Fractional quantum Hall effect of atoms: From state preparation to detection of anyons

Tobias Grass

ICFO

15th October 2020

Cold Atoms Workshop

https://sites.google.com/view/coldatoms2020/

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arXiv.org > cond-mat > arXiv:2009.08943

[Part I]

Condensed Matter > Quantum Gases

[Submitted on 18 Sep 2020]

Preparation of the 1/2-Laughlin state with atoms in a rotating trap

Bárbara Andrade, Valentin Kasper, Maciej Lewenstein, Christof Weitenberg, Tobias Graß

Barbara Andrade

Valentin Kasper

Christof Weitenberg (Uni Hamburg)

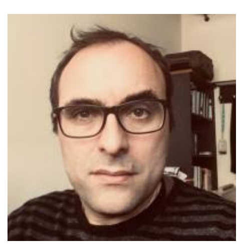
Maciej Lewenstein

[Part II]

Fractional Angular Momentum and Anyon Statistics of Impurities in Laughlin Liquids

Tobias Graß, Bruno Juliá-Díaz, Niccolò Baldelli, Utso Bhattacharya, and Maciej Lewenstein

Phys. Rev. Lett. **125**, 136801 – Published 21 September 2020



Bruno Julia-Diaz (U Barcelona)

Nicollo Baldelli

Utso Bhattacharya

Fractional quantum Hall effect

Originally: Transport phenomenon of 2d electron gas in strong magnetic field [1998: Tsui, Stoermer, Laughlin]

Laughlin wave function:

strong anticorrelations!

$$\Psi_{\mathrm{L}}^{(q)} = \prod_{i<j} (z_i - z_j)^q \exp[-\sum_i |z_i|^2/2]$$

$(z = x + iy)$

Fixes filling factor (within lowest Landau level): $\nu = 1/q$

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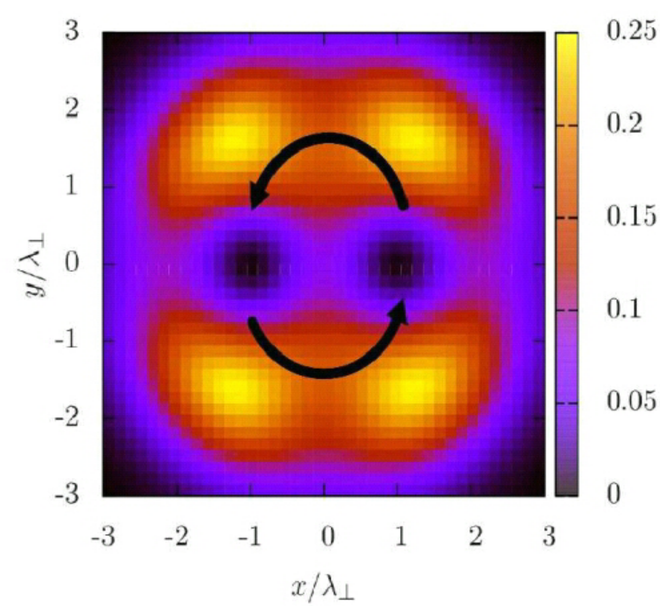
Anyonic bulk excitations:

Wave function for two quasiholes:

$$\Psi_{2\mathrm{qh}} \sim \prod_i (w_1 - z_i)(w_2 - z_i)\Psi_{\mathrm{L}}^{(q)}$$

acquires a **fractional statistical phase**

$$w_1 \leftrightarrow w_2 \Leftrightarrow \Psi_{2\mathrm{qh}} \to e^{i\alpha\pi}\Psi_{2\mathrm{qh}}$$ where $\alpha = 1/q$:



FQH physics in rotating atomic gas

Coriolis force is equivalent to Lorentz force.

