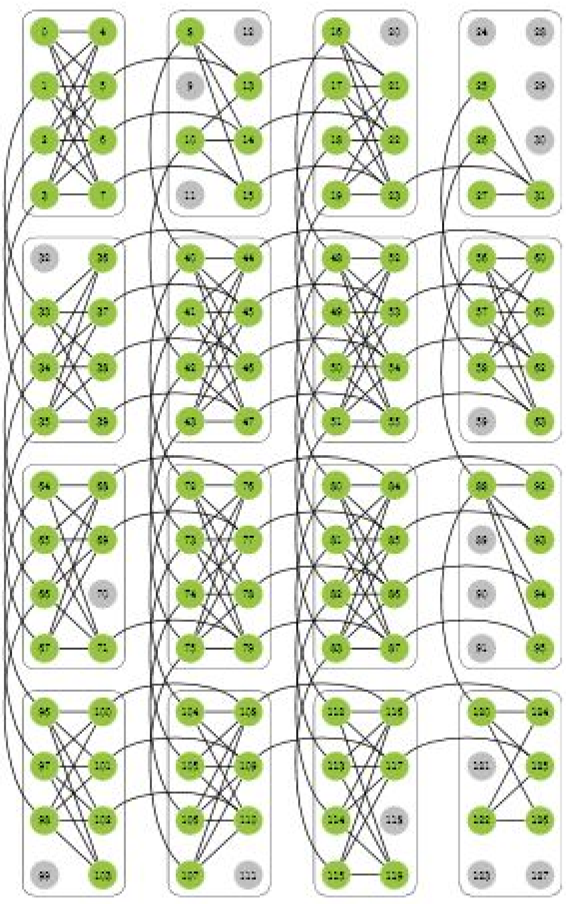
* Spin models with random couplings (aka spin glasses)

frontiers in PHYSICS
REVIEW ARTICLE
published: 12 February 2014
doi: 10.3389/fphy.2014.00005

Ising formulations of many NP problems

*Andrew Lucas**
Lyman Laboratory of Physics, Department of Physics, Harvard University, Cambridge, MA, USA

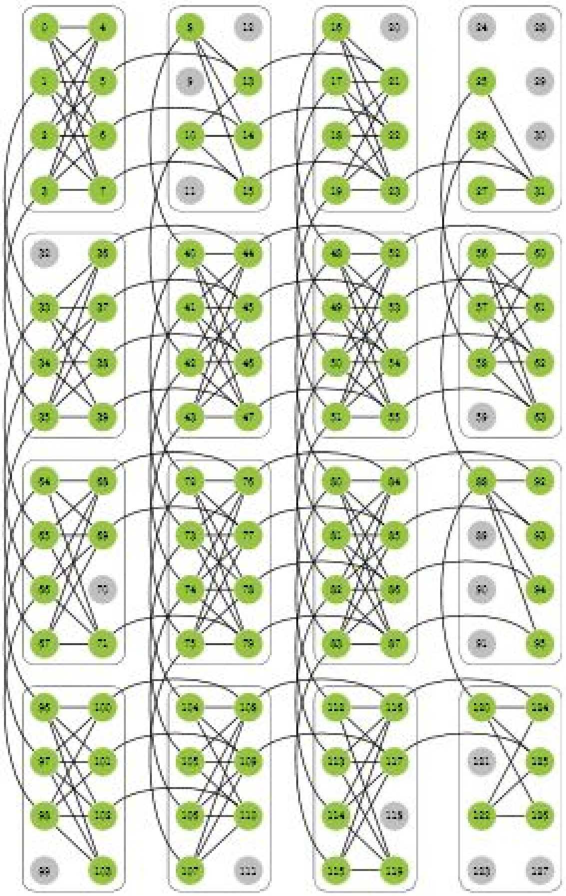
D-Wave machine



* chimera graph with up to 1024 qbits
* adjustable bimodal couplings
* quantum annealing of classical Ising spin glass

Spin glass solver

D-Wave machine



* chimera graph with up to 1024 qbits

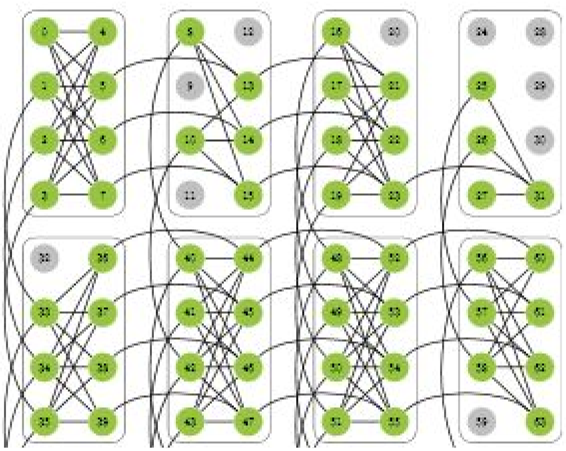
* adjustable bimodal couplings

* quantum annealing of classical Ising spin glass

→ Is it really quantum?
→ Is there quantum speed-up?

Spin glass solver

D-Wave machine



* chimera graph with up to 1024 qbits
* adjustable bimodal couplings
* quantum annealing of classical Ising spin glass

→ Is it really quantum?
→ Is there quantum speed-up?

RESEARCH | *REPORTS* Science (2014)

QUANTUM COMPUTING

Defining and detecting quantum speedup

Troels F. Rønnow,[1] Zhihui Wang,[2,3] Joshua Job,[3,4] Sergio Boixo,[5,6] Sergei V. Isakov,[7] David Wecker,[8] John M. Martinis,[9] Daniel A. Lidar,[2,3,4,6,10] Matthias Troyer[1]*

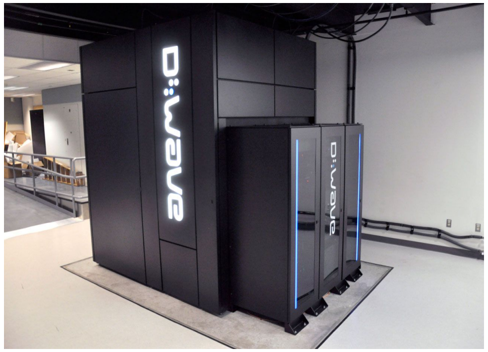
arXiv 1512.02206

What is the Computational Value of Finite Range Tunneling?

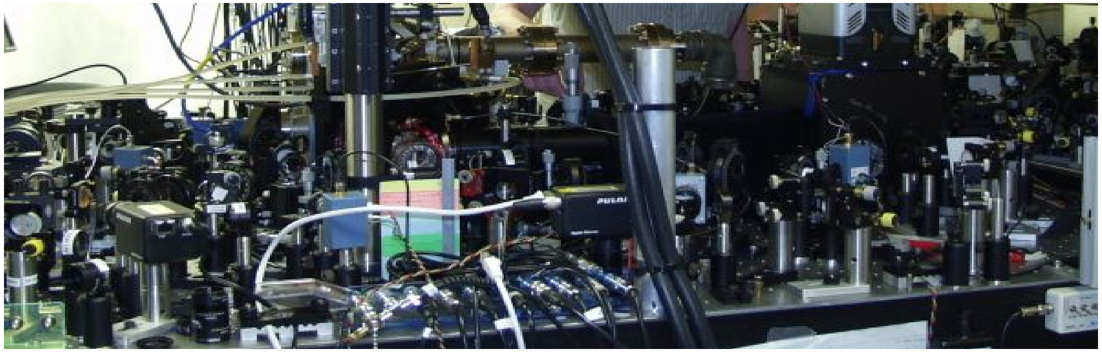
Vasil S. Denchev,[1] Sergio Boixo,[1] Sergei V. Isakov,[1] Nan Ding,[1] Ryan Babbush,[1] Vadim Smelyanskiy,[1] John Martinis,[2] and Hartmut Neven[1]

[1] *Google Inc., Venice, CA 90291, USA*
[2] *Google Inc., Santa Barbara, CA 93117, USA*
(Dated: December 31, 2015)

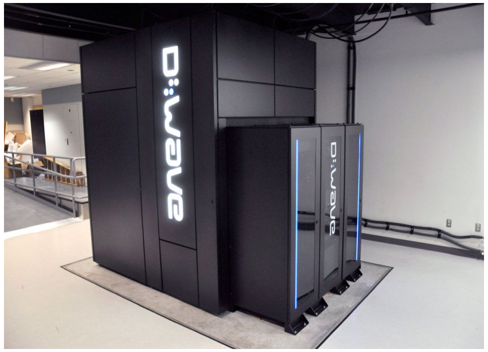
Trapped ions quantum annealer?



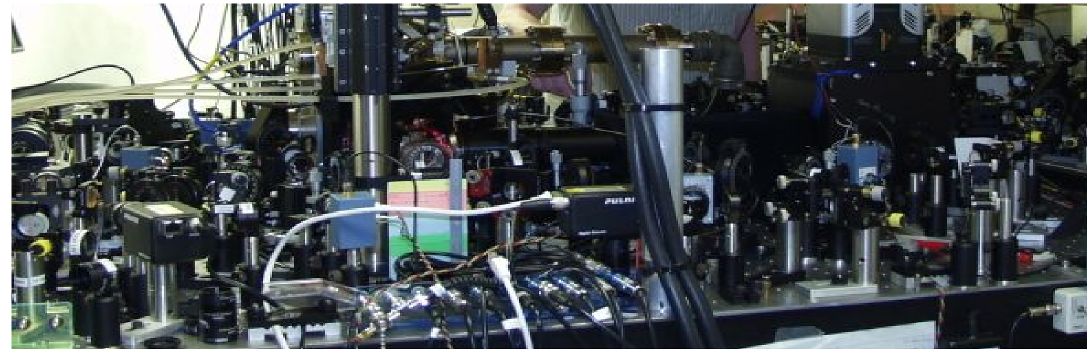
vs



Trapped ions quantum annealer?



vs



* Potential complexity due to very high connectivity

* Tunability of interactions

* Quantum annealing via transverse field

* Access to many observables (e.g. local spin polarization)

How to get complex Hamiltonians with ions?

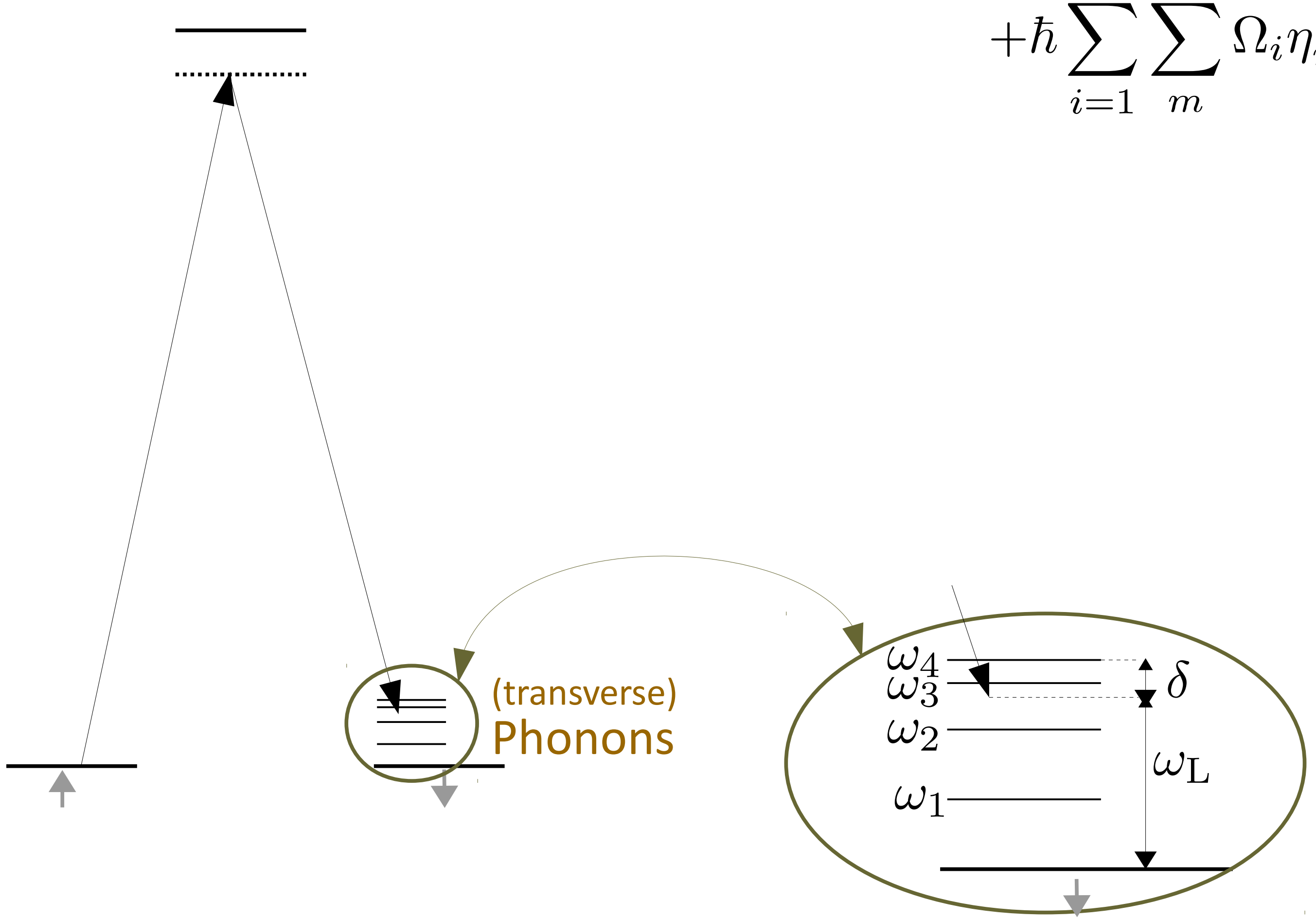
Spin-spin interactions

Raman coupling:

Ω_i : Rabi frequency (at ion *i*)
ω_{r} : recoil energy
ω_{L} : laser beatnote frequency

Interaction picture + rotating wave approximation:

$$H = \sum_{\text{phonons } m} \hbar(\omega_m - \omega_{\mathrm{L}})\hat{a}_m^\dagger \hat{a}_m + \hbar \sum_{i=1}^{N} \sum_m \Omega_i \eta_m^{(i)} (\hat{a}_m + \text{H.c.})\sigma_x^{(i)}$$



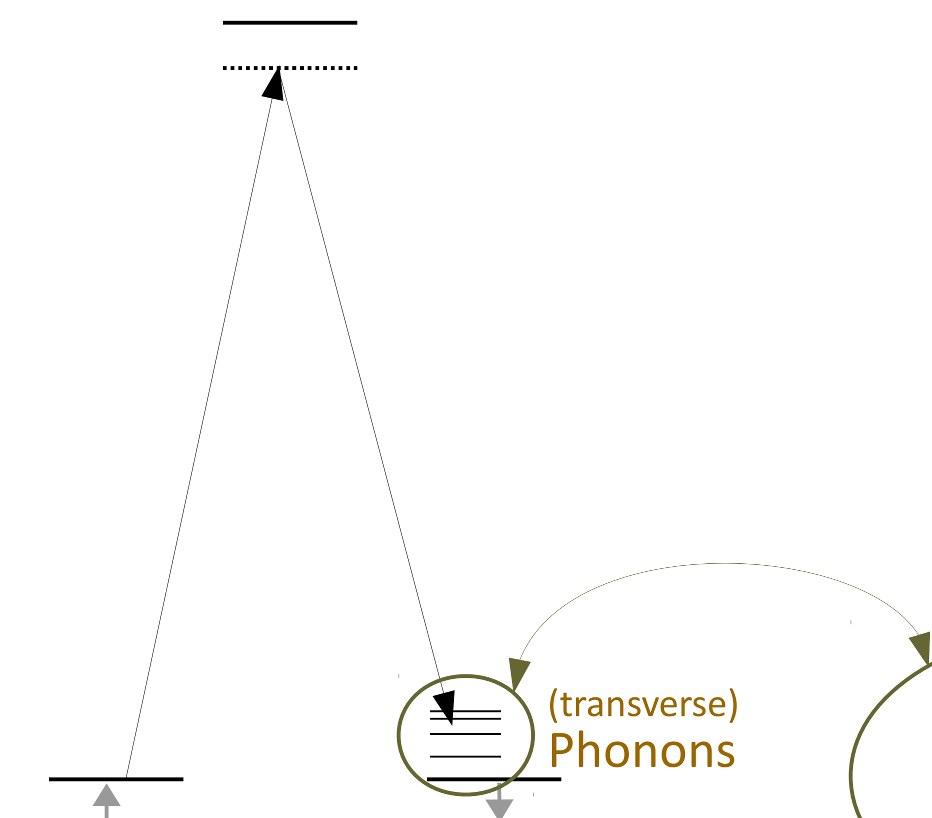
Spin-spin interactions

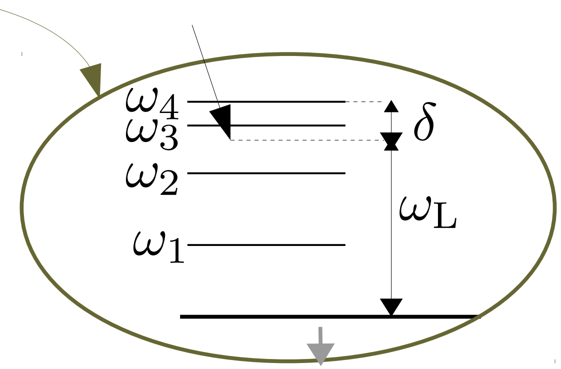
Raman coupling:

