International Workshop on Cold Gases in Quantum Information
Bilbao
01.07.2015

Atomic Quantum Hall Solvers

Tobias Grass (ICFO - Barcelona)

In collaboration with:

Bruno Juliá-Díaz (University Barcelona)

Maciej Lewenstein (ICFO)

Nuria Barberán (University Barcelona)

David Raventós (ICFO)

Experimental status

Selected for a Viewpoint in *Physics*

PRL **111,** 185301 (2013) PHYSICAL REVIEW LETTERS week ending 1 NOVEMBER 2013

Realization of the Hofstadter Hamiltonian with Ultracold Atoms in Optical Lattices

M. Aidelsburger,[1,2] M. Atala,[1,2] M. Lohse,[1,2] J. T. Barreiro,[1,2] B. Paredes,[3] and I. Bloch[1,2]

[1]*Fakultät für Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4, 80799 München, Germany*

[2]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, 85748 Garching, Germany*

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(Received 1 August 2013; published 28 October 2013)

arXiv.org > cond-mat > arXiv:1502.02496

Condensed Matter > Quantum Gases

Visualizing edge states with an atomic Bose gas in the quantum Hall regime

B. K. Stuhl, H.-I Lu, L. M. Aycock, D. Genkina, I. B. Spielman

(Submitted on 9 Feb 2015)

We engineered a two-dimensional magnetic lattice in an elongated strip geometry, with effective per-plaquette flux ~4/3 times the flux quanta. We ima

arXiv.org > cond-mat > arXiv:1502.02495

Condensed Matter > Quantum Gases

Observation of chiral edge states with neutral fermions in synthetic Hall ribbons

M. Mancini, G. Pagano, G. Cappellini, L. Livi, M. Rider, J. Catani, C. Sias, P. Zoller, M. Inguscio, M. Dalmonte, L. Fallani

(Submitted on 9 Feb 2015)

Chiral edge states are a hallmark of quantum Hall physics. In electronic systems, they appear as a macroscopic consequence of the cyclotron orbits i

Cold atomic quantum Hall effect – Why?

Systems with well controlled Hamiltonians

Quantum Hall solver

- Hard problem: Competition between different phases

Novel quantum Hall phases

- Quantum Hall effect of bosons
- *Interacting* integer quantum Hall phases
- Ideal phases (parent Hamiltonians)

Exploring anyonic properties

- First experimental detection of fractional statistics
- Braiding of (non-Abelian) anyons
- Anyon technologies

Outline

1. Quantum Hall Physics – in general:

- Single-particle physics: Landau levels
- Many-body effects: Trial states

2. Fractional quantum Hall physics of spin-1/2 bosons:

- Abelian vs. Non-Abelian phases
- Numerical results: ambiguous
- Prospects of a quantum simulation

3. *Integer* quantum Hall physics of spin-1/2 bosons:

- Interactions are crucial
- Edge spectrum as a fingerprint

4. Anyon braiding in small systems

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Quantum Hall Systems

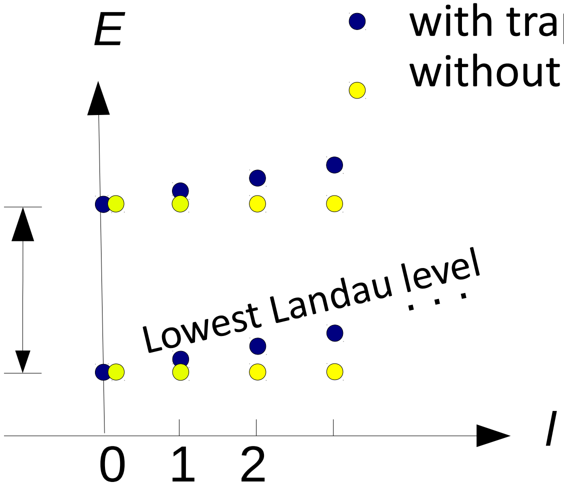
$$H = \frac{(\mathbf{p}+\mathbf{A})^2}{2M} + \frac{M}{2}\omega^2\mathbf{r}^2$$