Synthetic ‘magnetic’ field:

$$H_0 = \frac{\mathbf{p}^2}{2M} + \frac{M}{2}\omega^2\mathbf{r}^2 - \Omega L_z = \frac{|\mathbf{p} - M\mathbf{\Omega}\times\mathbf{r}|^2}{2M} + \frac{M}{2}\left(\omega^2 - \mathbf{\Omega}^2\right)\mathbf{r}^2$$

$$M\mathbf{\Omega}\times\mathbf{r} \equiv q\mathbf{A} \Rightarrow q\mathbf{B} = 2M\Omega\hat{z} \qquad (\mathbf{\Omega} = \Omega\hat{z})$$

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- Single-particle levels are Landau levels:

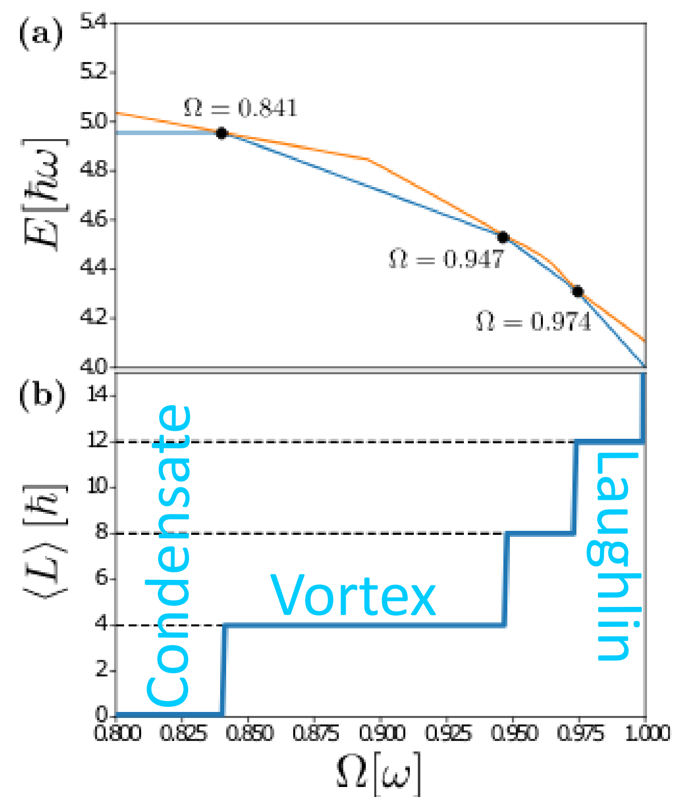
$$E_{nm} = \hbar\left[(\Omega + \omega)n + (\Omega - \omega)m\right] + \text{const.}$$

$$\text{"}\infty\text{"} \qquad \qquad 0$$

Lowest Landau Level – Flat !

- Interactions produce anticorrelations: vortices, vortex lattices, FQH phases

For $\Omega \to \omega$: Laughlin state!



Behavior of N=4 bosons with contact repulsion (g=1)

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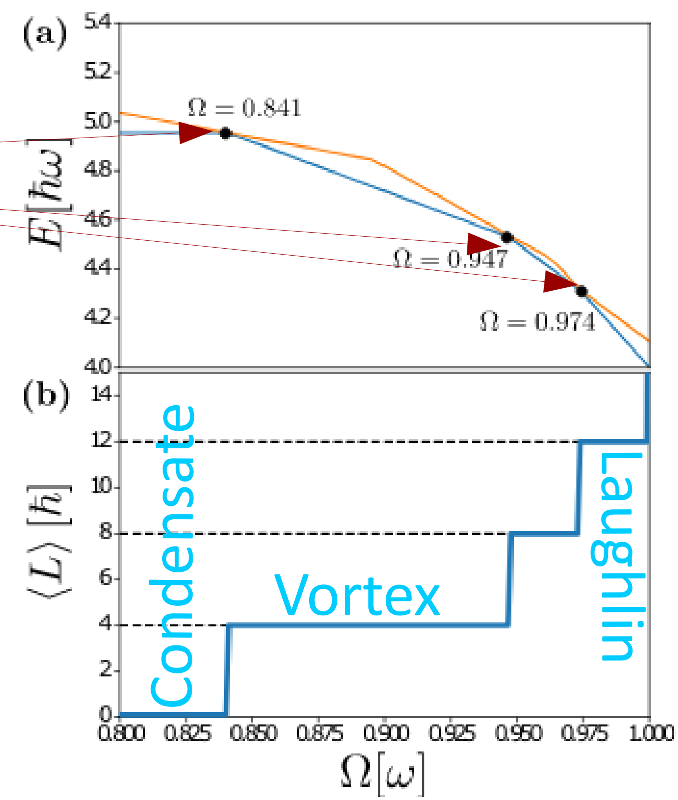
True level crossings