Ω_i : Rabi frequency (at ion *i*)

ω_{r} : recoil energy

ω_{L} : laser beatnote frequency





Interaction picture + rotating wave approximation:

$$H = \sum_{\text{phonons } m} \hbar(\omega_m - \omega_{\mathrm{L}})\hat{a}_m^\dagger \hat{a}_m + \hbar \sum_{i=1}^{N} \sum_m \Omega_i \eta_m^{(i)} (\hat{a}_m + \mathrm{H.c.})\sigma_x^{(i)}$$

Quadratic completion:

$$H = \sum_m \hbar(\omega_m - \omega_{\mathrm{L}}) \left(\hat{a}_m^\dagger + \sum_i \frac{\Omega_i \eta_m^{(i)}}{\omega_m - \omega_{\mathrm{L}}} \sigma_x^{(i)} \right) \times \left(\hat{a}_m + \sum_i \frac{\Omega_i \eta_m^{(i)}}{\omega_m - \omega_{\mathrm{L}}} \sigma_x^{(i)} \right) - \sum_m \sum_{ij} \frac{\hbar \Omega_i \Omega_j \eta_m^{(i)} \eta_m^{(j)}}{\omega_m - \omega_{\mathrm{L}}} \sigma_x^{(i)} \sigma_x^{(j)}$$

Spin-spin interactions

Raman coupling:

Ω_i : Rabi frequency (at ion *i*)

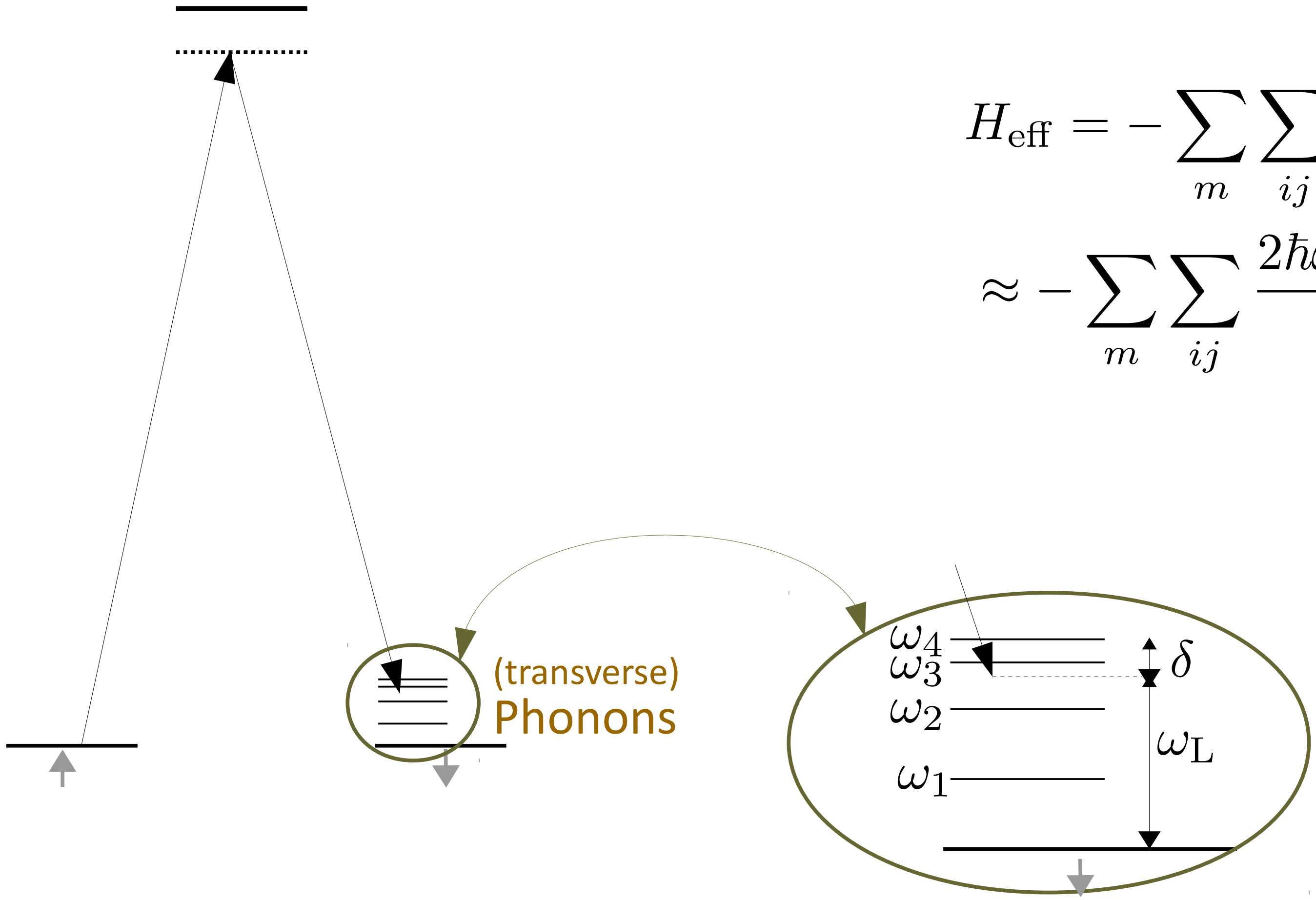
ω_r : recoil energy

ω_L : laser beatnote frequency

Effective Hamiltonian:

$$H_\mathrm{eff} = -\sum_m \sum_{ij} \frac{\hbar \Omega_i \Omega_j \eta_m^{(i)} \eta_m^{(j)}}{\omega_m - \omega_\mathrm{L}} \sigma_x^{(i)} \sigma_x^{(j)}$$

$$\approx -\sum_m \sum_{ij} \frac{2\hbar \omega_m \Omega_i \Omega_j \eta_m^{(i)} \eta_m^{(j)}}{\omega_m^2 - \omega_\mathrm{L}^2} \sigma_x^{(i)} \sigma_x^{(j)}$$



Phonon modes

Effective Hamiltonian:

$$H_{\rm eff} = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)} \qquad \text{with} \qquad J_{ij} = \sum_{m=1}^{N} 2\hbar\omega_m \Omega_i \Omega_j \frac{\eta_m^{(i)} \eta_m^{(j)}}{\omega_m^2 - \omega_{\rm L}^2}$$

Phonon modes

Effective Hamiltonian:

$$H_{\mathrm{eff}} = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)} \qquad \text{with} \qquad J_{ij} = \sum_{m=1}^{N} 2\hbar\omega_m\Omega_i\Omega_j \frac{\eta_m^{(i)}\eta_m^{(j)}}{\omega_m^2 - \omega_{\mathrm{L}}^2}$$

Lamb-Dicke parameter $\eta_m^{(i)}$ depend on phonon modes:

Phonon Hamiltonian:

$$H_{\mathrm{ph}} = \frac{m}{2}\sum_{ij} V_{ij} q_i q_j$$

Trap and Coulomb potential:

$$V_{ij} = \begin{cases} \omega_{\mathrm{trap}}^2 - \frac{e^2/m}{4\pi\epsilon_0}\sum_{i''(\neq i)} \frac{1}{d^3|i-i''|^3} & i=j \\ \frac{e^2/m}{4\pi\epsilon_0}\frac{1}{d^3|i-j|^3} & i\neq j \end{cases}$$

Phonon modes and frequencies are eigenvalues and eigenvectors of *V*:

$$\boldsymbol{\xi}_{m'}^T V \boldsymbol{\xi}_m = \omega_m^2 \delta_{m,m'}$$

$$\boldsymbol{\xi}_m = (\xi_m^{(1)}, \dots, \xi_m^{(N)})$$

$$\eta_m^{(i)} = \sqrt{\frac{\omega_{\mathrm{r}}}{\omega_m}}\xi_m^{(i)}$$

Phonon modes

Effective Hamiltonian:

$$H_{\text{eff}} = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)} \qquad \text{with} \qquad J_{ij} = \sum_{m=1}^{N} 2\hbar\omega_m\Omega_i\Omega_j \frac{\eta_m^{(i)}\eta_m^{(j)}}{\omega_m^2 - \omega_{\text{L}}^2}$$

Lamb-Dicke parameter $\eta_m^{(i)}$ depend on phonon modes:

Phonon Hamiltonian:

$$H_{\text{ph}} = \frac{m}{2}\sum_{ij} V_{ij} q_i q_j$$

Trap and Coulomb potential:

$$V_{ij} = \begin{cases} \omega_{\text{trap}}^2 - \frac{e^2/m}{4\pi\epsilon_0}\sum_{i''(\neq i)} \frac{1}{d^3|i-i''|^3} & i=j \\ \frac{e^2/m}{4\pi\epsilon_0}\frac{1}{d^3|i-j|^3} & i\neq j \end{cases}$$

Phonon modes and frequencies are eigenvalues and eigenvectors of *V*:

$$\boldsymbol{\xi}_{m'}^T V \boldsymbol{\xi}_m = \omega_m^2 \delta_{m,m'}$$

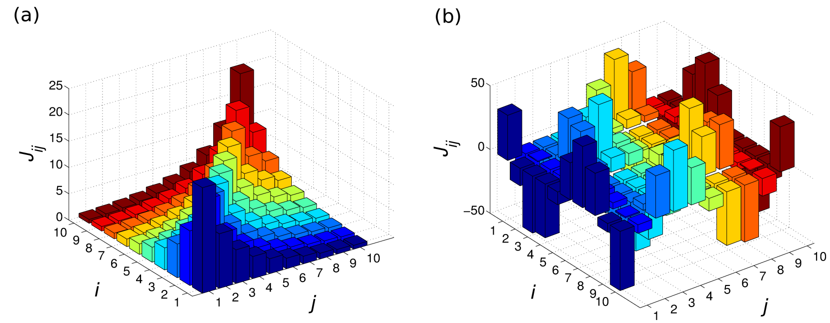
$$\boldsymbol{\xi}_m = (\xi_m^{(1)}, \ldots, \xi_m^{(N)})$$

$$\eta_m^{(i)} = \sqrt{\frac{\omega_{\text{r}}}{\omega_m}}\xi_m^{(i)}$$

Coupling:

* at constant Rabi frequency
* adjustable via laser frequency

$$J_{ij} \propto \sum_m \frac{\xi_m^{(i)}\xi_m^{(j)}}{\omega_m^2 - \omega_{\text{L}}^2}$$



Mattis model

Effective ion Hamiltonian: $H_{\text{eff}} = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)}$ with $J_{ij} \propto \sum_m \frac{\xi_m^{(i)} \xi_m^{(j)}}{\omega_m^2 - \omega_{\text{L}}^2}$

Special cases: Mattis model

$\omega_{\text{L}} \to \omega_m - \epsilon \quad \Rightarrow H_{\text{eff}} \propto -\sum_{ij} \xi_m^{(i)} \xi_m^{(j)} \sigma_x^{(i)} \sigma_x^{(j)}$

Ferromagnetic coupling to mode *m*

$\omega_{\text{L}} \to \omega_m + \epsilon \quad \Rightarrow H_{\text{eff}} \propto +\sum_{ij} \xi_m^{(i)} \xi_m^{(j)} \sigma_x^{(i)} \sigma_x^{(j)}$

Antiferromagnetic coupling to mode *m*

Two-fold degenerate ground state defined by pattern: $\langle \sigma_x^{(i)} \rangle = \pm \text{sign}(\xi_m^{(i)})$

ω_4 — $\xi_4 = (0.5, 0.5, 0.5, 0.5)$ GS1= ↑↑↑↑ GS2= ↓↓↓↓

ω_3 — $\xi_3 = (0.66, 0.26, -0.26, -0.66)$ GS1= ↑↑↓↓ GS2= ↓↓↑↑

ω_2 — $\xi_2 = (0.5, -0.5, -0.5, 0.5)$ GS1= ↑↓↓↑ GS2= ↓↑↑↓

ω_1 — $\xi_1 = (-0.26, 0.66, -0.66, 0.26)$ GS1= ↑↓↑↓ GS2= ↓↑↓↑

Energy is cost function of *number partitioning*:

$$E = \left(\sum_{i \in \uparrow} \xi_m^{(i)} - \sum_{i \in \downarrow} \xi_m^{(i)} \right)^2$$

Optimized by ground states – parity eigenstates:

ω_4 — ↑↑↓↓ ↓↓↑↑ ↑↓↑↓ ↓↑↓↑ — $\xi_4 = (0.5, 0.5, 0.5, 0.5)$

ω_3 — ↑↓↓↑ ↓↑↑↓ ↑↑↑↑ ↓↓↓↓ — $\xi_3 = (0.66, 0.26, -0.26, -0.66)$

ω_2 — ↑↑↓↓ ↓↓↑↑ ↑↓↑↓ ↓↑↓↑ — $\xi_2 = (0.5, -0.5, -0.5, 0.5)$

ω_1 — ↑↓↓↑ ↓↑↑↓ ↑↑↑↑ ↓↓↓↓ — $\xi_1 = (-0.26, 0.66, -0.66, 0.26)$

Mattis model

Effective ion Hamiltonian:

$$H_{\text{eff}} = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)} \qquad \text{with} \qquad J_{ij} \propto \sum_m \frac{\xi_m^{(i)}\xi_m^{(j)}}{\omega_m^2 - \omega_{\text{L}}^2}$$

Special cases: Mattis model

$$\omega_{\text{L}} \to \omega_m - \epsilon \quad \Rightarrow H_{\text{eff}} \propto -\Big(\sum_i \xi_m^{(i)}\sigma_x^{(i)}\Big)^2$$

Ferromagnetic coupling to mode *m*

$$\omega_{\text{L}} \to \omega_m + \epsilon \quad \Rightarrow H_{\text{eff}} \propto +\Big(\sum_i \xi_m^{(i)}\sigma_x^{(i)}\Big)^2$$

Antiferromagnetic coupling to mode *m*

Two-fold degenerate ground state defined by pattern: $\langle\sigma_x^{(i)}\rangle = \pm\text{sign}(\xi_m^{(i)})$

ω_4 — $\xi_4 = (0.5, 0.5, 0.5, 0.5)$
GS1= ↑↑↑↑ GS2=↓↓↓↓

ω_3 — $\xi_3 = (0.66, 0.26, -0.26, -0.66)$
GS1= ↑↑↓↓ GS2=↓↓↑↑

ω_2 — $\xi_2 = (0.5, -0.5, -0.5, 0.5)$
GS1= ↑↓↓↑ GS2=↓↑↑↓

ω_1 — $\xi_1 = (-0.26, 0.66, -0.66, 0.26)$
GS1= ↑↓↑↓ GS2=↓↑↓↑

Energy is cost function of *number partitioning*:

$$E = \Big(\sum_{i\in\uparrow}\xi_m^{(i)} - \sum_{i\in\downarrow}\xi_m^{(i)}\Big)^2$$

Optimized by ground states – parity eigenstates:

