CECAM Workshop, Munich, 08.04.2015

Solving Quantum Hall Problems With Cold Atoms

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In collaboration with:

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Quantum Simulations – Why?

Two reasons for a cold-atom implementation of Fractional Quantum Hall Physics

1. Hard problem:

- Strongly correlated many-body system
- Competing effective theories
- Competing trial states
- Restricted size of exactly solved systems

2. Intriguing physics:

- anyons: fractional quantum statistics
- no experimental observation so far
- topologically protected quantum states

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Outline

1. Quantum Hall Physics – in general:

- Single-particle physics: highly degenerate Landau levels
- Many-body effects: Trial states

2. *Fractional* Quantum Hall physics of (pseudo)spin-1/2 bosons:

- NASS states (Non-Abelian Spin Singlets)
- Unsolved competition between NASS and Abelian CF states
- Quantum simulation: signature via correlation function

3. *Integer* Quantum Hall physics of (pseudo)spin-1/2 bosons:

- interacting (!) IQH phases
- Edge states as a fingerprint of the phase

Quantum Hall Systems

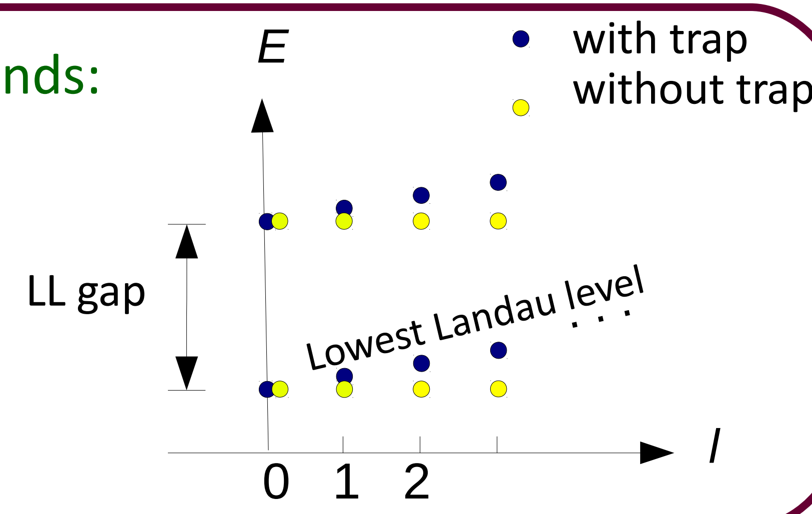
$$H = \frac{(\mathbf{p}+\mathbf{A})^2}{2M} + \frac{M}{2}\omega^2\mathbf{r}^2$$

Gauge potential:

$$\mathbf{A} = \frac{B}{2}(y, -x)$$

Trapping potential

Flat bands:



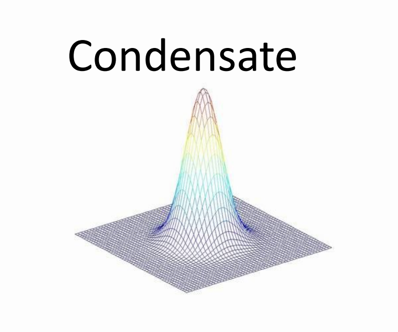
Fermions:

Integer filling:
→ effectively non-interacting
→ Integer Quantum Hall Phases

Fractional filling:
→ Gapped phases due to interactions
→ Fractional Quantum Hall Phases

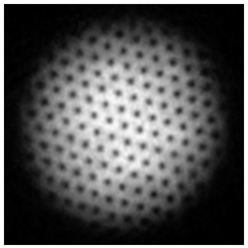
Bosons:

Condensate



symmetry breaking

Vortex lattice



melting

Interacting Quantum Hall Phases:

→ fractional
(like the fermionic ones , with or without spin)

→ integer
(no fermionic counterpart, needs spin)

Trial states: Laughlin and Halperin

Wave functions with “zeros” for all particle pairs:

→ Laughlin wave function (spinless system)

$$\Psi_{\mathrm{L}}^{(q)} = \prod_{i<j} (z_i - z_j)^q \exp[-\sum_i z_i^2/2] \qquad \text{filling} \quad \nu = 1/q$$

$$(z = x + iy)$$

→ Halperin wave function (two-component system)

$$\Psi_{\mathrm{H}}^{(lmn)} \sim \prod_{1\leq i<j\leq N_\uparrow} (z_{i\uparrow} - z_{j\uparrow})^l \prod_{1\leq i<j\leq N_\downarrow} (z_{i\downarrow} - z_{j\downarrow})^m \prod_{\substack{1\leq i\leq N_\uparrow \\ 1\leq j\leq N_\downarrow}} (z_{i\uparrow} - z_{j\downarrow})^n$$

fillings $\nu_\uparrow = \dfrac{l-n}{lm-n^2}$ and $\nu_\downarrow = \dfrac{m-n}{lm-n^2}$

Exact zero-energy solutions in contact potential!