- States protected by rotational symmetry
- To pick up angular momentum:
  Rotational symmetry must be broken!
- Elliptic trap deformation:

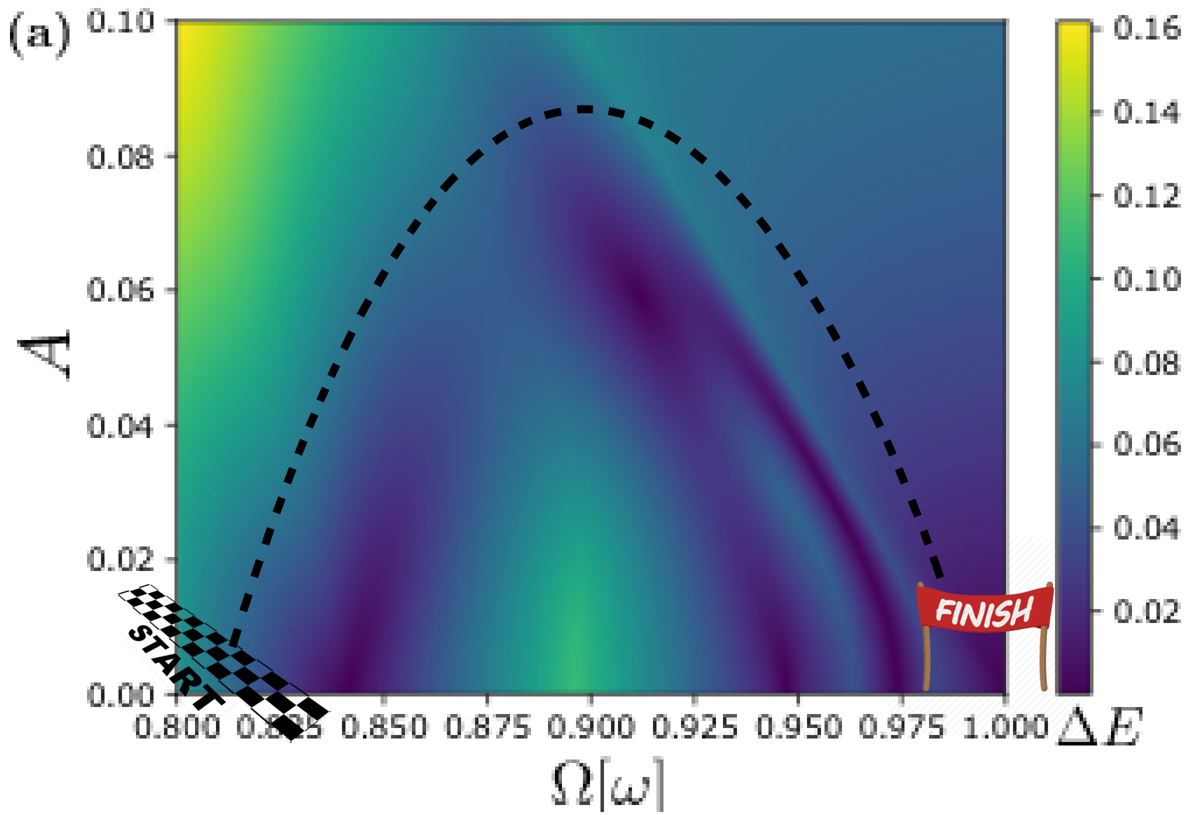
$$V(t) = A(t) M\omega^2 (x^2 - y^2)$$



Behavior of N=4 bosons with contact repulsion (g=1)