ω_4 — ↑↑↓↓ ↓↓↑↑ ↑↓↑↓ ↓↑↓↑
$\xi_4 = (0.5, 0.5, 0.5, 0.5)$

ω_3 — ↑↓↓↑ ↓↑↑↓ ↑↑↑↑ ↓↓↓↓
$\xi_3 = (0.66, 0.26, -0.26, -0.66)$

ω_2 — ↑↑↓↓ ↓↓↑↑ ↑↓↑↓ ↓↑↓↑
$\xi_2 = (0.5, -0.5, -0.5, 0.5)$

ω_1 — ↑↓↓↑ ↓↑↑↓ ↑↑↑↑ ↓↓↓↓
$\xi_1 = (-0.26, 0.66, -0.66, 0.26)$

Mattis model

Effective ion Hamiltonian:

$$H_{\text{eff}} = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)} \quad \text{with} \quad J_{ij} \propto \sum_m \frac{\xi_m^{(i)}\xi_m^{(j)}}{\omega_m^2 - \omega_{\text{L}}^2}$$

Special cases: Mattis model

$$\omega_{\text{L}} \to \omega_m - \epsilon \quad \Rightarrow H_{\text{eff}} \propto -\Big(\sum_i \xi_m^{(i)}\sigma_x^{(i)}\Big)^2$$

Ferromagnetic coupling to mode *m*

$$\omega_{\text{L}} \to \omega_m + \epsilon \quad \Rightarrow H_{\text{eff}} \propto +\Big(\sum_i \xi_m^{(i)}\sigma_x^{(i)}\Big)^2$$

Antiferromagnetic coupling to mode *m*

Two-fold degenerate ground state defined by pattern: $\langle\sigma_x^{(i)}\rangle = \pm\text{sign}(\xi_m^{(i)})$

ω_4 $\xi_4 = (0.5, 0.5, 0.5, 0.5)$
GS1= ↑↑↑↑ GS2=↓↓↓↓

ω_3 $\xi_3 = (0.66, 0.26, -0.26, -0.66)$
GS1= ↑↑↓↓ GS2=↓↓↑↑

ω_2 $\xi_2 = (0.5, -0.5, -0.5, 0.5)$
GS1= ↑↓↓↑ GS2=↓↑↑↓

ω_1 $\xi_1 = (-0.26, 0.66, -0.66, 0.26)$
GS1= ↑↓↑↓ GS2=↓↑↓↑

Energy is cost function of *number partitioning*:

$$E = \Big(\sum_{i\in\uparrow}\xi_m^{(i)} - \sum_{i\in\downarrow}\xi_m^{(i)}\Big)^2$$

Optimized by ground states – parity eigenstates:

ω_4 ↑↑↓↓ ↓↓↑↑ ↑↓↑↓ ↓↑↓↑
$\xi_4 = (0.5, 0.5, 0.5, 0.5)$

ω_3 ↑↓↓↑ ↓↑↑↓ ↑↑↑↑ ↓↓↓↓
$\xi_3 = (0.66, 0.26, -0.26, -0.66)$

ω_2 ↑↑↓↓ ↓↓↑↑ ↑↓↑↓ ↓↑↓↑
$\xi_2 = (0.5, -0.5, -0.5, 0.5)$

ω_1 ↑↓↓↑ ↓↑↑↓ ↑↑↑↑ ↓↓↓↓
$\xi_1 = (-0.26, 0.66, -0.66, 0.26)$

Mattis model

Effective ion Hamiltonian:

$$H_{\text{eff}} = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)} \quad \text{with} \quad J_{ij} \propto \sum_m \frac{\xi_m^{(i)} \xi_m^{(j)}}{\omega_m^2 - \omega_{\text{L}}^2}$$

Special cases: Mattis model

$$\omega_{\text{L}} \to \omega_m - \epsilon \quad \Rightarrow H_{\text{eff}} \propto -\Big(\sum_i \xi_m^{(i)} \sigma_x^{(i)}\Big)^2$$

Ferromagnetic coupling to mode *m*

$$\omega_{\text{L}} \to \omega_m + \epsilon \quad \Rightarrow H_{\text{eff}} \propto +\Big(\sum_i \xi_m^{(i)} \sigma_x^{(i)}\Big)^2$$

Antiferromagnetic coupling to mode *m*

Two-fold degenerate ground state defined by pattern: $\langle \sigma_x^{(i)} \rangle = \pm \text{sign}(\xi_m^{(i)})$

ω_4 — $\xi_4 = (0.5, 0.5, 0.5, 0.5)$ GS1= ↑↑↑↑ GS2=↓↓↓↓

ω_3 — $\xi_3 = (0.66, 0.26, -0.26, -0.66)$ GS1= ↑↑↓↓ GS2=↓↓↑↑

ω_2 — $\xi_2 = (0.5, -0.5, -0.5, 0.5)$ GS1= ↑↓↓↑ GS2=↓↑↑↓

ω_1 — $\xi_1 = (-0.26, 0.66, -0.66, 0.26)$ GS1= ↑↓↑↓ GS2=↓↑↓↑

Energy is cost function of *number partitioning*:

$$E = \Big(\sum_{i\in\uparrow} \xi_m^{(i)} - \sum_{i\in\downarrow} \xi_m^{(i)}\Big)^2$$

Optimized by ground states – parity eigenstates:

ω_4 ↑↑↓↓ ↓↓↑↑ ↑↓↑↓ ↓↑↓↑
$\xi_4 = (0.5, 0.5, 0.5, 0.5)$

ω_3 ↑↓↓↑ ↓↑↑↓ ↑↑↑↑ ↓↓↓↓
$\xi_3 = (0.66, 0.26, -0.26, -0.66)$

ω_2 ↑↑↓↓ ↓↓↑↑ ↑↓↑↓ ↓↑↓↑
$\xi_2 = (0.5, -0.5, -0.5, 0.5)$

ω_1 ↑↓↓↑ ↓↑↑↓ ↑↑↑↑ ↓↓↓↓
$\xi_1 = (-0.26, 0.66, -0.66, 0.26)$

Mattis model

Effective ion Hamiltonian: $H_{\text{eff}} = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)}$ with $J_{ij} \propto \sum_m \frac{\xi_m^{(i)} \xi_m^{(j)}}{\omega_m^2 - \omega_L^2}$

Special cases: Mattis model

$\omega_L \to \omega_m - \epsilon \quad \Rightarrow H_{\text{eff}} \propto -\left(\sum_{ij} \xi_m^{(i)} \sigma_x^{(i)}\right)^2$

Ferromagnetic coupling to mode *m*

$\omega_L \to \omega_m + \epsilon \quad \Rightarrow H_{\text{eff}} \propto +\left(\sum_i \xi_m^{(i)} \sigma_x^{(i)}\right)^2$

Antiferromagnetic coupling to mode *m*

Two-fold degenerate ground state defined by pattern: $\langle \sigma_x^{(i)} \rangle = \pm \text{sign}(\xi_m^{(i)})$

ω_4 — $\xi_4 = (0.5, 0.5, 0.5, 0.5)$
GS1= ↑↑↑↑ GS2=↓↓↓↓

ω_3 — $\xi_3 = (0.66, 0.26, -0.26, -0.66)$
GS1= ↑↑↓↓ GS2=↓↓↑↑

ω_2 — $\xi_2 = (0.5, -0.5, -0.5, 0.5)$
GS1= ↑↓↓↑ GS2=↓↑↑↓

ω_1 — $\xi_1 = (-0.26, 0.66, -0.66, 0.26)$
GS1= ↑↓↑↓ GS2=↓↑↓↑

Energy is cost function of *number partitioning*:

$$E = \left(\sum_{i \in \uparrow} \xi_m^{(i)} - \sum_{i \in \downarrow} \xi_m^{(i)}\right)^2$$

Optimized by ground states – parity eigenstates:

ω_4 ↑↑↓↓ ↓↓↑↑ ↑↓↑↓ ↓↑↓↑
$\xi_4 =$

ω_3 ↑↓ ... ↓↓
$\xi_3 =$

ω_2 ↑↑ ... ↓↑
$\xi_2 =$

ω_1 ↑↓↓↑ ↓↑↑↓ ↑↑↑↑ ↓↓↓↓
$\xi_1 = (-0.26, 0.66, -0.66, 0.26)$

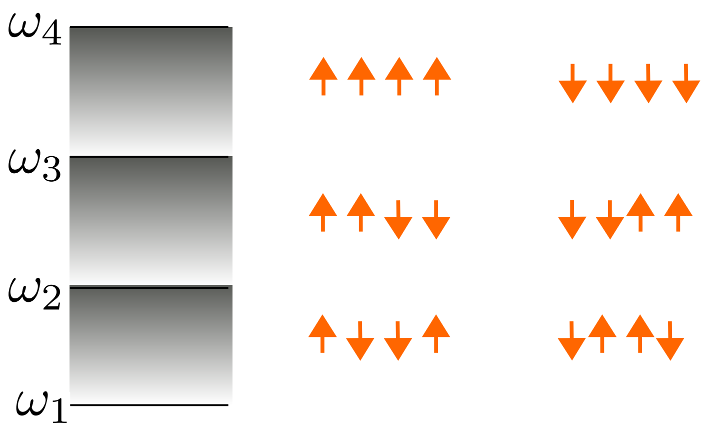
Exponentially large ground state manifold:

$$2^{N/2}$$

→ Mattis spin glass

Mattis model

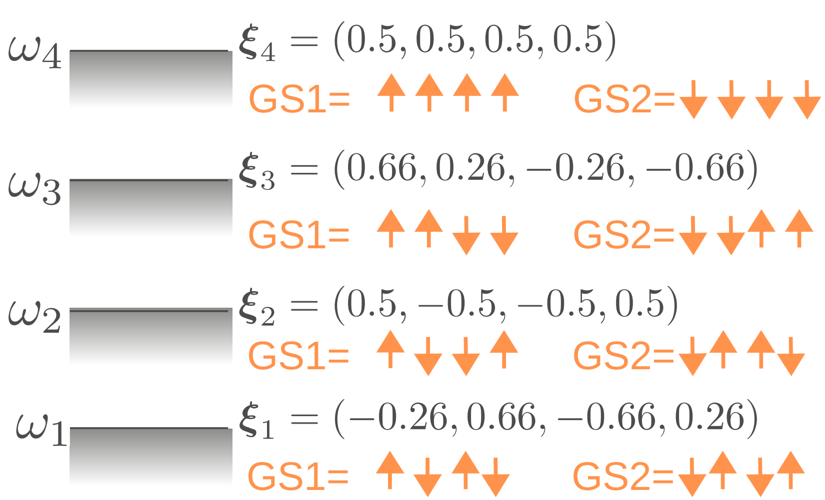
Effective ion Hamiltonian: $H_{\text{eff}} = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)}$ with $J_{ij} \propto \sum_m \frac{\xi_m^{(i)} \xi_m^{(j)}}{\omega_m^2 - \omega_L^2}$

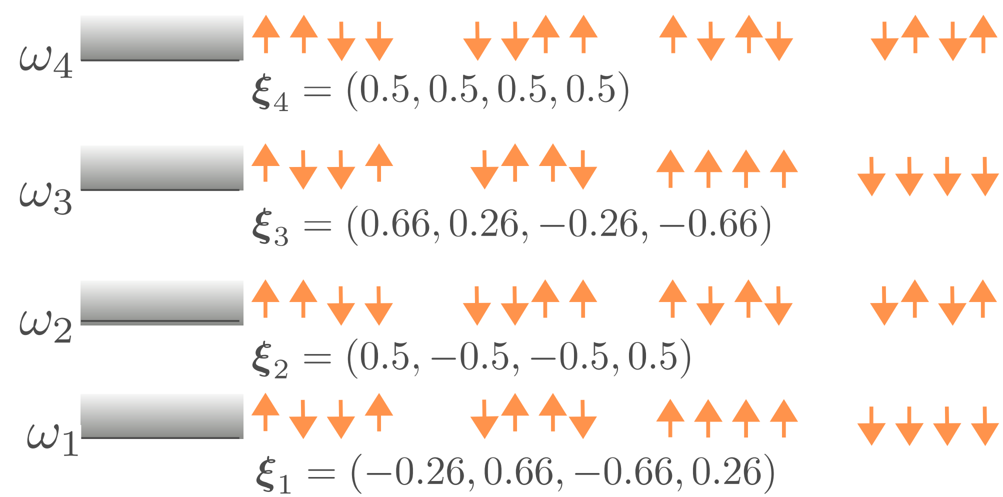


From single-mode to two-mode approximation:

Ferromagnetic and antiferromagnetic couplings can be satisfied simultaneously.

Verified by exact numerical calculations for small systems (*N=10*).





Complexity of the problem

- Two-mode approximation yields only trivial problems:
  - Analytical solution is simple
  - Experimental solution (via quantum simulation) still carries all difficulties of complex spin glass problems

- Enhancing complexity via influence of additional modes:
  - Increasing ion number
  - Multiple Raman couplings: $J_{ij} \propto \sum_\mu \Omega_\mu^2 \sum_m \frac{\xi_m^{(i)} \xi_m^{(j)}}{\omega_m^2 - \omega_{\mathrm{L},\mu}^2}$

- Enhancing complexity within single-mode approximation:
  - Number partitioning is potentially NP-complete
  - Precision of the numbers must scale with the number of spins
  - Still not all instances are difficult to solve → trivial instances!

Complexity of the problem

- Two-mode approximation yields only trivial problems:
  - Analytical solution is simple
  - Experimental solution (via quantum simulation) still carries all difficulties of complex spin glass problems

- Enhancing complexity via influence of additional modes:
  - Increasing ion number
  - Multiple Raman couplings: $J_{ij} \propto \sum_\mu \Omega_\mu^2 \sum_m \frac{\xi_m^{(i)} \xi_m^{(j)}}{\omega_m^2 - \omega_{\mathrm{L},\mu}^2}$

- Enhancing complexity within single-mode approximation:
  - Number partitioning is potentially NP-complete
  - Precision of the numbers must scale with the number of spins
  - Still not all instances are difficult to solve → trivial instances!

Complexity of the problem

→ Two-mode approximation yields only trivial problems:

- Analytical solution is simple
- Experimental solution (via quantum simulation) still carries all difficulties of complex spin glass problems

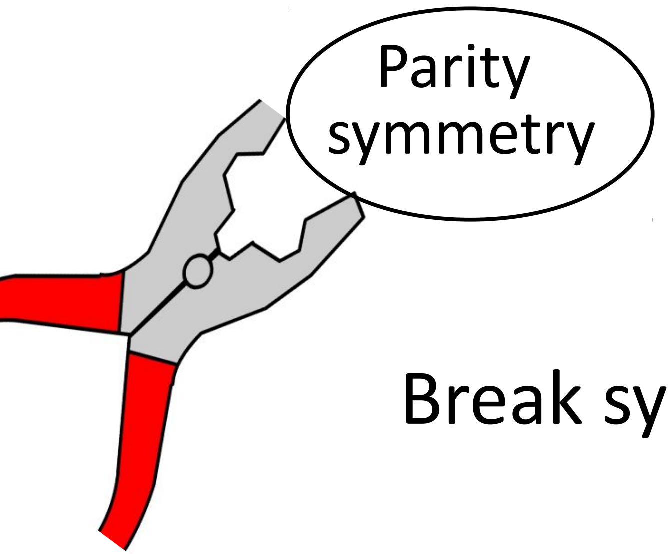
→ Enhancing complexity via influence of additional modes:

- Increasing ion number
- Multiple Raman couplings: $J_{ij} \propto \sum_{\mu} \Omega_{\mu}^2 \sum_{m} \frac{\xi_m^{(i)} \xi_m^{(j)}}{\omega_m^2 - \omega_{\mathrm{L},\mu}^2}$

→ Enhancing complexity within single-mode approximation:

- Number partitioning is potentially NP-complete
- Precision of the numbers must scale with the number of spins
- Instances become non-trivial if parity symmetry is broken

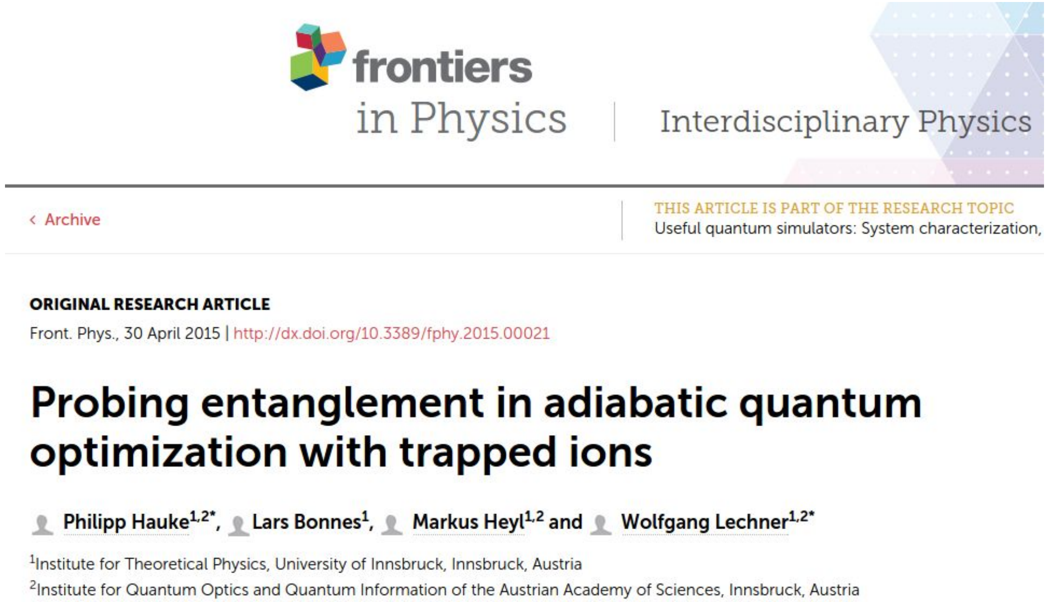
From trivial to complex



$\Rightarrow$ Number partitioning is trivial.