Trial states: Pairing states

1) Divide system into k clusters.
2) Each cluster forms a Laughlin/Halperin state.
3) (Anti-)Symmetrize over all possible clusters.

Moore/Read (1991)
Read/Rezayi (1999)

→ Read-Rezayi series (spinless):

$$\Psi_{\rm RR}^{(k)} \sim \mathcal{S}[\Psi_{\rm L}^{(2)}(z_{i_1},\ldots,z_{i_M})\Psi_{\rm L}^{(2)}(z_{i_{M+1}},\ldots,z_{i_{2M}})\ldots]$$

filling $\nu = k/2$

Ardonne/Schoutens (1999)

→ Non-Abelian spin singlet (NASS) series

$$\Psi_{\rm NASS}^{(k)} \sim \mathcal{S}[\Psi_{\rm H}^{(221)}(z_{i_1\uparrow},\ldots,z_{i_M\uparrow},z_{i_1\downarrow},\ldots,z_{i_M\downarrow})\Psi_{\rm H}^{(221)}(z_{i_{M+1}\uparrow},\ldots,z_{i_{2M}\uparrow},z_{i_{M+1}\downarrow},\ldots,z_{i_{2M}\downarrow})\ldots]$$

filling $\nu = 2k/3$

Exact ground states for (*k+1*)-body contact interactions!

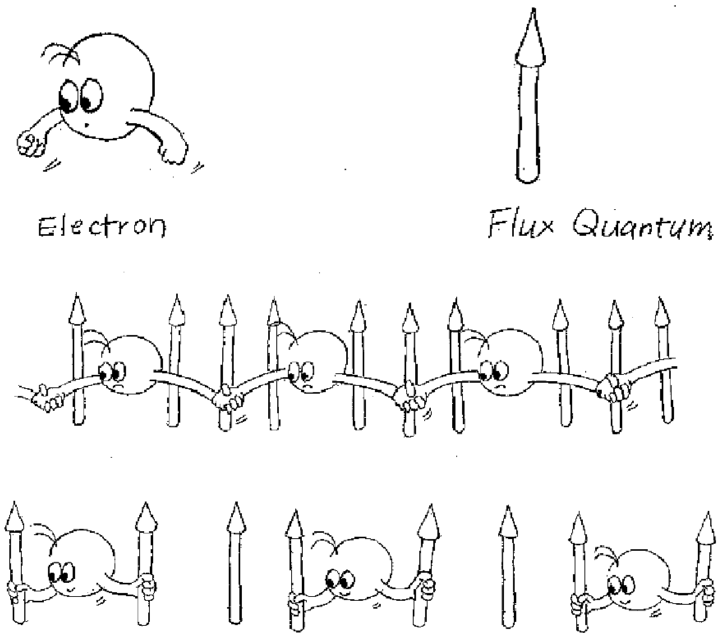
Trial states: Composite fermion states

Construction Recipe:

Jain/Kawamura (1995)
Cooper/Wilkin (1999)

1. Composite fermion = particle + *m* magnetic fluxes
   → Jastrow factor: $J(z) = \prod_{i>j}(z_i - z_j)^m$
2. CFs fill Landau levels at modified magnetic field
   → Slater determinant ϕ of filled LLs
3. Project back into low-energy space:
   Lowest Landau level of the original system

$$\Psi_{\mathrm{CF}} = \mathcal{P}_{\mathrm{LLL}} \Phi(z) J(z)$$



(from Jain book)

Construction works for fermionic and bosonic systems with or without spin, at filling factors $\nu = \frac{n}{mn \pm 1}$ where the number *m* of attached fluxes per particle must be even for fermions or odd for bosons.

Trial states: Overview

	Spinless fermions	Spinless bosons	Two-component bosons (fully unpolarized)
Abelian Fractional Quantum Hall States	Laughlin $\nu = 1/q, \ q \text{ odd}$ CF states $\nu = \frac{n}{mn+1}, \ m \text{ even}$	Laughlin $\nu = 1/q, \ q \text{ even}$ CF states $\nu = \frac{n}{mn+1}, \ m \text{ odd}$	Halperin $\nu = \frac{2}{m+n}, \ m \text{ even}$ CF states $\nu = \frac{n}{n \pm 1} \notin \mathbb{N}$
Non-Abelian Fractional Quantum Hall States	Read-Rezayi $\nu = \frac{k}{k+2}$	Read-Rezayi $\nu = \frac{k}{2}$	NASS $\nu = \frac{2k}{3}$
Integer Quantum Hall States	trivial	✗	CF state $\nu = 2$

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The System

We now focus on:

- bosons
- pseudospin-1/2
- in the lowest Landau level
- with contact interactions:

$$H = \sum_{i<j} \left[g_{\uparrow\uparrow}\delta(z_{i\uparrow} - z_{j\uparrow}) + g_{\downarrow\downarrow}\delta(z_{i\downarrow} - z_{j\downarrow}) + g_{\uparrow\downarrow}\delta(z_{i\uparrow} - z_{j\downarrow}) + g_{\uparrow\downarrow}\delta(z_{i\downarrow} - z_{j\uparrow}) \right]$$

- SU(2)-symmetric: $g_{\uparrow\uparrow} = g_{\uparrow\downarrow} = g_{\downarrow\downarrow}$

Numerical studies on different geometries:

Disk	Torus	Sphere
· most realistic · edge effects	· Purely bulk physics · Complicated wave functions	· Purely bulk physics · Relatively simple wave func. · Shifted filling factors

NASS series on the torus?