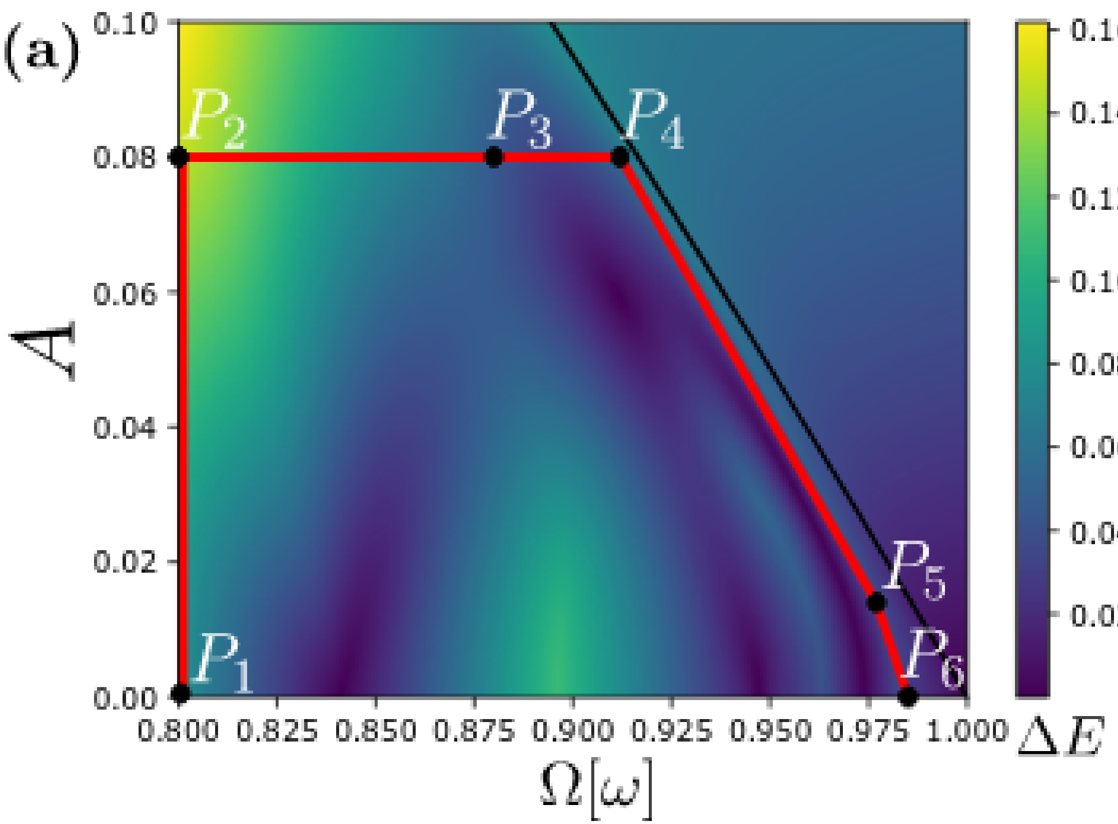
Adiabatic path to Laughlin state

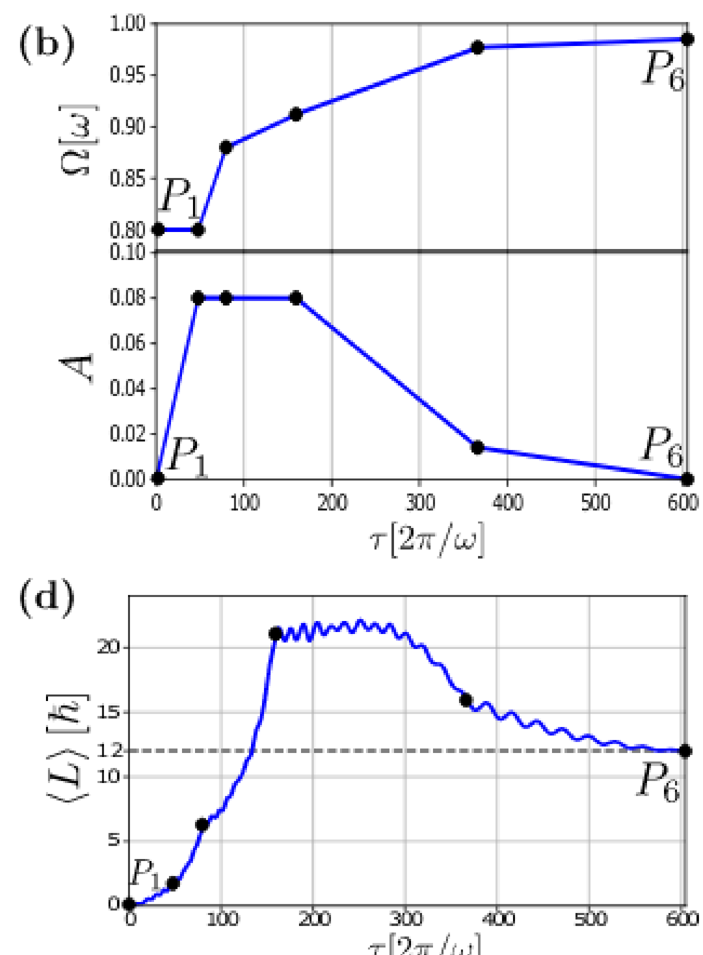
- Tunable parameters: ellipticity *A* and rotation frequency Ω
- Energy gap above ground state:



Adiabatic path to Laughlin state

- Tunable parameters: ellipticity *A* and rotation frequency Ω
- Adjust speed of parameter changes to the gap!





- Final fidelity with Laughlin state: 0.99
- Preparation time: 600 trapping periods