Break symmetry to make things complex!

Option III: Rabi frequencies


frontiers in Physics | Interdisciplinary Physics

‹ Archive

THIS ARTICLE IS PART OF THE RESEARCH TOPIC
Useful quantum simulators: System characterization,

ORIGINAL RESEARCH ARTICLE

Front. Phys., 30 April 2015 | http://dx.doi.org/10.3389/fphy.2015.00021

Probing entanglement in adiabatic quantum optimization with trapped ions

Philipp Hauke[1,2*], Lars Bonnes[1], Markus Heyl[1,2] and Wolfgang Lechner[1,2*]

[1]Institute for Theoretical Physics, University of Innsbruck, Innsbruck, Austria
[2]Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, Innsbruck, Austria

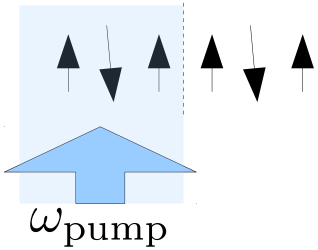
Option I: Microtraps


letters to nature

A scalable quantum computer with ions in an array of microtraps

J. I. Cirac & P. Zoller

Option II: Fast pulses



$$\sigma_x^{(i)} \rightarrow \sigma_x^{(i)} e^{i\omega_{\rm pump} t}$$

From classical to quantum

Classical Hamiltonian

$$H = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)}$$

$\longrightarrow$

Quantum Hamiltonian

$$H = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)} + B \sum_i \sigma_z^{(i)}$$

From classical to quantum

Classical Hamiltonian

$$H = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)}$$

Spin glass *or* "ferromagnet"

$$\longrightarrow$$

Quantum Hamiltonian

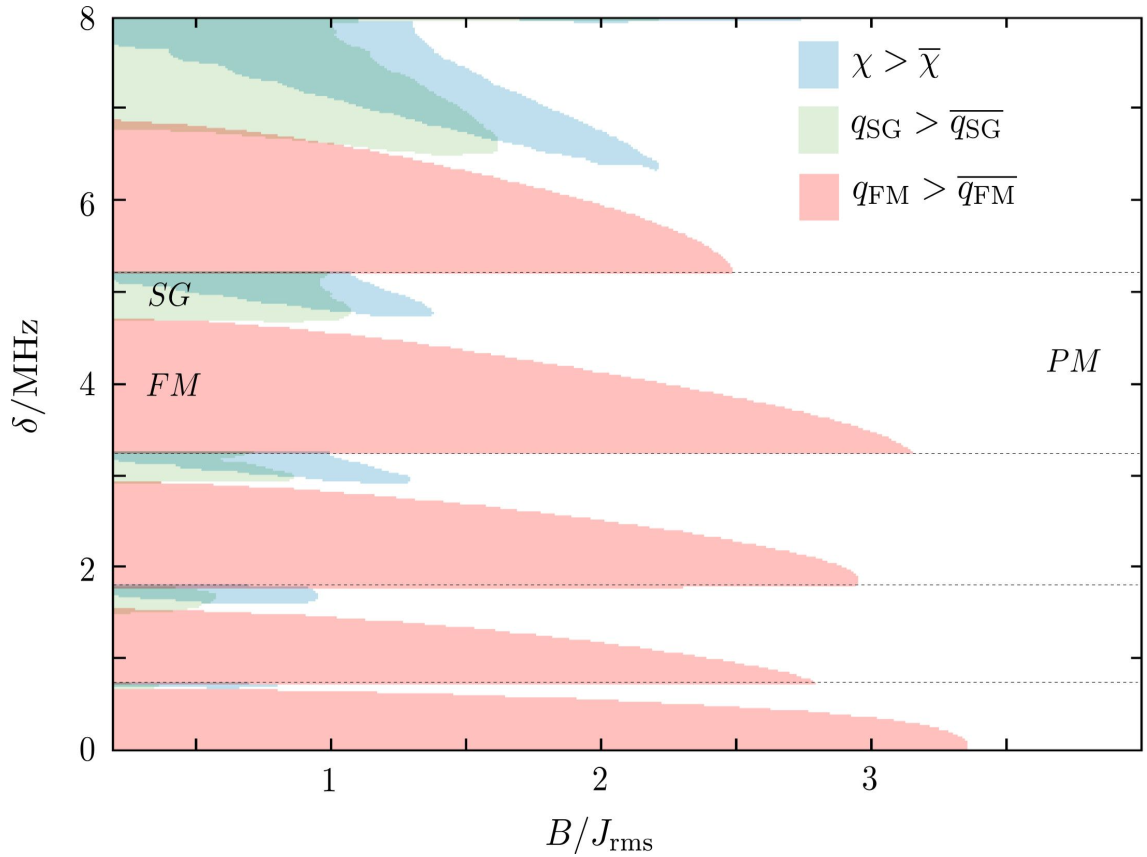
$$H = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)} + B \sum_i \sigma_z^{(i)}$$

Quantum spin glass
or "ferromagnet"
or paramagnet

$$?$$

“Phase diagram”

System properties upon varying detuning and transverse field (N=6):



Useful thermal averages:

$$q_{\rm FM} = \frac{1}{N}\sum_i \langle\langle \sigma_x^i \rangle\rangle_T^2$$

$$q_{\rm EA} = \frac{1}{N}\sum_i \langle\langle \sigma_x^i \rangle^2\rangle_T$$

$$q_{\rm SG} = q_{\rm EA}/q_{\rm FM}$$

(should be calculated for $k_{\rm B}T \approx J$ in the presence of a Z_2 breaking field)

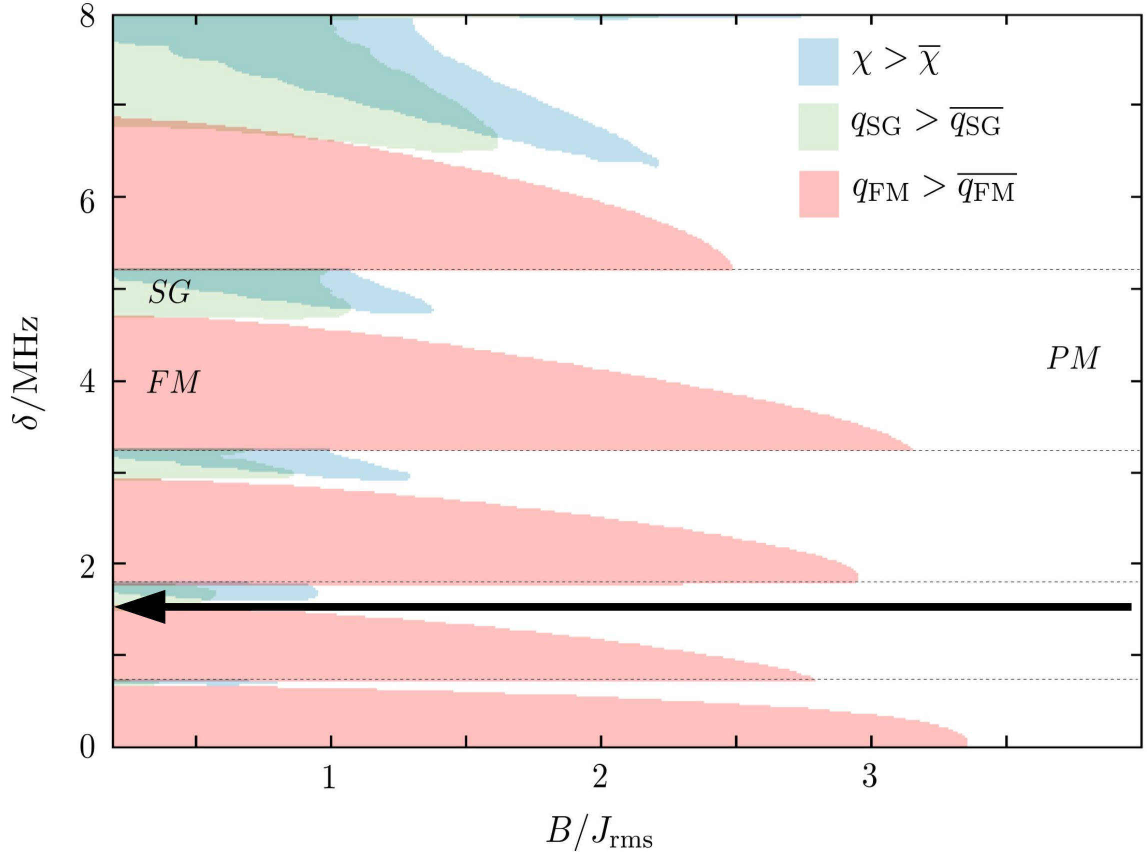
Magnetic susceptibility:

$$\chi = \frac{1}{N}\sum_{ij}\left(\frac{\partial\langle\sigma_x^i\rangle}{\partial h_x^j}\right)^2$$

(small longitudinal field *h* plus Z_2 breaking field)

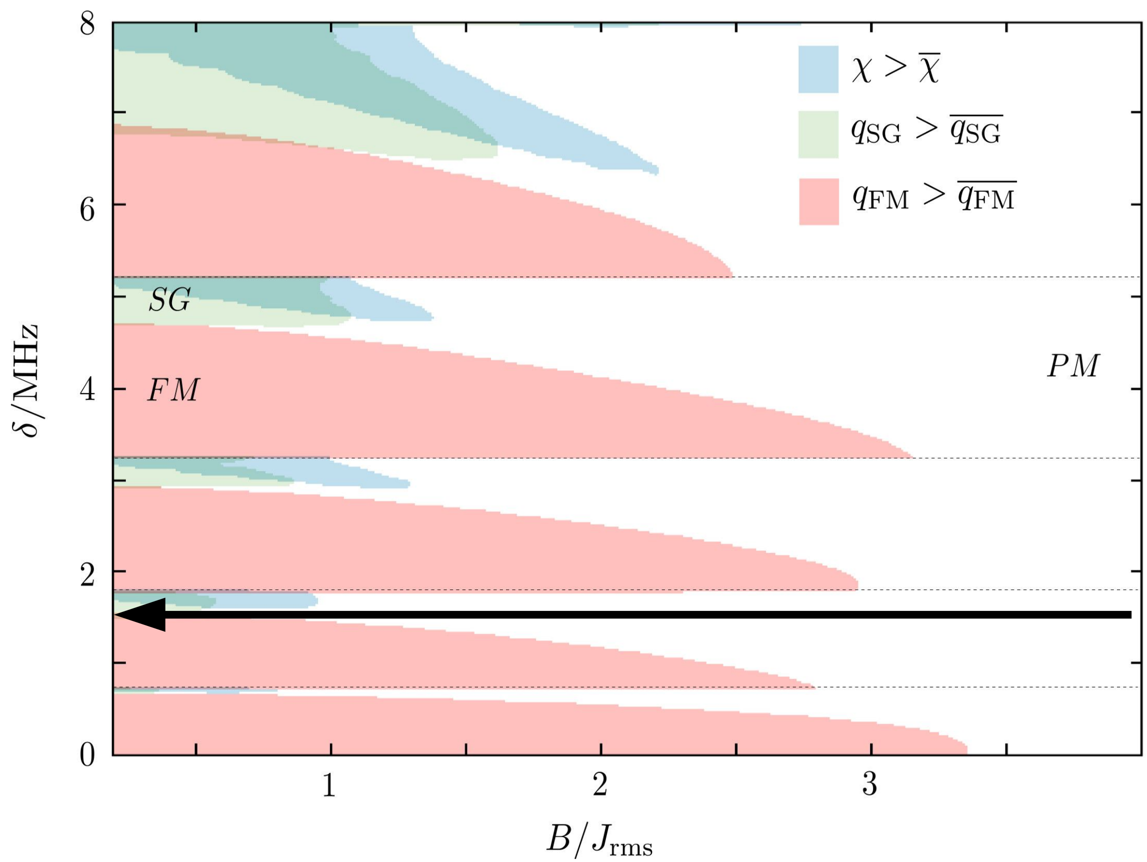
Quantum annealer

Can we reach the ground state in the glassy regime starting from the paramagnetic configuration?



Quantum annealer

Can we reach the ground state in the glassy regime starting from the paramagnetic configuration?



Time-dependent magnetic field:

$$B(t) = B_0 \exp(-t/\tau)$$

How slow does it have to be?

How slow can it be (dissipation)?

Which role do phonons play?

Closed system dynamics

Phonons and spin-phonon coupling:

$$H_0(t) = \sum_m \hbar\omega_m a_m^\dagger a_m + \sum_{i,m} \hbar\Omega_i \sqrt{\frac{\omega_{\rm recoil}}{\omega_m}} \xi_{im} \sin(\omega_{\rm L} t) \times \sigma_x^i (a_m + a_m^\dagger)$$

With time-dependent transverse field (annealing) and symmetry-breaking bias:

$$H = H_0(t) + B(t) \sum_i \sigma_z^{(i)} + \epsilon_{\rm bias} \sigma_x^{(1)}$$

Closed system dynamics

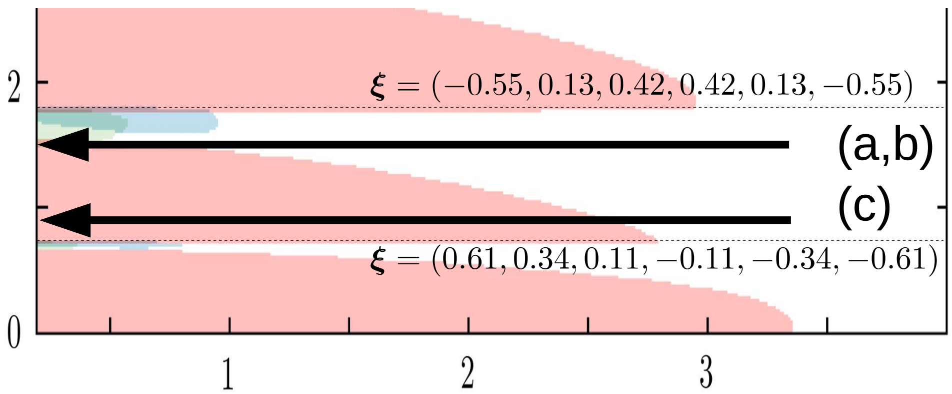
Phonons and spin-phonon coupling:

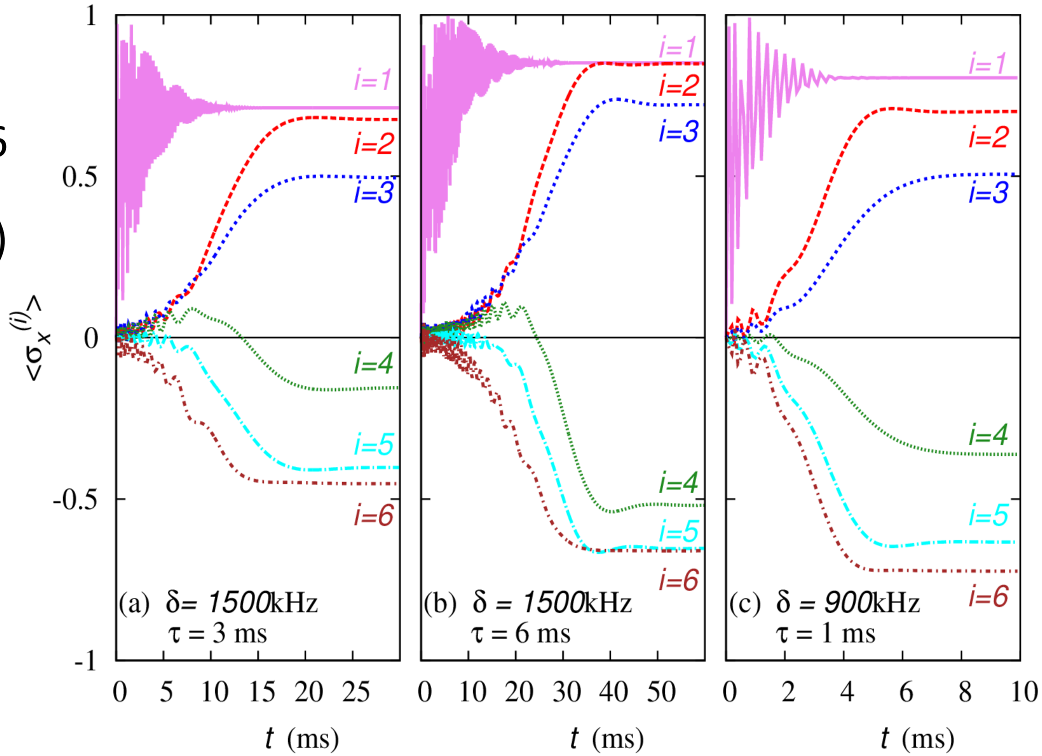
$$H_0(t) = \sum_m \hbar\omega_m a_m^\dagger a_m + \sum_{i,m} \hbar\Omega_i \sqrt{\frac{\omega_{\text{recoil}}}{\omega_m}} \xi_{im} \sin(\omega_{\text{L}} t) \times \sigma_x^i (a_m + a_m^\dagger)$$

With time-dependent transverse field (annealing) and symmetry-breaking bias:

$$H = H_0(t) + B(t) \sum_i \sigma_z^{(i)} + \epsilon_{\text{bias}} \sigma_x^{(1)}$$

Results for N=6 (using Krylov subspace method)





Open system dynamics

Dissipative processes:

Spontaneous emission: σ_x flip

Dephasing: σ_z flip

Open system dynamics

Dissipative processes:

Spontaneous emission: σ_x flip

Dephasing: σ_z flip

Monte Carlo wave function method:

Unitary evolution interrupted by random quantum jumps

Averaged over many runs

524 a reprint from Journal of the Optical Society of America B

Monte Carlo wave-function method in quantum optics

Klaus Mølmer

Institute of Physics and Astronomy, University of Aarhus, DK-8000 Aarhus C, Denmark

Yvan Castin and Jean Dalibard

Laboratoire de Spectroscopie Hertzienne de l'Ecole Normale Supérieure, 24 rue Lhomond, F-75231 Paris Cedex 05, France

Received April 7, 1992; revised manuscript received July 8, 1992

Open system dynamics

Dissipative processes:

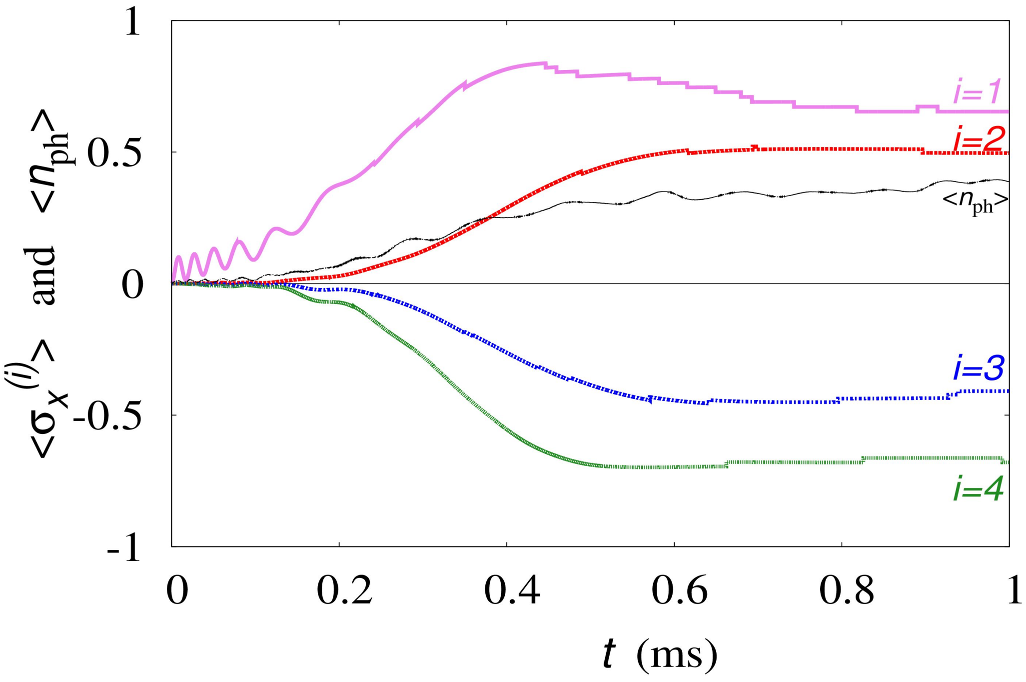
Spontaneous emission: σ_x flip

Dephasing: σ_z flip

Monte Carlo wave function method

Unitary evolution interrupted by random quantum jumps

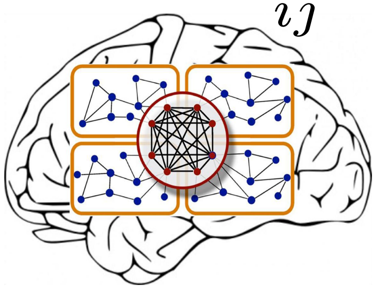
Averaged over many runs



Example of a simple instance with *N*=4.
Noise rate: 1 flip per ms.

Connection to neural networks

$$H = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)}$$

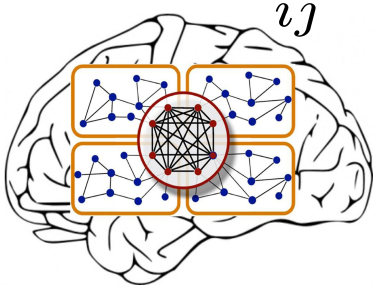


The brain as a spin model:

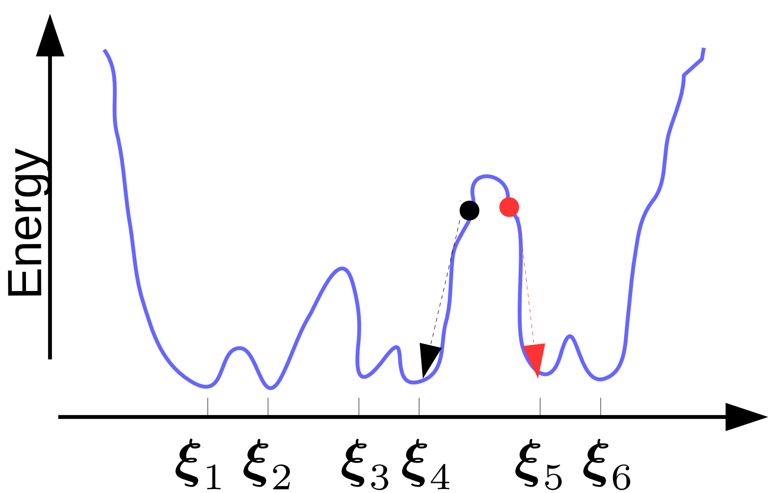
* neurons: firing or not → "spin-1/2" $\sigma_x^{(i)}$
* synapsis: connection between two neurons → coupling J_{ij}
    exitatory synapsis: ferromagnetic
    inhibitory synapsis: antiferromagnetic

Connection to neural networks

$$H = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)}$$



$$J_{ij} = \sum_m \xi_m^{(i)}\xi_m^{(j)}$$



The brain as a spin model:

* neurons: firing or not → "spin-1/2" $\sigma_x^{(i)}$
* synapsis: connection between two neurons → coupling J_{ij}
  exitatory synapsis: ferromagnetic
  inhibitory synapsis: antiferromagnetic

Associative memory:

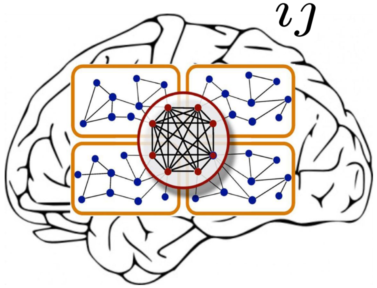
2*N* classical ground states given by patterns $\xi_m^{(i)} = \pm 1$:

$\langle\sigma_x^{(i)}\rangle = \pm\xi_m^{(i)}$

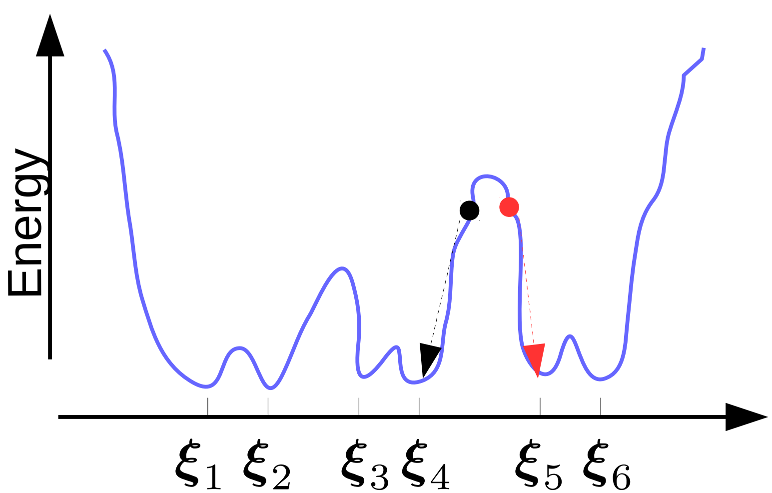
Memorized information (pattern) is retrieved through the dynamics of the model

Connection to neural networks

$$H = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)}$$



$$J_{ij} = \sum_m \xi_m^{(i)}\xi_m^{(j)}$$



The brain as a spin model:

* neurons: firing or not → "spin-1/2" $\sigma_x^{(i)}$
* synapsis: connection between two neurons → coupling J_{ij}
    exitatory synapsis: ferromagnetic
    inhibitory synapsis: antiferromagnetic

Associative memory:

2*N* classical ground states given by patterns $\xi_m^{(i)} = \pm 1$:

$\langle \sigma_x^{(i)} \rangle = \pm \xi_m^{(i)}$

Memorized information (pattern) is retrieved through the dynamics of the model

<u>Selected references:</u>

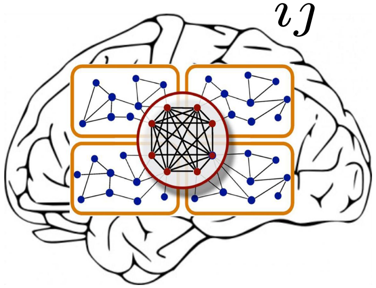
Model: J. Hopfield (PNAS 1982)

Connection to spin glasses: D. Amit, H. Gutfreund, H. Sompolinsky (PRL 1985)

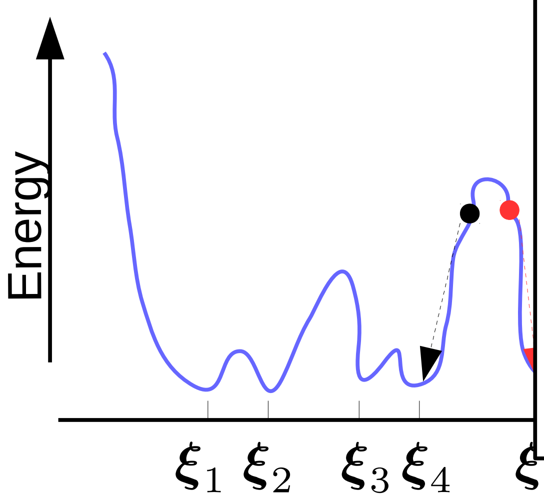
Connection to Dicke models: P. Strack & S. Sachdev (PRL 2011), S. Gopalakrishnan, B. Lev, P. Goldbart (PRL 2011), P. Rotondo, M. Lagomarsino, G. Viola (PRL 2015)

Connection to neural networks

$$H = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)}$$



$$J_{ij} = \sum_m \xi_m^{(i)} \xi$$



The brain as a spin model:

* neurons: firing or not → "spin-1/2" $\sigma_x^{(i)}$
* synapsis: connection between two neurons → coupling J_{ij}

agnetic

rromagnetic

ry:

ven by patterns $\xi_m^{(i)} = \pm 1$:

tern) is retrieved through

982)

: D. Amit, H. Gutfreund, H.

s: P. Strack & S. Sachdev (PRL

B. Lev, P. Goldbart (PRL 2011),

P. Rotondo, M. Lagomarsino, G. Viola (PRL 2015)

Trapped ion devices with associative memory, e.g. for pattern recognition?

$$J_{ij} \propto \sum_\mu \Omega_\mu^2 \sum_m \frac{\xi_m^{(i)}\xi_m^{(j)}}{\omega_m^2 - \omega_{\mathrm{L},\mu}^2}$$

Quantum ground state patterns

Classical Hamiltonian

$$H = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)} \longrightarrow H = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)} + B \sum_i \sigma_z^{(i)}$$

Quantum Hamiltonian

Classical ground state reflects the mode pattern.

Symmetry $\sigma_x \to -\sigma_x$ maintained, but 2-fold degeneracy broken: $\Rightarrow \langle \sigma_x^{(i)} \rangle = 0.$

Quantum ground state patterns

Classical Hamiltonian

$$H = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)} \longrightarrow$$

Classical ground state reflects the mode pattern.

Quantum Hamiltonian

$$H = -\sum_{ij} J_{ij} \sigma_x^{(i)} \sigma_x^{(j)} + B \sum_i \sigma_z^{(i)}$$

Symmetry $\sigma_x \to -\sigma_x$ maintained, but 2-fold degeneracy broken:

$$\Rightarrow \langle \sigma_x^{(i)} \rangle = 0.$$

GS for strong *B*-field: $|\mathrm{GS}\rangle = | \uparrow \ldots \uparrow \rangle_z$

Hamiltonian for low excitations (one spin flip):

$$\tilde{J}_{ij} = -\xi_m^{(i)} \xi_m^{(j)}$$

Has one eigenvector with non-zero eigenvalue:

$$\tilde{J} \mathbf{x} = -(\boldsymbol{\xi}_m \cdot \mathbf{x}) \boldsymbol{\xi}_m = -\boldsymbol{\xi}_m \Leftrightarrow \mathbf{x} = \boldsymbol{\xi}_m$$

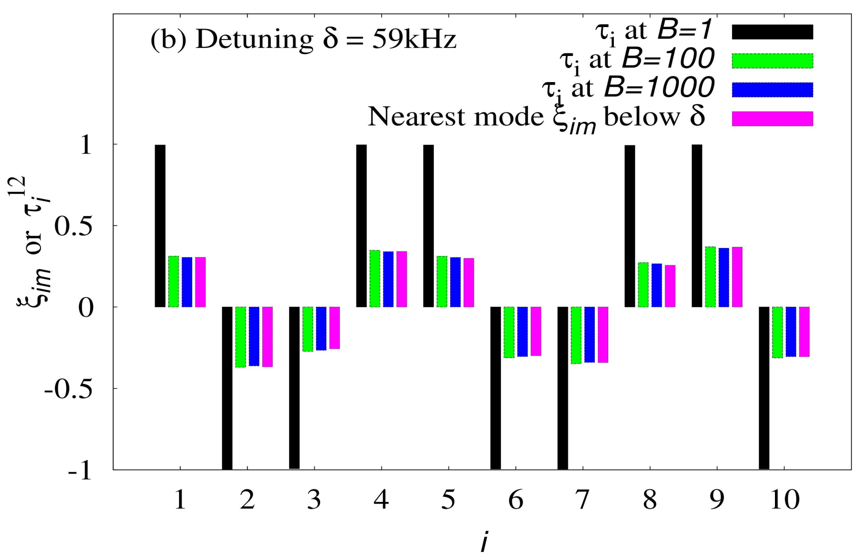
$$\Rightarrow \tau_i \equiv \langle \mathrm{GS} | \sigma_x^{(i)} | 1\mathrm{EX} \rangle = \xi_m^{(i)}$$

Quantum ground state patterns

Classical Hamiltonian

$$H = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)}$$

Ground state pattern given by nearest ferromagnetically coupled mode.