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **86**, 031604(R) (2012)

Quantum Hall states in rapidly rotating two-component Bose gases

Shunsuke Furukawa and Masahito Ueda
Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **86**, 021603(R) (2012)

Non-Abelian spin-singlet states of two-component Bose gases in artificial gauge fields

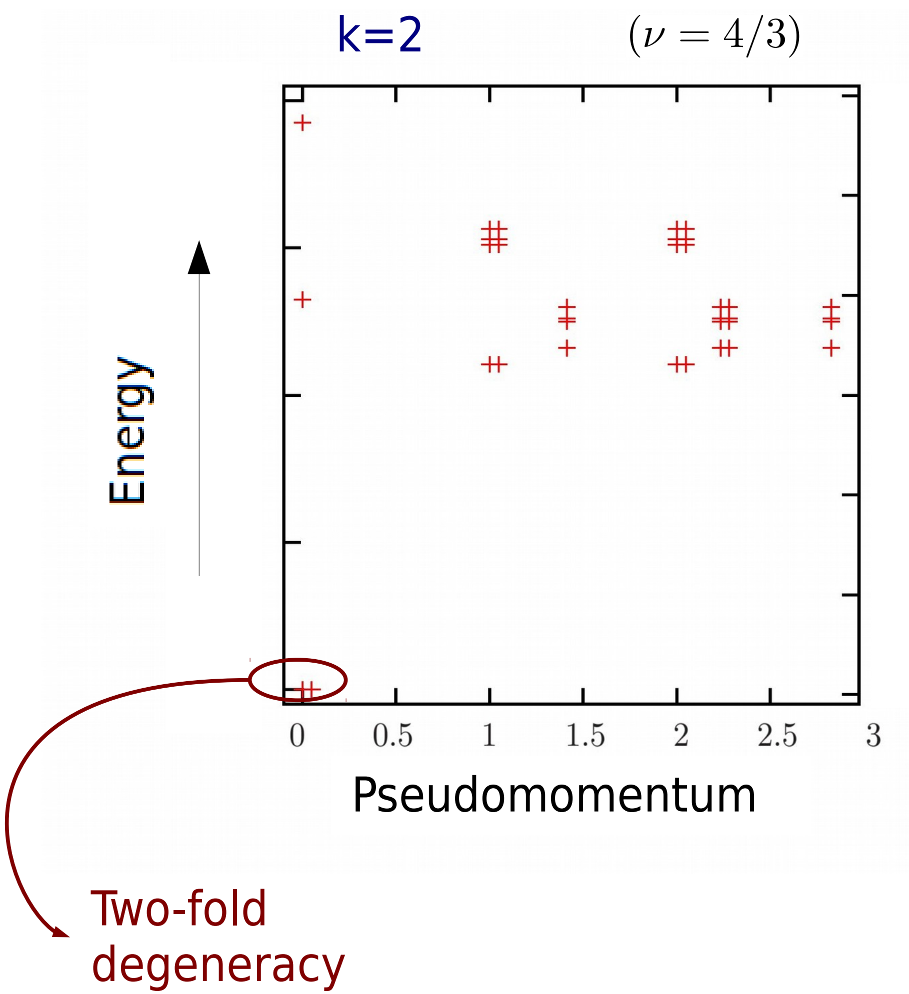
T. Graß,[1] B. Juliá-Díaz,[1] N. Barberán,[2] and M. Lewenstein[1,3]
[1]*ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain*
[2]*Departament ECM, Facultat de Física, Universitat de Barcelona, 08028 Barcelona, Spain*
[3]*ICREA-Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*