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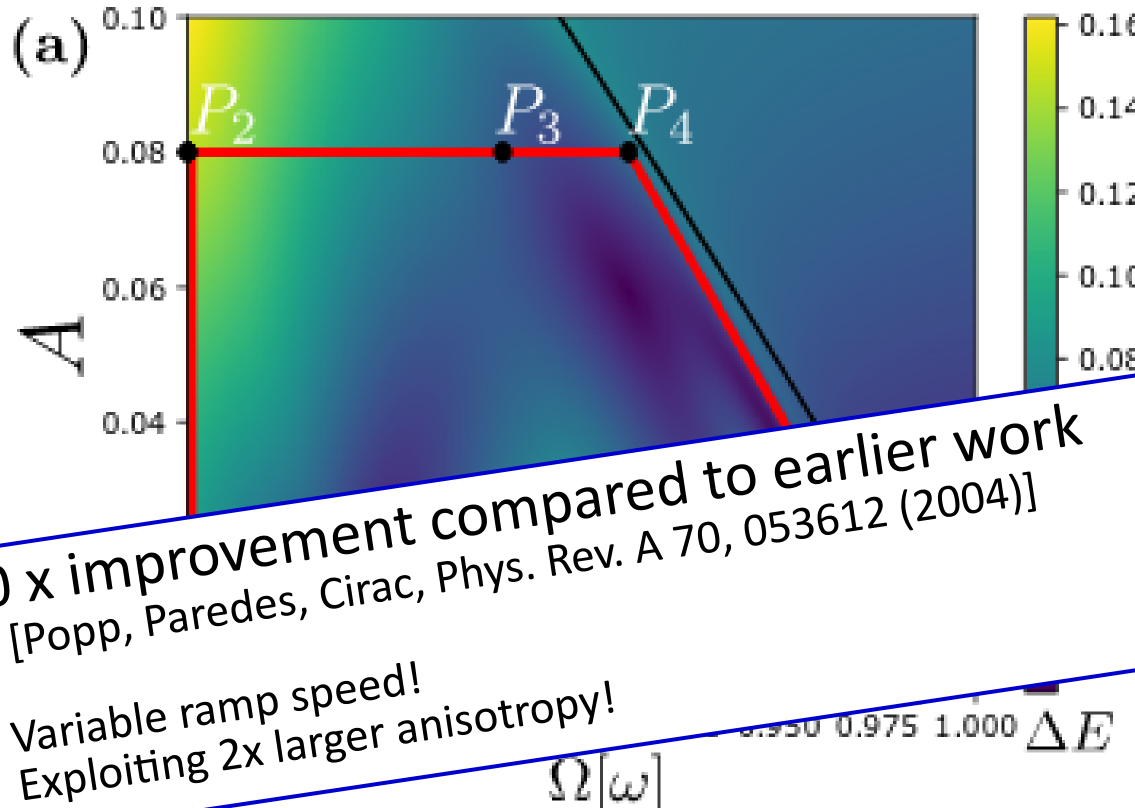
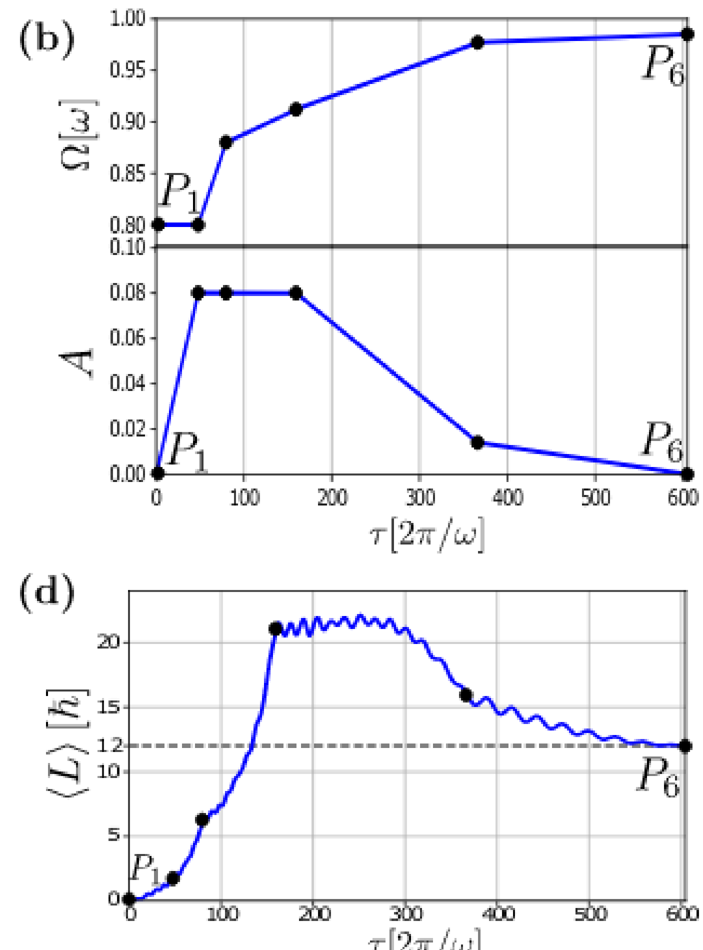


10 x improvement compared to earlier work
[Popp, Paredes, Cirac, Phys. Rev. A 70, 053612 (2004)]

- Variable ramp speed!
- Exploiting 2x larger anisotropy!