Quantum Hamiltonian

$$\longrightarrow \quad H = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)} + B\sum_i \sigma_z^{(i)}$$

Symmetry $\sigma_x \to -\sigma_x$ maintained, but 2-fold degeneracy broken:

$$\Rightarrow \langle \sigma_x^{(i)} \rangle = 0.$$

GS for strong B-field: $|\mathrm{GS}\rangle = |\uparrow \ldots \uparrow\rangle_z$

Hamiltonian for low excitations (one spin flip):

$$\tilde{J}_{ij} = -\xi_m^{(i)}\xi_m^{(j)}$$

Has one eigenvector with non-zero eigenvalue:

$$\tilde{J}\mathbf{x} = -(\boldsymbol{\xi}_m \cdot \mathbf{x})\boldsymbol{\xi}_m = -\boldsymbol{\xi}_m \Leftrightarrow \mathbf{x} = \boldsymbol{\xi}_m$$

$$\Rightarrow \tau_i \equiv \langle \mathrm{GS}|\sigma_x^{(i)}|1\mathrm{EX}\rangle = \xi_m^{(i)}$$

Quantum ground state patterns

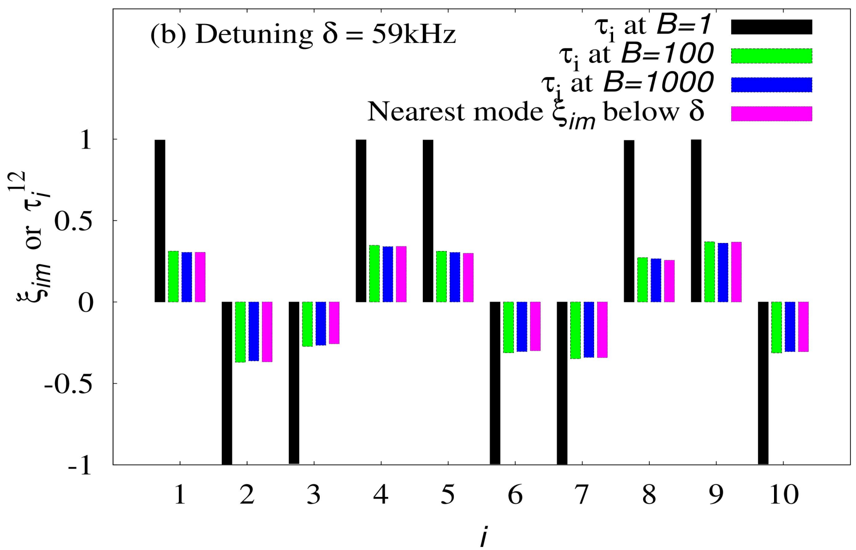
Classical Hamiltonian

$$H = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)} \longrightarrow H = -\sum_{ij} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)} + B\sum_i \sigma_z^{(i)}$$

Quantum Hamiltonian

Binary memory

Real valued memory



GS for strong *B*-field: $|\mathrm{GS}\rangle = |\uparrow \ldots \uparrow\rangle_z$

Hamiltonian for low excitations (one spin flip):

$$\tilde{J}_{ij} = -\xi_m^{(i)}\xi_m^{(j)}$$

Has one eigenvector with non-zero eigenvalue:

$$\tilde{J}\mathbf{x} = -(\boldsymbol{\xi}_m \cdot \mathbf{x})\boldsymbol{\xi}_m = -\boldsymbol{\xi}_m \Leftrightarrow \mathbf{x} = \boldsymbol{\xi}_m$$

$$\Rightarrow \tau_i \equiv \langle \mathrm{GS}|\sigma_x^{(i)}|1\mathrm{EX}\rangle = \xi_m^{(i)}$$

Summary & Outlook

arXiv.org > cond-mat > arXiv:1507.07863

Condensed Matter > Quantum Gases

Controlled complexity in trapped ions: from quantum Mattis glasses to number partitioning

Tobias Graß, David Raventós, Bruno Juliá-Díaz, Christian Gogolin, Maciej Lewenstein

(Submitted on 28 Jul 2015)

David Raventós (ICFO)

Bruno Juliá-Díaz (UB, ICFO)

Christian Gogolin (ICFO,MPQ)

Maciej Lewenstein (ICFO, ICREA)

Trapped ions can be used as:

→ spin glass solver (classical or quantum Mattis glass)

→ solver of number partitioning problem
(either trivial with parity symmetry or NP-hard)

→ flexible quantum annealer (alternatives to D-Wave)
Test and optimize annealing protocols!

→ (quantum) neural network
Pattern recognition (with real-valued data sets)

My work on ions

SU(3) models and quantum chaos

PRL 111, 090404 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Quantum Chaos in SU(3) Models with Trapped Ions

Tobias Graß,[1] Bruno Juliá-Díaz,[1,2] Marek Kuś,[3] and Maciej Lewenstein[1,4]
[1]ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain
[2]Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, 08028 Barcelona, Spain
[3]Center for Theoretical Physics, Polish Academy of Sciences, 02-668 Warszawa, Poland
[4]ICREA—Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain
(Received 29 May 2013; published 28 August 2013)

Artificial magnetic fluxes

PHYSICAL REVIEW A 91, 063612 (2015)

Synthetic magnetic fluxes and topological order in one-dimensional spin systems

Tobias Graß,[1] Christine Muschik,[1,2,3] Alessio Celi,[1] Ravindra W. Chhajlany,[1,4] and Maciej Lewenstein[1,5]
[1]ICFO-Institut de Ciències Fotòniques, Av. Carl Friedrich Gauss 3, 08860 Barcelona, Spain
[2]Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria
[3]Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria
[4]Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland
[5]ICREA-Institució Catalana de Recerca i Estudis Avançats, Lluis Campanys 23, 08010 Barcelona, Spain
(Received 13 January 2015; revised manuscript received 8 April 2015; published 11 June 2015)

Heisenberg models

Graß and Lewenstein *EPJ Quantum Technology* 2014, 1:8
http://www.epjquantumtechnology.com/content/1/1/8

EPJ.org

EPJ Quantum Technology
a SpringerOpen Journal

RESEARCH Open Access

Trapped-ion quantum simulation of tunable-range Heisenberg chains

Tobias Graß[1*] and Maciej Lewenstein[1,2]

Mattis glass and number partitioning

arXiv.org > cond-mat > arXiv:1507.07863

Condensed Matter > Quantum Gases

Controlled complexity in trapped ions: from quantum Mattis glasses to number partitioning

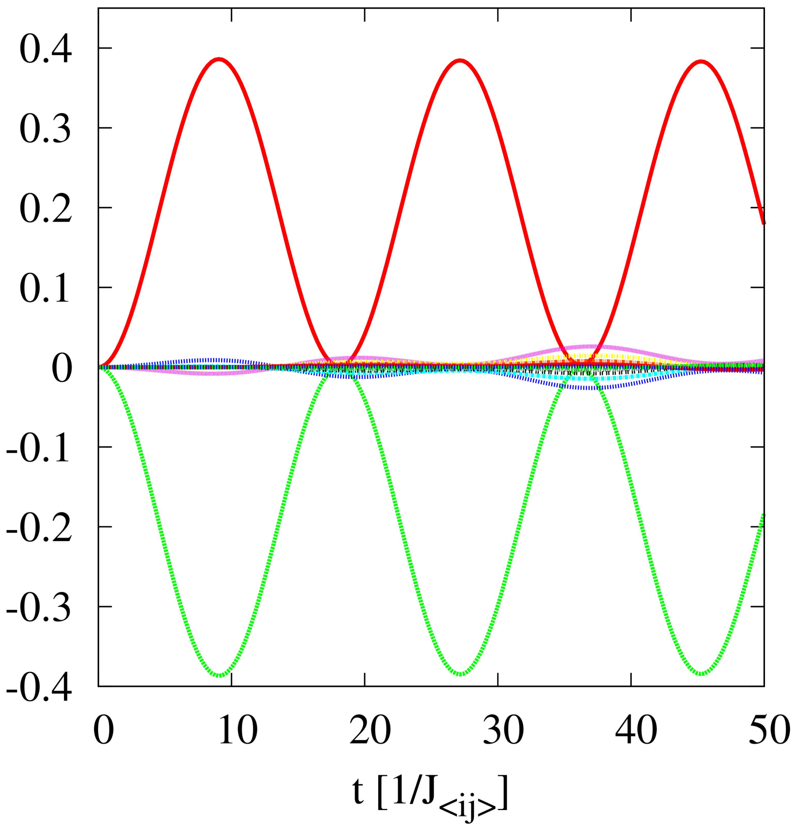
Tobias Graß, David Raventós, Bruno Juliá-Díaz, Christian Gogolin, Maciej Lewenstein

(Submitted on 28 Jul 2015)

Tunable-range Heisenberg chains

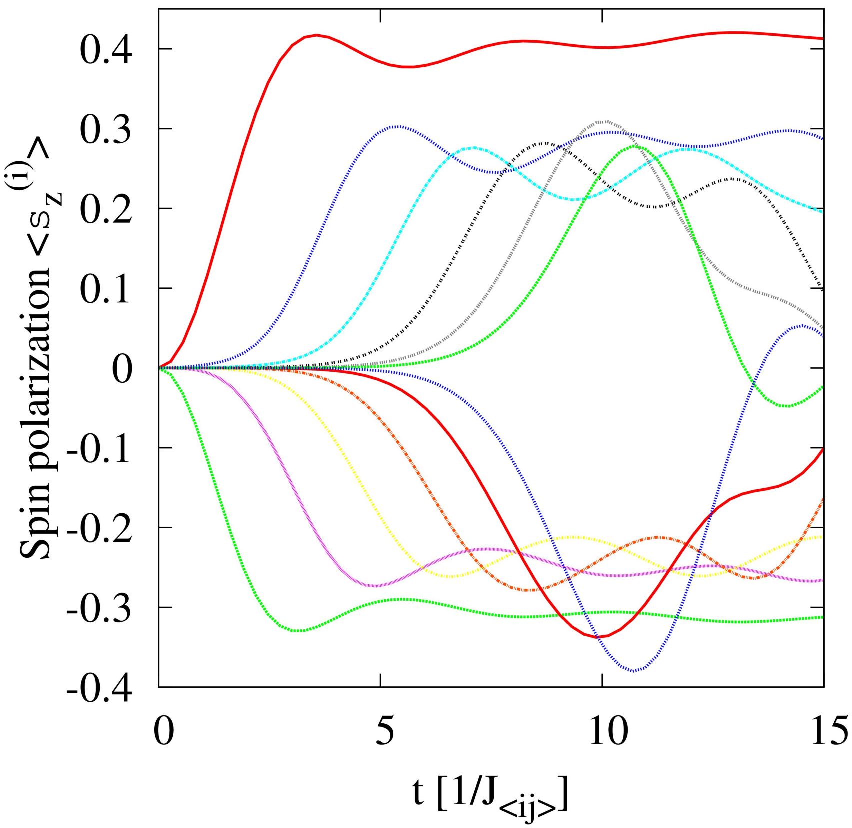
Long-range side $(\sim r^0)$

→ Dimerization
→ Excitations localize



Short-range side $(\sim r^{-3})$

→ Quasi-long-range order
→ Fast propagation of excitations



My work on ions

SU(3) models and quantum chaos

PRL 111, 090404 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Quantum Chaos in SU(3) Models with Trapped Ions

Tobias Graß,[1] Bruno Juliá-Díaz,[1,2] Marek Kuś,[3] and Maciej Lewenstein[1,4]
[1]*ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain*
[2]*Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, 08028 Barcelona, Spain*
[3]*Center for Theoretical Physics, Polish Academy of Sciences, 02-668 Warszawa, Poland*
[4]*ICREA—Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*
(Received 29 May 2013; published 28 August 2013)

Artificial magnetic fluxes

PHYSICAL REVIEW A 91, 063612 (2015)

Synthetic magnetic fluxes and topological order in one-dimensional spin systems

Tobias Graß,[1] Christine Muschik,[1,2,3] Alessio Celi,[1] Ravindra W. Chhajlany,[1,4] and Maciej Lewenstein[1,5]
[1]*ICFO-Institut de Ciències Fotòniques, Av. Carl Friedrich Gauss 3, 08860 Barcelona, Spain*
[2]*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria*
[3]*Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria*
[4]*Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*
[5]*ICREA-Institució Catalana de Recerca i Estudis Avançats, Lluis Campanys 23, 08010 Barcelona, Spain*
(Received 13 January 2015; revised manuscript received 8 April 2015; published 11 June 2015)

Heisenberg models

Graß and Lewenstein *EPJ Quantum Technology* 2014, 1:8
http://www.epjquantumtechnology.com/content/1/1/8
EPJ.org
EPJ Quantum Technology
a SpringerOpen Journal
RESEARCH Open Access

Trapped-ion quantum simulation of tunable-range Heisenberg chains

Tobias Graß[1*] and Maciej Lewenstein[1,2]

Mattis glass and number partitioning

arXiv.org > cond-mat > arXiv:1507.07863
Condensed Matter > Quantum Gases

Controlled complexity in trapped ions: from quantum Mattis glasses to number partitioning

Tobias Graß, David Raventós, Bruno Juliá-Díaz, Christian Gogolin, Maciej Lewenstein

(Submitted on 28 Jul 2015)

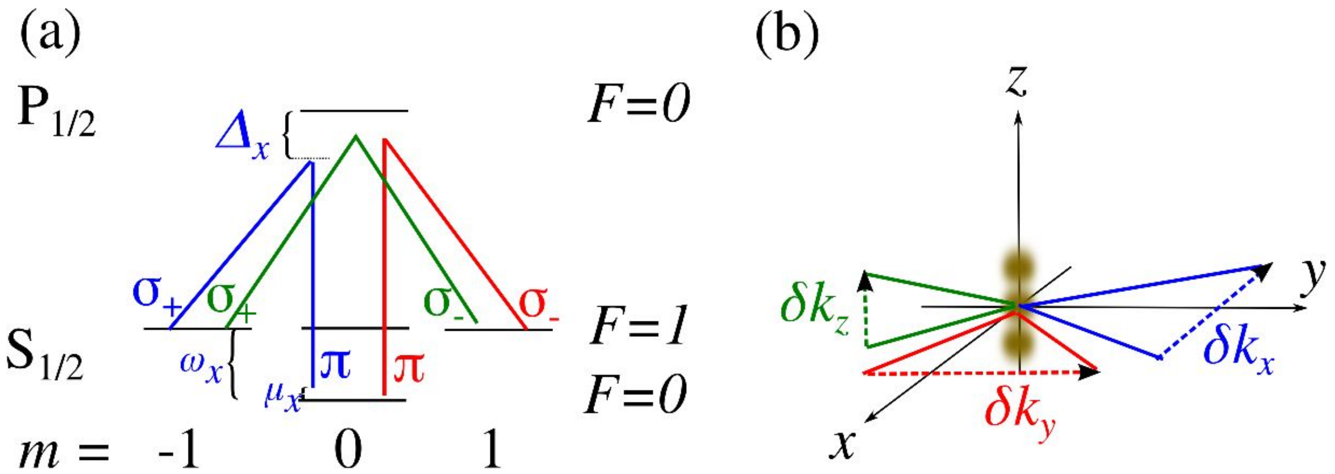
Quantum chaos in an SU(3) model

Motivation

System with SU(3) algebra do not have a unique classical limit. Dynamics of the system (chaotic or regular?) depends on the representation. [Gnutzmann, Haake, Kuś, J. Phys. A (2000)]