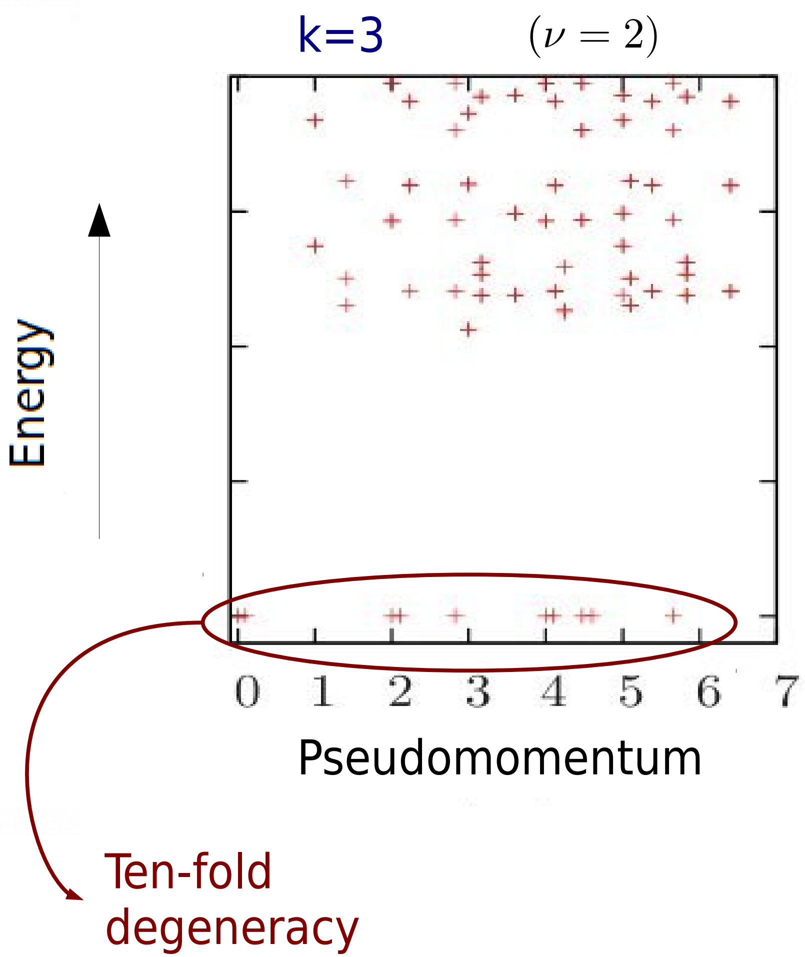
Exact diagonalization on the torus:

- Evidence of incompressible (gapped) phases at $\nu = \frac{2k}{3}$ for $k = 1, 2, 3.$
- NASS series?

NASS series on the torus?

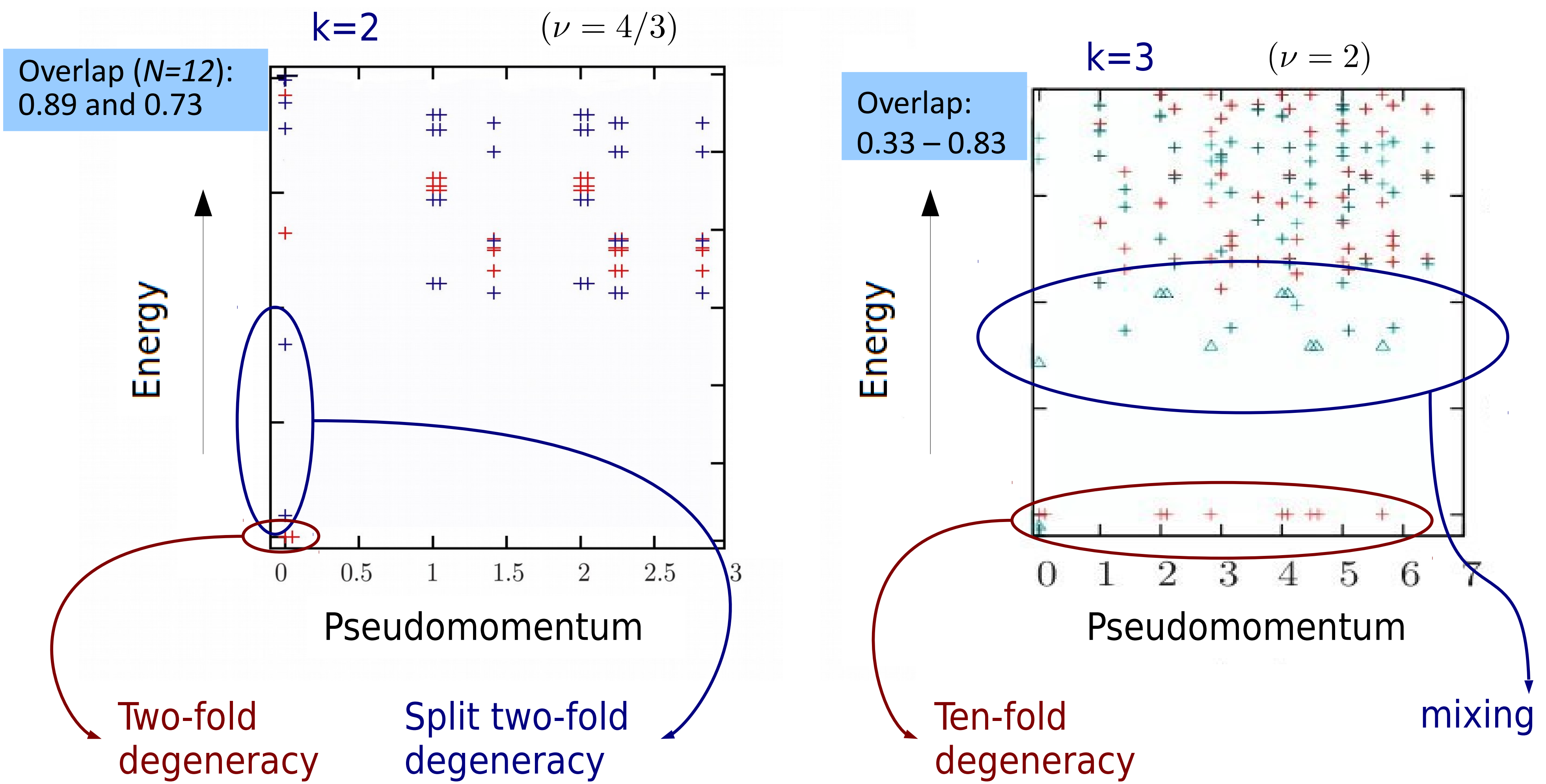
Spectra of *(k+1)*-body contact interaction