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Detection of anyons using impurities

- Impurities in FQH liquid: Bound states with quasiholes

- Y. Zhang, G. J. Sreejith, N. D. Gemelke, and J. K. Jain, Fractional Angular Momentum in Cold-Atom Systems, Phys. Rev. Lett. **113**, 160404 (2014)
- D. Lundholm and N. Rougerie, Emergence of Fractional Statistics for Tracer Particles in a Laughlin Liquid, Phys. Rev. Lett. **116**, 170401 (2016)
- F. Grusdt, N. Y. Yao, D. Abanin, M. Fleischhauer, and E. Demler, Interferometric measurements of many-body topological invariants using mobile impurities, Nat. Commun. **7**, 11994 (2016)
- E. Yakaboylu and M. Lemeshko, Anyonic statistics of quantum impurities in two dimensions, Phys. Rev. B **98**, 045402 (2018)

- Screening of magnetic field due to the liquid:

At filling $\nu = \frac{N}{N_B}$, the effective magnetic field for the impurity is:

$$B^* = B(1-\nu) \;\Rightarrow\; l_B^* = l_B/\sqrt{1-\nu}$$

- Effective Landau level wave functions for the impurities:

$$\tilde{\varphi}_m(w) \sim w^m e^{-(1-\nu)|w|^2/4}$$

→ have average angular momentum: $L_b^m = \frac{m+\nu}{1-\nu}$

- Integer *m* is an “effective quantum number”

Verifying the mean-field prediction

- Simple model: Parent Hamiltonian for Laughlin liquid + contact interaction with impurity

- Analytic expression for zero-energy eigenstates:

$$\Psi_{1\text{imp},m} \sim w^m \prod_i (w - z_i)\; f_d(\{z_i\})\; \Psi_{\text{L}}^{(q)}$$

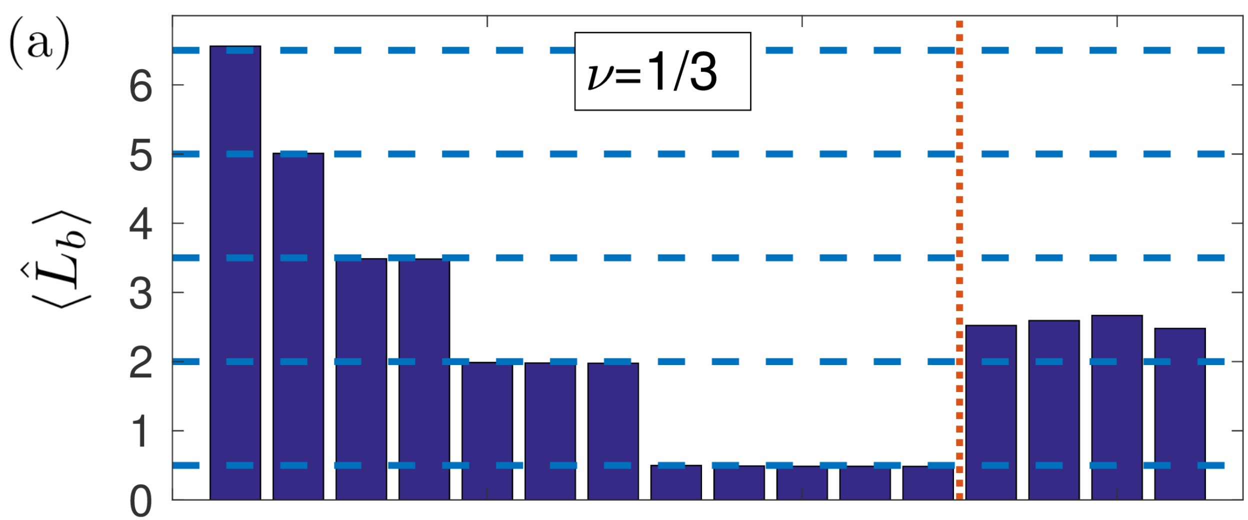
Excitation of impurity

Impurity bound to quasihole

Symmetric polynomial (edge excitations of liquid)

Laughlin state

- Total angular momentum $L = L_{\text{Laughlin}} + L_{\text{quasihole}} + d + m$