Goal

Develop quantum simulations with SU(3) systems



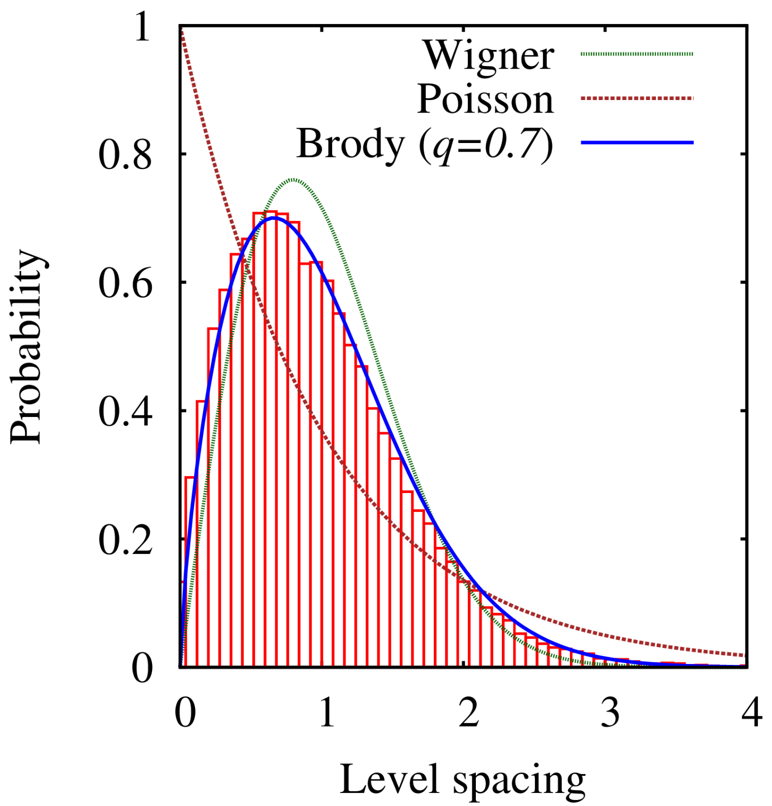
Yields Lipkin-Meshkov-Glick model

$$H_{\mathrm{LMG}} = J \sum_{i,j} \{\text{spin flip site } i\} \times \{\text{same flip site } j\} + \text{magnetic field}$$

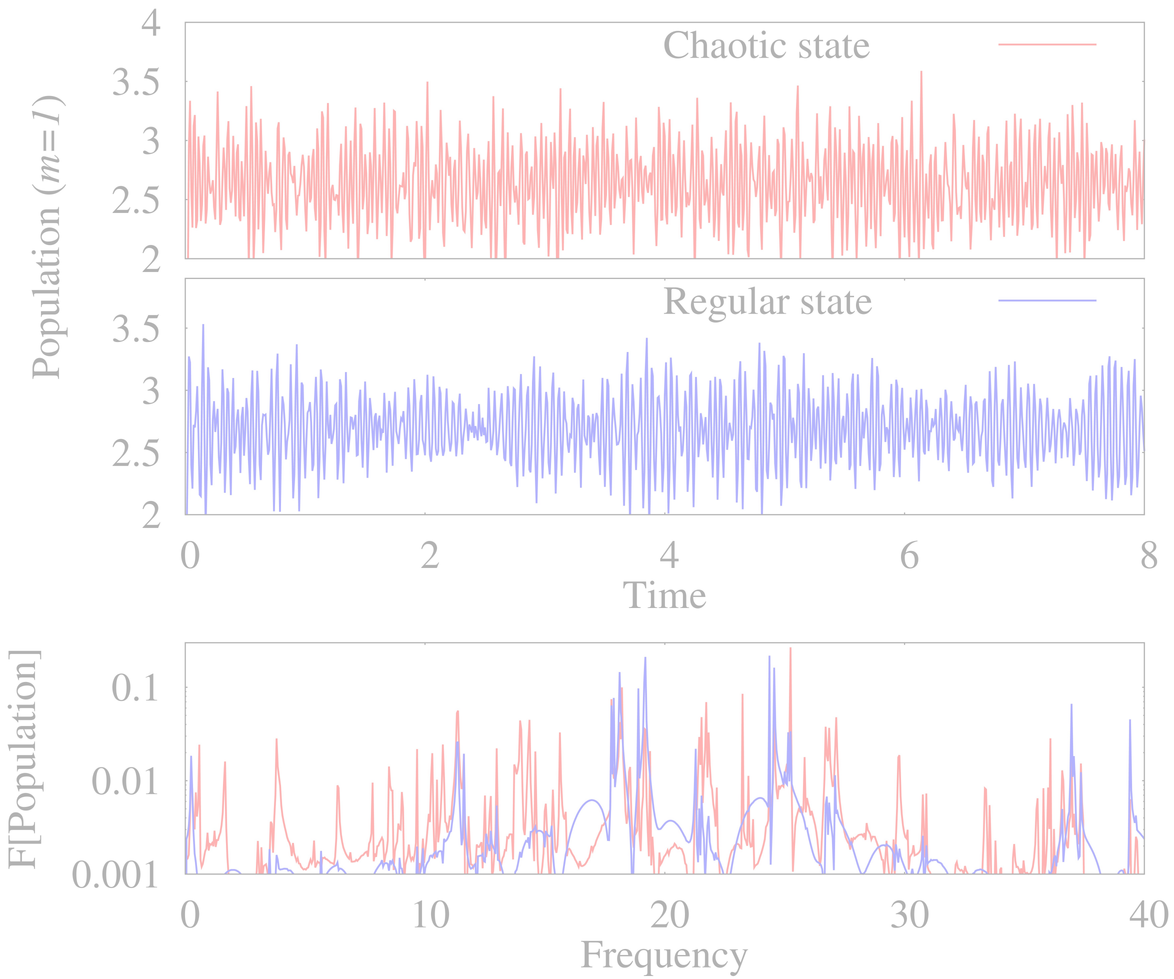
Quantum chaos in an SU(3) model

Signatures of quantum chaos

Level spacing distribution:



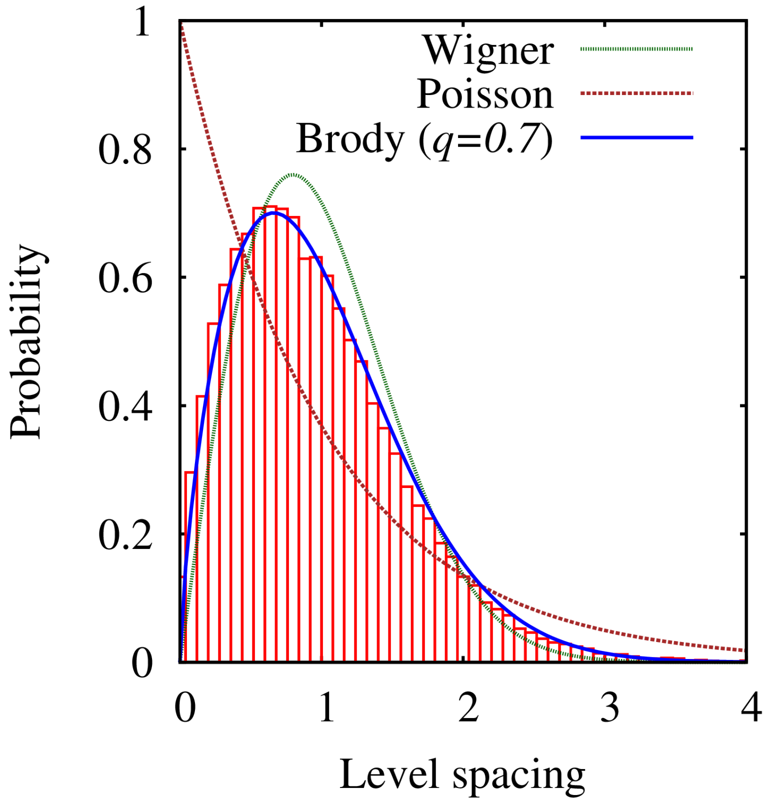
Time evolution of observables:



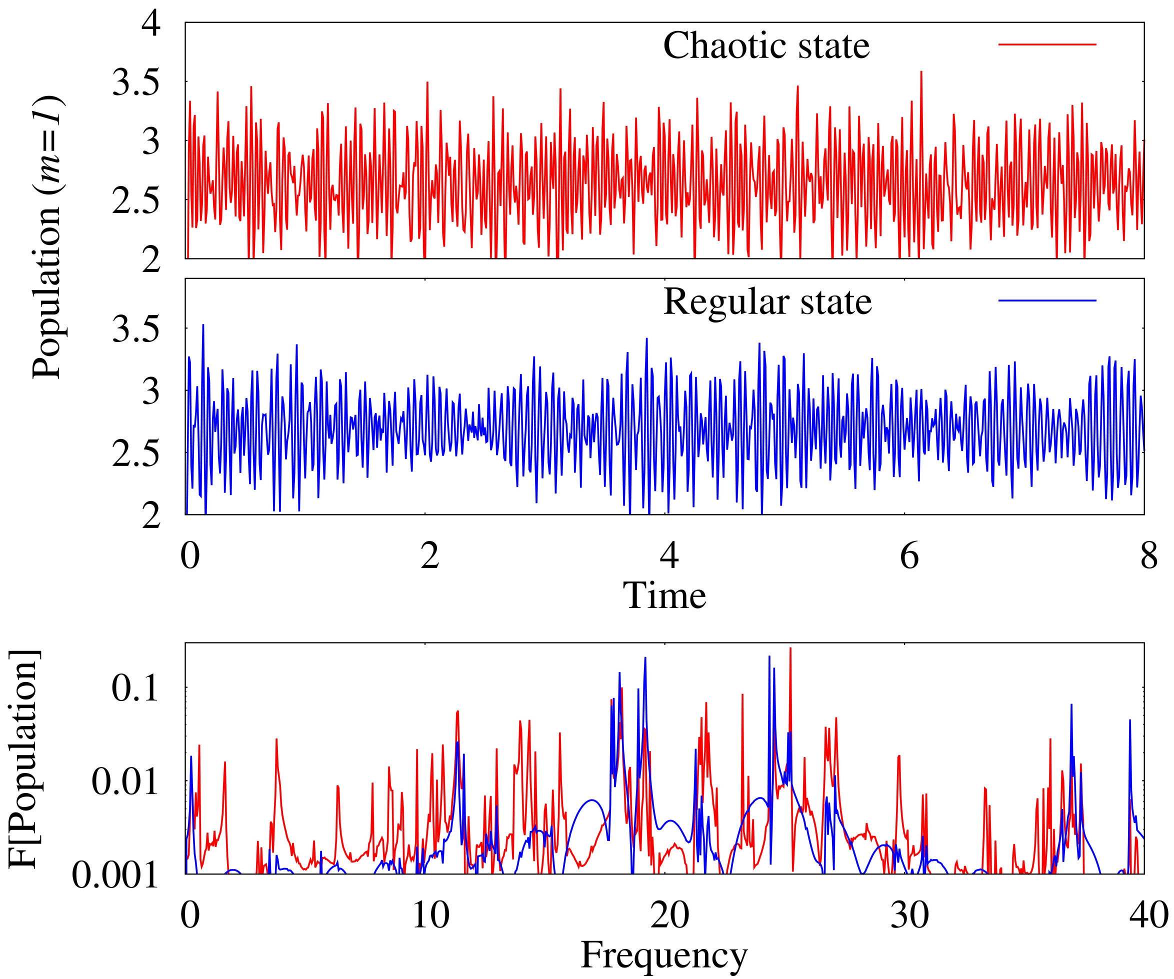
Quantum chaos in an SU(3) model

Signatures of quantum chaos

Level spacing distribution:



Time evolution of observables:



My work on ions

SU(3) models and quantum chaos

PRL 111, 090404 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Quantum Chaos in SU(3) Models with Trapped Ions

Tobias Graß,[1] Bruno Juliá-Díaz,[1,2] Marek Kuś,[3] and Maciej Lewenstein[1,4]
[1]ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain
[2]Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, 08028 Barcelona, Spain
[3]Center for Theoretical Physics, Polish Academy of Sciences, 02-668 Warszawa, Poland
[4]ICREA—Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain
(Received 29 May 2013; published 28 August 2013)

Artificial magnetic fluxes

PHYSICAL REVIEW A **91**, 063612 (2015)

Synthetic magnetic fluxes and topological order in one-dimensional spin systems

Tobias Graß,[1] Christine Muschik,[1,2,3] Alessio Celi,[1] Ravindra W. Chhajlany,[1,4] and Maciej Lewenstein[1,5]
[1]*ICFO-Institut de Ciències Fotòniques, Av. Carl Friedrich Gauss 3, 08860 Barcelona, Spain*
[2]*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria*
[3]*Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria*
[4]*Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*
[5]*ICREA-Institució Catalana de Recerca i Estudis Avançats, Lluis Campanys 23, 08010 Barcelona, Spain*
(Received 13 January 2015; revised manuscript received 8 April 2015; published 11 June 2015)

Heisenberg models

Graß and Lewenstein *EPJ Quantum Technology* 2014, 1:8
http://www.epjquantumtechnology.com/content/1/1/8
EPJ .org
EPJ Quantum Technology
a SpringerOpen Journal
RESEARCH Open Access

Trapped-ion quantum simulation of tunable-range Heisenberg chains

Tobias Graß[1*] and Maciej Lewenstein[1,2]

Mattis glass and number partitioning

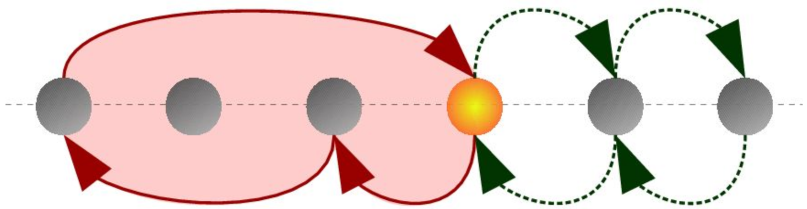
arXiv.org > cond-mat > arXiv:1507.07863
Condensed Matter > Quantum Gases

Controlled complexity in trapped ions: from quantum Mattis glasses to number partitioning

Tobias Graß, David Raventós, Bruno Juliá-Díaz, Christian Gogolin, Maciej Lewenstein
(Submitted on 28 Jul 2015)

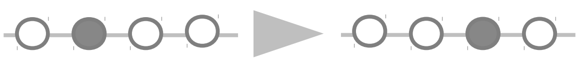
Artificial magnetic fluxes in 1D systems

Non-trivial loops in 1D via long-range links

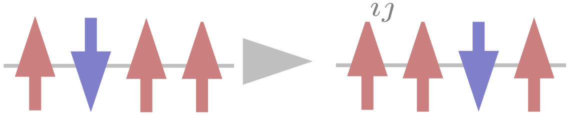


Mapping: XY model ↔ hopping hard-core bosons

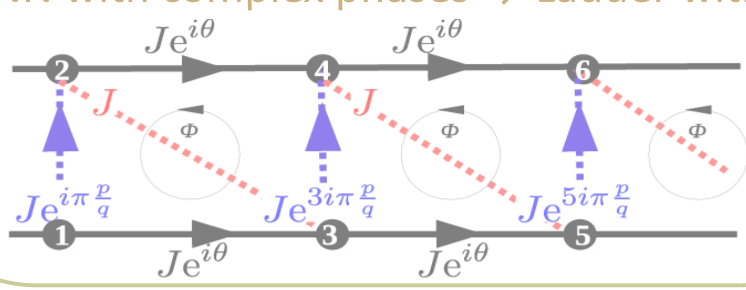
Hopping: $H = -J \sum_{ij} a_i^\dagger a_j$