NASS series on the torus?

Spectra of *(k+1)*-body contact interaction
versus
Spectra of two-body contact interaction



CF states on the sphere?

ED on torus:

NASS phase
at *ν=4/3*

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Different picture on the sphere!

PHYSICAL REVIEW B **87**, 245123 (2013)

Quantum Hall effect of two-component bosons at fractional and integral fillings

Ying-Hai Wu and Jainendra K. Jain
Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA

CF states on the sphere?

Overlaps on the sphere:

- with NASS state: 0.918
- with CF state: 0.985

for *N=12* at filling *ν=4/3.*

BUT: Filling factor is biased on the sphere.

$$\nu = \frac{N}{N_{\mathrm{V}}} + \delta$$

Direct competition between NASS and CF is not possible on the sphere.
(Neither on small disks!)

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Quantum Hall states in rapidly rota

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?

To solve the competition, one might:

- Look at overlaps with CF states on the torus: LLL projection on the torus?
- Study bigger systems: Quantum simulation?

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Cold atom quantum simulation

All ingredients of the Hamiltonian are available:

- ✓ Synthetic magnetic fields
- ✓ 2-body contact potential

But how could a quantum simulation distinguish between different states?

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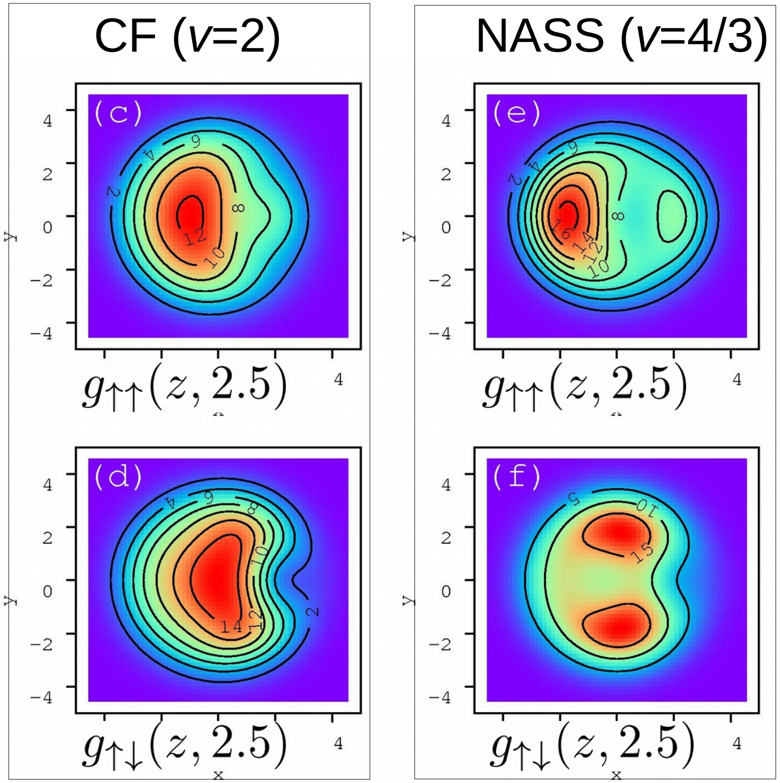
Example:

$$N_\uparrow = 4$$
$$N_\downarrow = 4$$
$$L = 16$$



Correlation functions: $C(z_1, z_2) = \langle \Psi | \hat{\psi}^\dagger(z_1) \hat{\psi}^\dagger(z_2) \hat{\psi}(z_1) \hat{\psi}(z_2) | \Psi \rangle$