12 zero-energy eigenstates for $d + m = 4$

- Impurity angular momentum (average):

$$L_b^m = \frac{m+\nu}{1-\nu}$$

Anyonic quantum statistics of impurities

- Given the effective single-impurity levels $L_b^m = \dfrac{m+\nu}{1-\nu}$,

  a system of N_b non-interacting impurities should have:

$$L_{\mathrm{F}} \equiv \langle L_b \rangle = \sum_{m=0}^{N_b-1} L_b^m$$ if impurities are fermionic

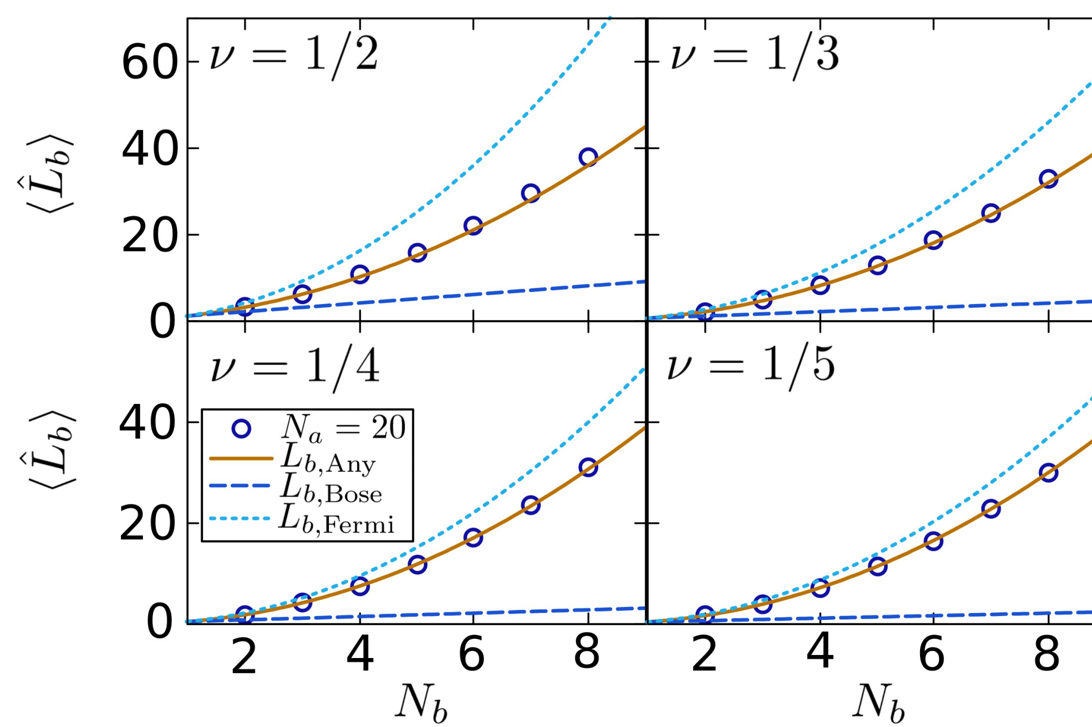
or

$$L_{\mathrm{B}} \equiv \langle L_b \rangle = N_b L_b^0$$ if impurities are bosonic

- Binding to quasiholes makes (formerly fermionic) impurities exhibit anyonic statistics:

$$L_{\mathrm{A}} = (1-\alpha)L_{\mathrm{F}} + \alpha L_{\mathrm{B}}$$

where α=ν is the statistical parameter of the quasiholes



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Preparation of the 1/2-Laughlin state with atoms in a rotating trap

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Laughlin state of rotating bosons:

“Fast” adiabatic preparation is possible exploiting large trap anisotropies and variable ramp speeds.

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Impurities bound to quasiholes are described by effective single-particle levels characterized by their average angular momentum.

Filling of these effective levels is neither bosonic nor fermionic: Impurity angular momentum reflects anyonic statistics.

THANK YOU!

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