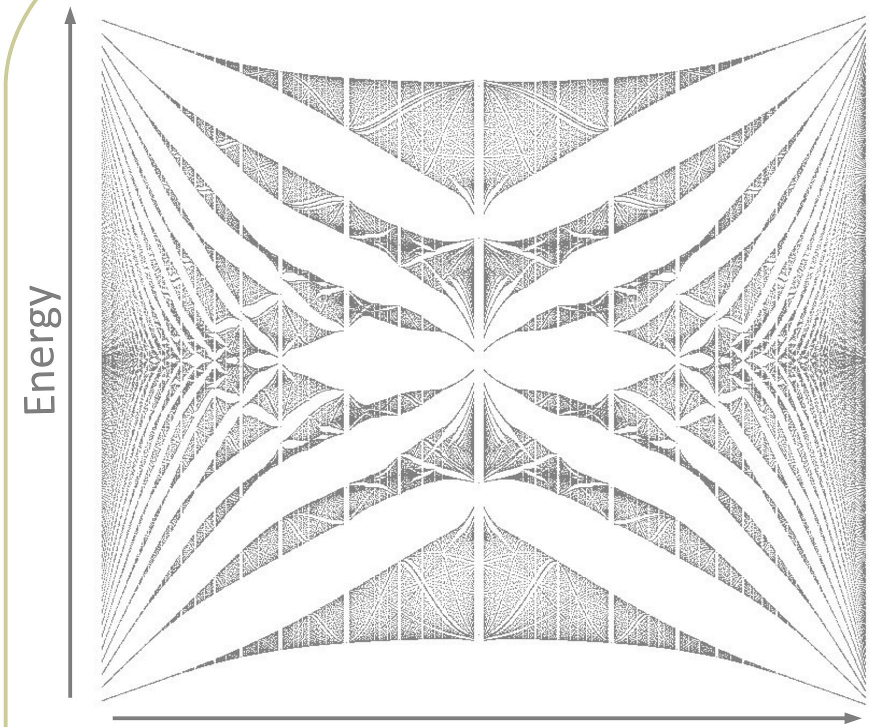
Spin flip: $H = -J \sum_{ij} \sigma_i^+ \sigma_j^-$



NN+NNN with complex phases → Ladder with fluxes



Butterfly spectrum

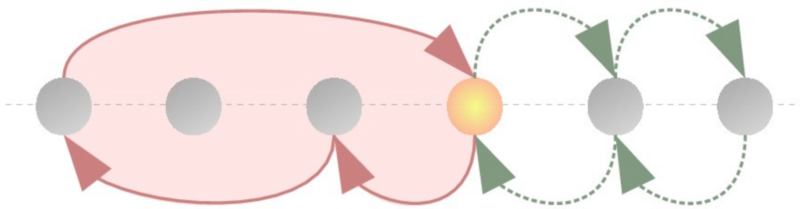


→ Topological band structure (edge states, Chern numbers)

→ Chern insulating phases of bosons

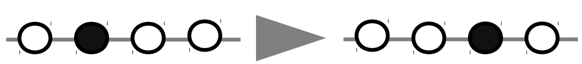
Artificial magnetic fluxes in 1D systems

Non-trivial loops in 1D via long-range links

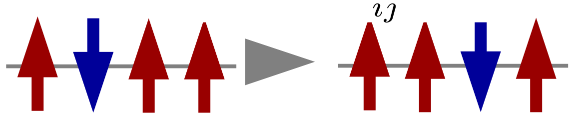


Mapping: XY model ↔ hopping hard-core bosons

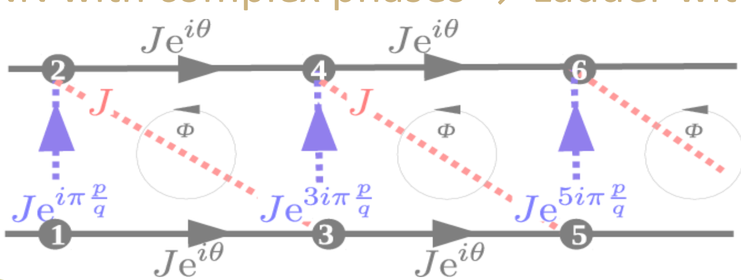
Hopping: $H = -J \sum_{ij} a_i^\dagger a_j$



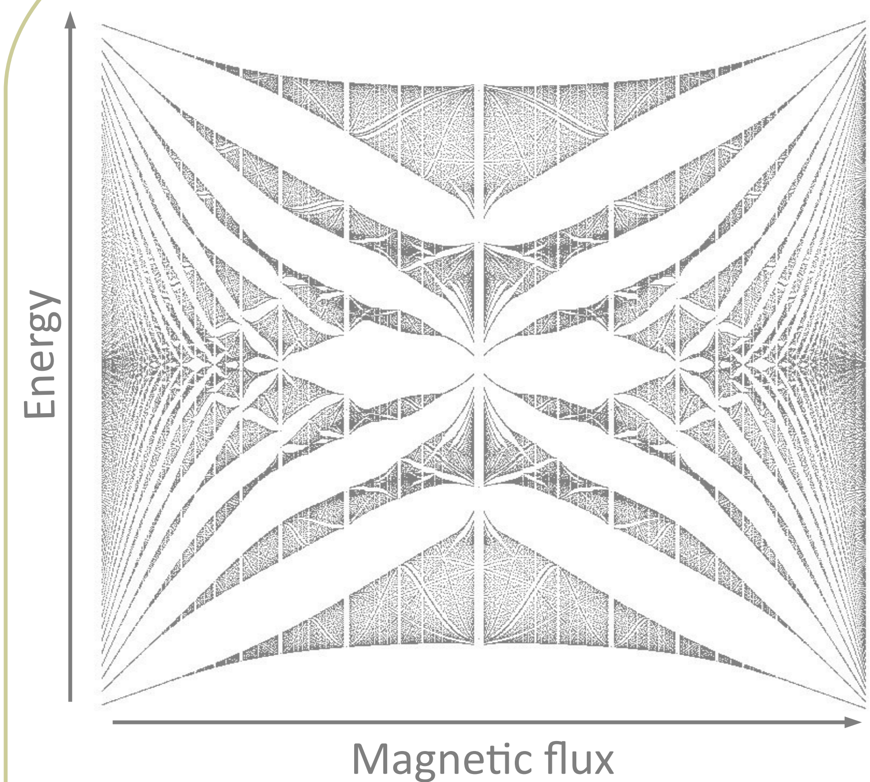
Spin flip: $H = -J \sum_{ij} \sigma_i^+ \sigma_j^-$



NN+NNN with complex phases → Ladder with fluxes



Butterfly spectrum

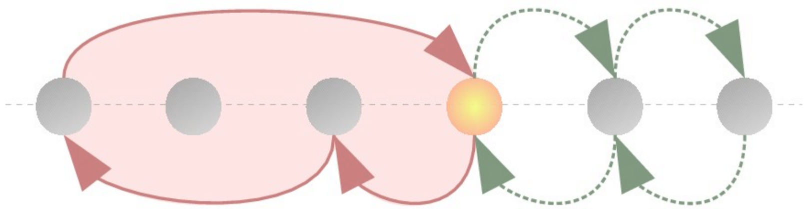


→ Topological band structure (edge states, Chern numbers)

→ Chern insulating phases of bosons

Artificial magnetic fluxes in 1D systems

Non-trivial loops in 1D via long-range links

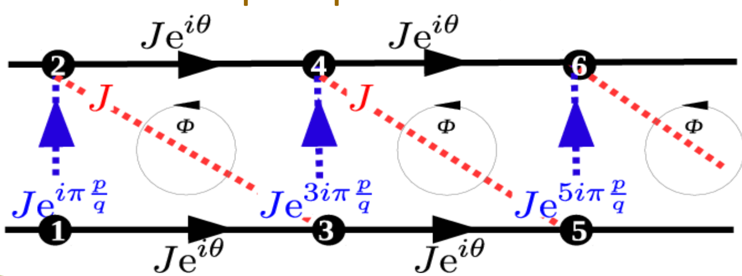


Mapping: XY model ↔ hopping hard-core bosons

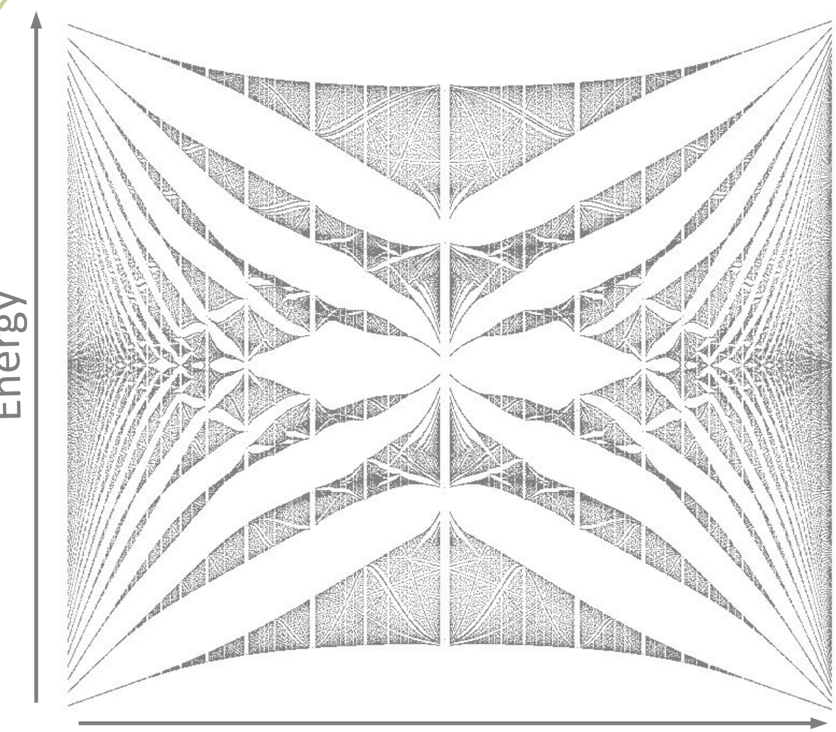
Hopping: $H = -J \sum_{ij} a_i^\dagger a_j$

Spin flip: $H = -J \sum_{ij} \sigma_i^+ \sigma_j^-$

NN+NNN with complex phases → Ladder with fluxes



Butterfly spectrum

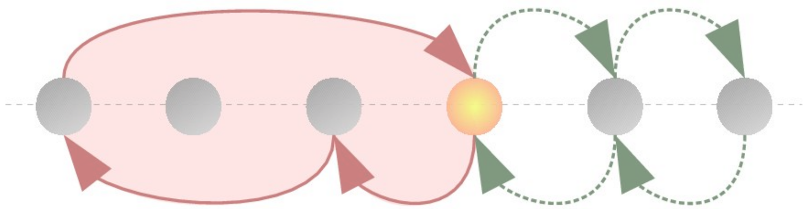


→ Topological band structure (edge states, Chern numbers)

→ Chern insulating phases of bosons

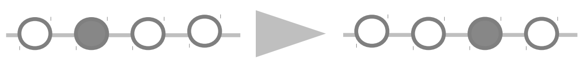
Artificial magnetic fluxes in 1D systems

Non-trivial loops in 1D via long-range links

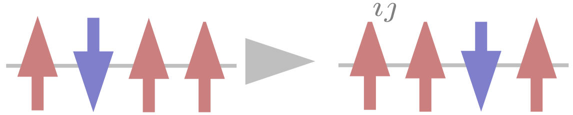


Mapping: XY model ↔ hopping hard-core bosons

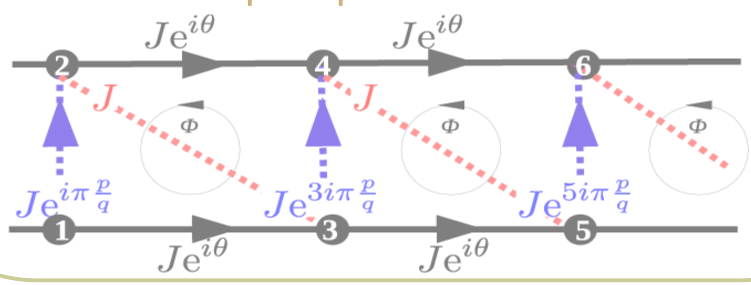
Hopping: $H = -J \sum_{ij} a_i^\dagger a_j$



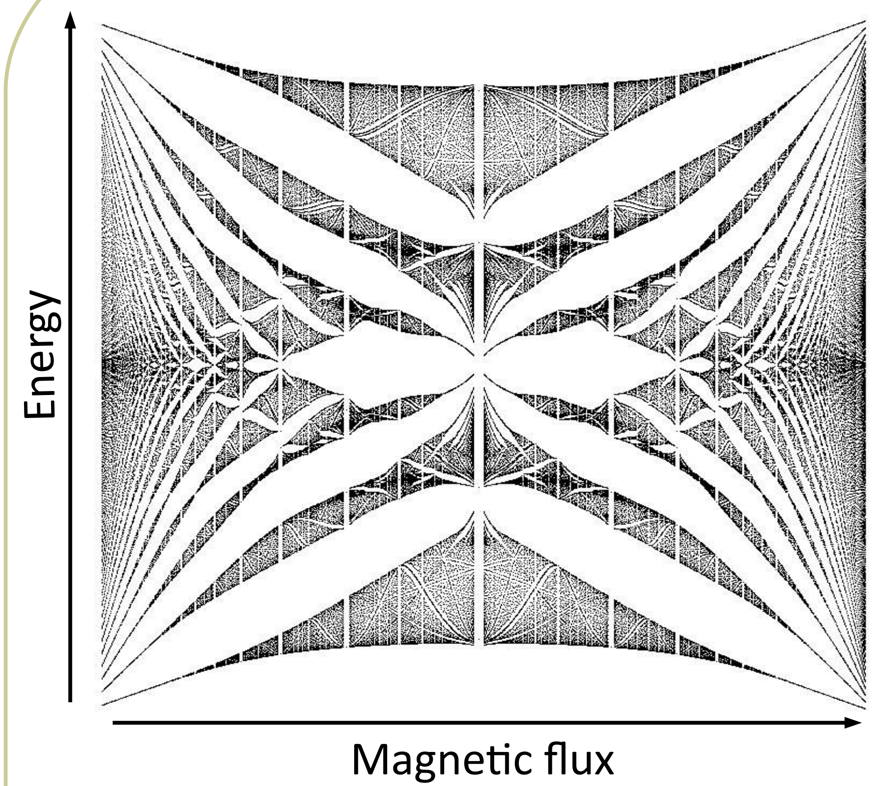
Spin flip: $H = -J \sum_{ij} \sigma_i^+ \sigma_j^-$



NN+NNN with complex phases → Ladder with fluxes



Butterfly spectrum



→ Topological band structure (edge states, Chern numbers)

→ Chern insulating phases of bosons

Summary

SU(3) models and quantum chaos

PRL **111,** 090404 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Quantum Chaos in SU(3) Models with Trapped Ions

Tobias Graß,[1] Bruno Juliá-Díaz,[1,2] Marek Kuś,[3] and Maciej Lewenstein[1,4]
[1]*ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain*
[2]*Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, 08028 Barcelona, Spain*
[3]*Center for Theoretical Physics, Polish Academy of Sciences, 02-668 Warszawa, Poland*
[4]*ICREA—Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*
(Received 29 May 2013; published 28 August 2013)

Artificial magnetic fluxes

PHYSICAL REVIEW A **91**, 063612 (2015)

Synthetic magnetic fluxes and topological order in one-dimensional spin systems

Tobias Graß,[1] Christine Muschik,[1,2,3] Alessio Celi,[1] Ravindra W. Chhajlany,[1,4] and Maciej Lewenstein[1,5]
[1]*ICFO-Institut de Ciències Fotòniques, Av. Carl Friedrich Gauss 3, 08860 Barcelona, Spain*
[2]*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria*
[3]*Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria*
[4]*Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*
[5]*ICREA-Institució Catalana de Recerca i Estudis Avançats, Lluis Campanys 23, 08010 Barcelona, Spain*
(Received 13 January 2015; revised manuscript received 8 April 2015; published 11 June 2015)

Heisenberg models

Graß and Lewenstein *EPJ Quantum Technology* 2014, 1:8
http://www.epjquantumtechnology.com/content/1/1/8
EPJ .org
EPJ Quantum Technology
a SpringerOpen Journal
RESEARCH Open Access

Trapped-ion quantum simulation of tunable-range Heisenberg chains

Tobias Graß[1*] and Maciej Lewenstein[1,2]

Mattis glass and number partitioning

arXiv.org > cond-mat > arXiv:1507.07863
Condensed Matter > Quantum Gases

Controlled complexity in trapped ions: from quantum Mattis glasses to number partitioning

Tobias Graß, David Raventós, Bruno Juliá-Díaz, Christian Gogolin, Maciej Lewenstein

(Submitted on 28 Jul 2015)