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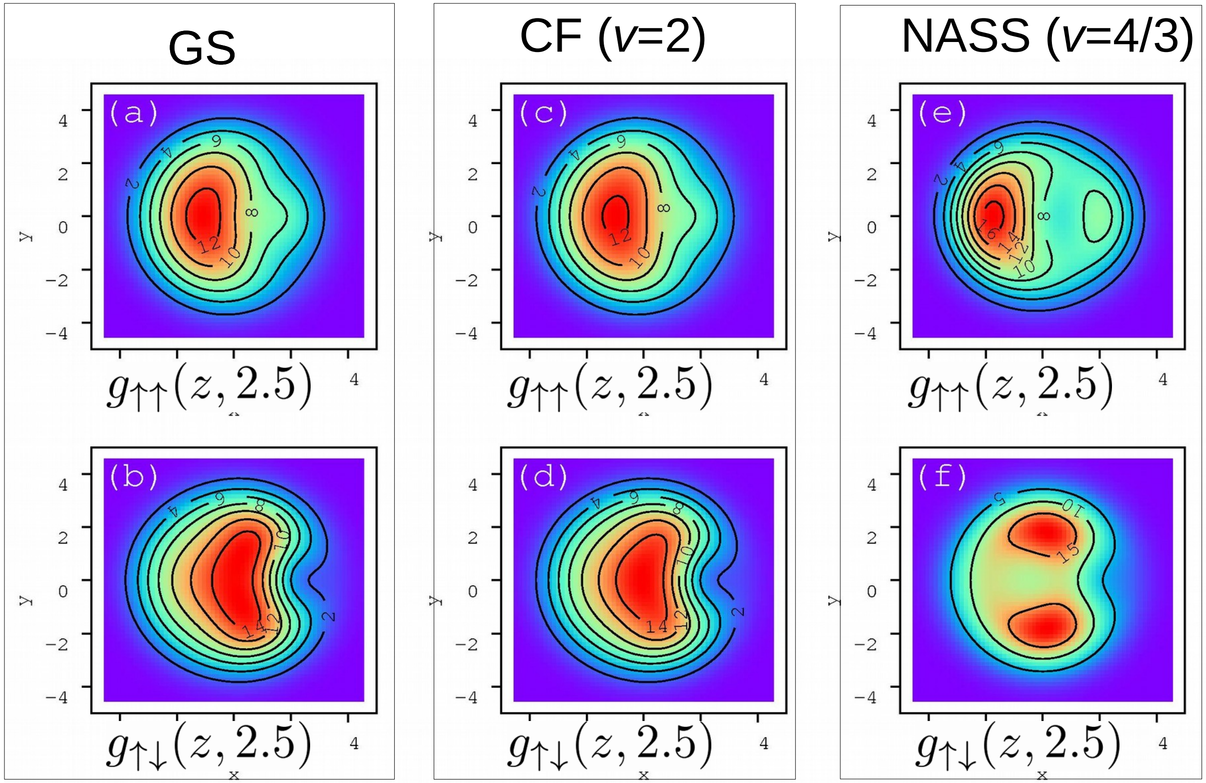

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What happens at *ν=2* ?

PRL **110,** 046801 (2013) PHYSICAL REVIEW LETTERS week ending 25 JANUARY 2013

Integer Quantum Hall Effect for Bosons

T. Senthil[1] and Michael Levin[2]
[1]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[2]*Department of Physics, Condensed Matter Theory Center, University of Maryland, College Park, Maryland 20742, USA*

Effective field theory: Possibility of an interacting integer quantum Hall effect for two-component bosons at *ν=2.*

PRL **111,** 090401 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

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PHYSICAL REVIEW B **88**, 161106(R) (2013)

Microscopic model for the boson integer quantum Hall effect

N. Regnault[1,2] and T. Senthil[3]
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[2]*Laboratoire Pierre Aigrain, ENS and CNRS, 24 rue Lhomond,*
[3]*Department of Physics, Massachusetts Institute of Technology, Cambrid*

PHYSICAL REVIEW B **89**, 045114 (2014)

Quantum Hall phases of two-component bosons

T. Graß,[1] D. Raventós,[2] M. Lewenstein,[1,3] and B. Juliá-Díaz[1,2]
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PRL **110,** 046801 (2013) PHYSICAL REVIEW LETTERS week ending 25 JANUARY 2013

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What happens at ν=2 ?

Torus:
Grass, Julia-Diaz, Barberan, Lewenstein (PRA, 2012)
Regnault & Senthil (PRB, 2013)
Furukawa & Ueda (PRL, 2013)

→ no NASS phase
→ unique, gapped GS

Sphere:
Furukawa & Ueda (PRL, 2013)
Wu & Jain (PRB, 2013)

Entanglement spectra:
→ edge physics of iIQHE

Overlap: with CF state
0.888 (N=14)

Disk:
Wu & Jain (PRB, 2013)
Grass, Raventos, Julia-Diaz, Lewenstein (PRB, 2014)

Edge spectrum agrees with IQH theory.

Overlap with CF state:
0.970 (N=8, L=16)

Edge spectrum at ν=2

Effective edge Hamiltonian of singlet state

[J.E. Moore, F.D.M. Haldane, PRB **55** 7818 (1997)]

$$H_{\text{edge}} \propto v_s \left(S_z^2 + \sum_l l b_l^\dagger b_l \right) + v_c \sum_l l c_l^\dagger c_l$$

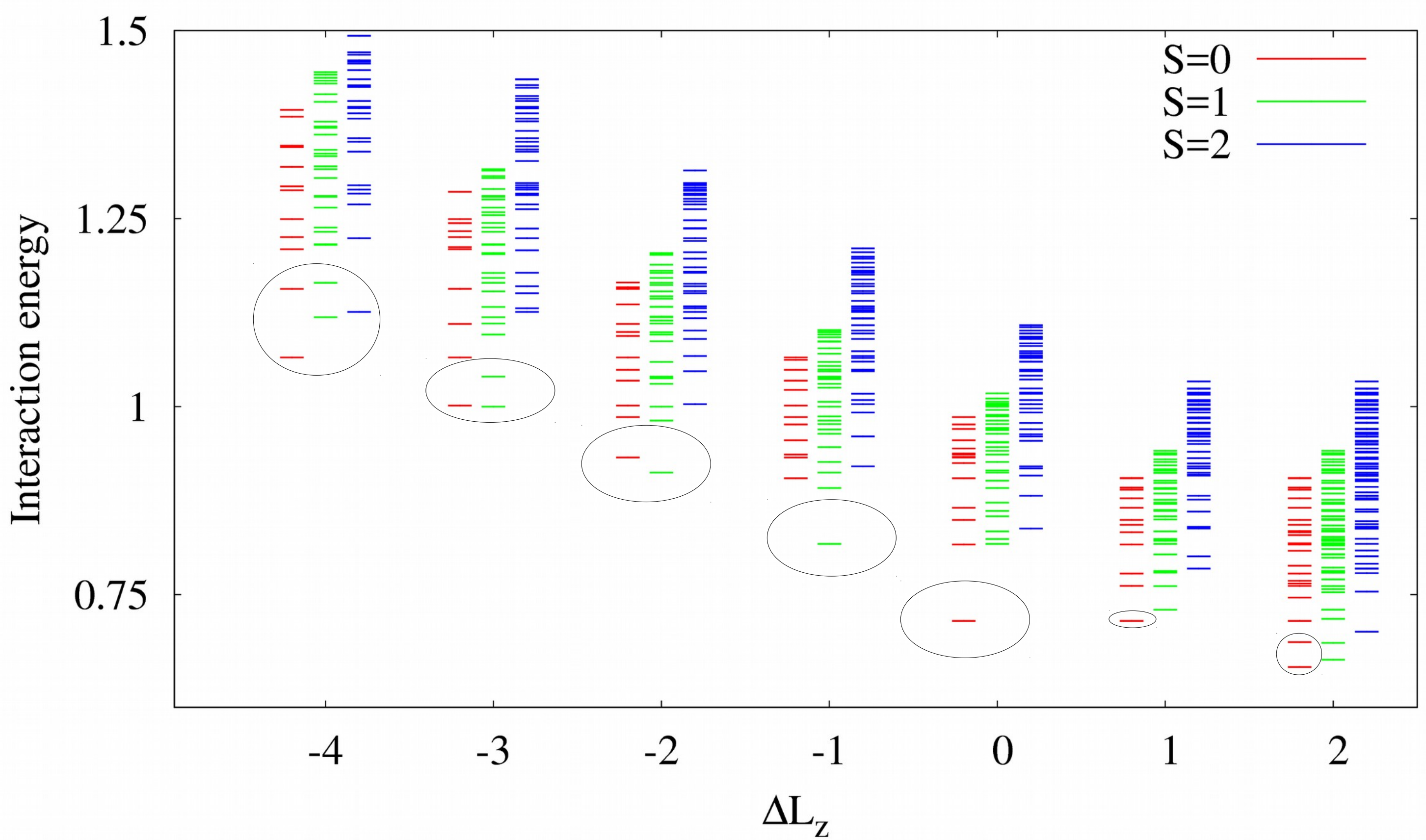
TABLE I. Number of modes of H_{edge} with $v_s < 0$ and $v_c > 0$.

ΔL_z	-4	-3	-2	-1	$+1$	$+2$	$+3$	$+4$
Number of singlets	2	1	1	0	1	2	3	5
Number of triplets	2	2	1	1	0	0	0	0
Number of quintets	1	0	0	0	0	0	0	0

Numerical results on a disk

[T. Grass *et al.*, PRB **89** 045114 (2014)]

$$N_\uparrow = 4$$
$$N_\downarrow = 4$$
$$L_z = 16 + \Delta L_z$$



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$$N_\uparrow = 4$$
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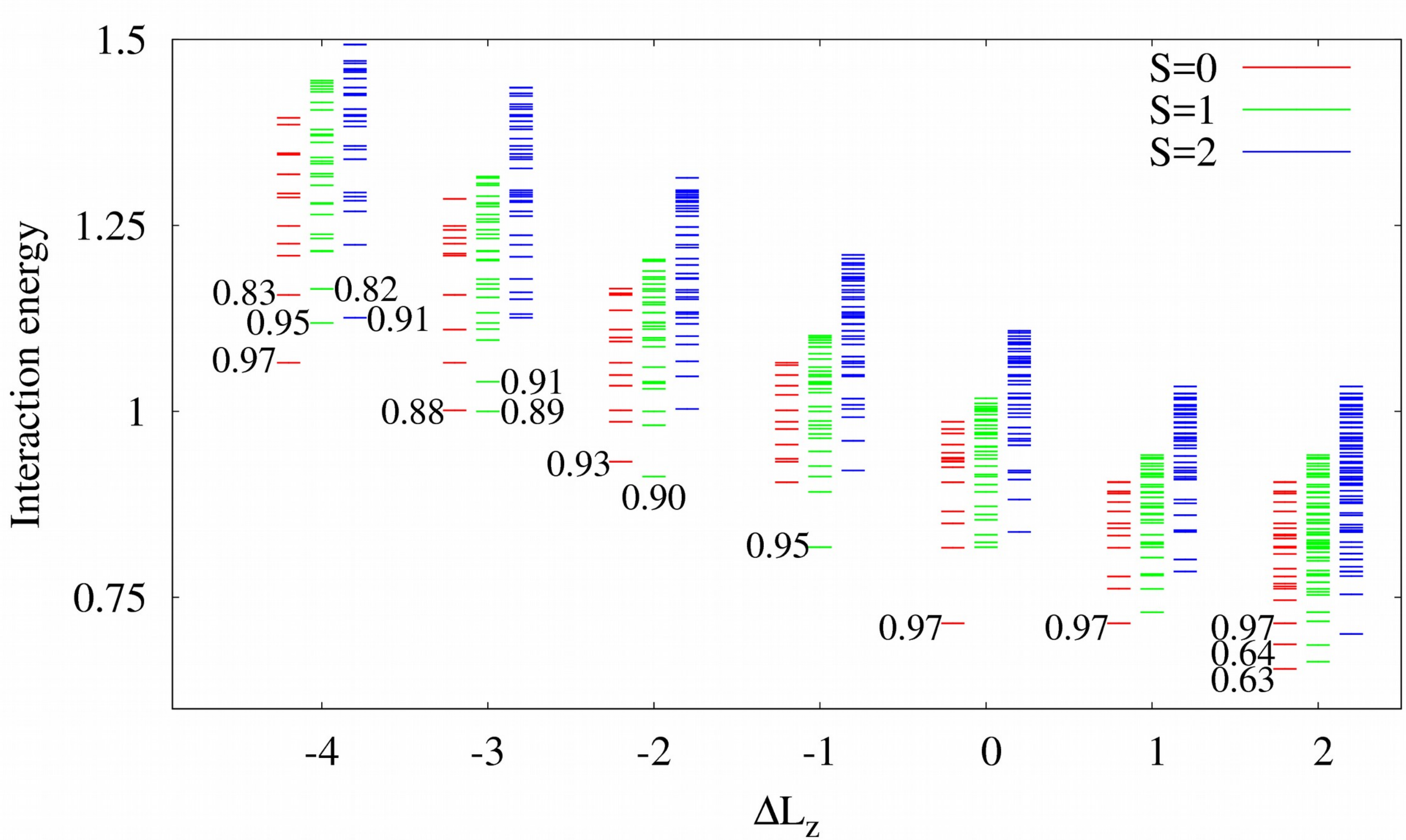
Explicit construction of edge states provides the same counting, and good overlaps:

- Backward states: Excite CFs in flux-reversed LLs
- Forward states: Symmetric polynomial



Summary

Rich variety of phases of two-component bosons:

- → Condensate
- → Vortex Lattices
- → Fractional Quantum Hall Phases (Abelian and/or Non-Abelian?)
- → Integer Quantum Hall Phase

Unresolved competition between phases even for simple contact potential:

- → Non-Abelian vs. Abelian phase at $\nu=4/3$
- → Interesting candidate for quantum simulations with cold atoms
- → Distinction between phases via correlation functions

Interacting integer quantum Hall phase at $\nu=2$:

- → Interactions crucial to avoid condensation
- → New quantum Hall phase without fermionic counterpart
- → Edge spectrum as a fingerprint (state counting and explicit overlaps)

T. Grass, B. Juliá-Díaz, N. Barberán, M. Lewenstein, PRA **86** 021603(R) (2012)

T. Grass, D. Raventós, M. Lewenstein, B. Juliá-Díaz, PRB **89** 045114 (2014)