Gauge potential:

$$\mathbf{A} = \frac{B}{2}(y, -x)$$

Trapping potential

Flat bands:



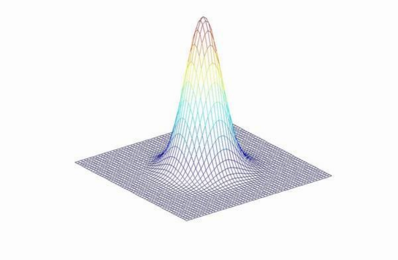
Fermions:

Integer filling:
→ effectively non-interacting
→ Integer Quantum Hall Phases

Fractional filling:
→ Gapped phases due to interactions
→ Fractional Quantum Hall Phases

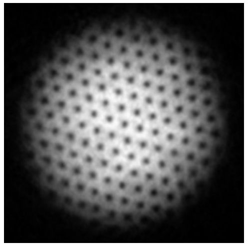
Bosons:

Condensate



symmetry breaking

Vortex lattice



melting

Interacting Quantum Hall Phases:

→ fractional
(like the fermionic ones , with or without spin)

→ integer
(no fermionic counterpart, needs spin)

Trial states: Laughlin and Halperin

Wave functions with “zeros” for all particle pairs:

- Laughlin wave function (spinless system)

$$\Psi_{\mathrm{L}}^{(q)} = \prod_{i<j}(z_i - z_j)^q \exp[-\sum_i |z_i|^2/2] \qquad \text{filling} \quad \nu = 1/q$$

$$(z = x + iy)$$

- Halperin wave function (two-component system)

$$\Psi_{\mathrm{H}}^{(lmn)} \sim \prod_{1\leq i<j\leq N_\uparrow}(z_{i\uparrow} - z_{j\uparrow})^l \prod_{1\leq i<j\leq N_\downarrow}(z_{i\downarrow} - z_{j\downarrow})^m \prod_{\substack{1\leq i\leq N_\uparrow \\ 1\leq j\leq N_\downarrow}}(z_{i\uparrow} - z_{j\downarrow})^n$$

fillings $\nu_\uparrow = \frac{l-n}{lm-n^2}$ and $\nu_\downarrow = \frac{m-n}{lm-n^2}$

Exact zero-energy solutions in contact potential!

Trial states: Pairing states

1) Divide system into k clusters.
2) Each cluster forms a Laughlin/Halperin state.
3) (Anti-)Symmetrize over all possible clusters.

Moore/Read (1991)
Read/Rezayi (1999)

→ Read-Rezayi series (spinless):

$$\Psi_{\mathrm{RR}}^{(k)} \sim \mathcal{S}[\Psi_{\mathrm{L}}^{(2)}(z_{i_1}, \ldots, z_{i_M})\Psi_{\mathrm{L}}^{(2)}(z_{i_{M+1}}, \ldots, z_{i_{2M}}) \ldots]$$

filling $\nu = k/2$

→ Non-Abelian spin singlet (NASS) series

Ardonne/Schoutens (1999)

$$\Psi_{\mathrm{NASS}}^{(k)} \sim \mathcal{S}[\Psi_{\mathrm{H}}^{(221)}(z_{i_1\uparrow}, \ldots, z_{i_M\uparrow}, z_{i_1\downarrow}, \ldots, z_{i_M\downarrow})\Psi_{\mathrm{H}}^{(221)}(z_{i_{M+1}\uparrow}, \ldots, z_{i_{2M}\uparrow}, z_{i_{M+1}\downarrow}, \ldots, z_{i_{2M}\downarrow}) \ldots]$$

filling $\nu = 2k/3$

Exact ground states for (*k+1*)-body contact interactions!

Trial states: Composite fermion states

Construction Recipe:

Jain/Kawamura (1995)
Cooper/Wilkin (1999)

1. Composite fermion = particle + *m* magnetic fluxes

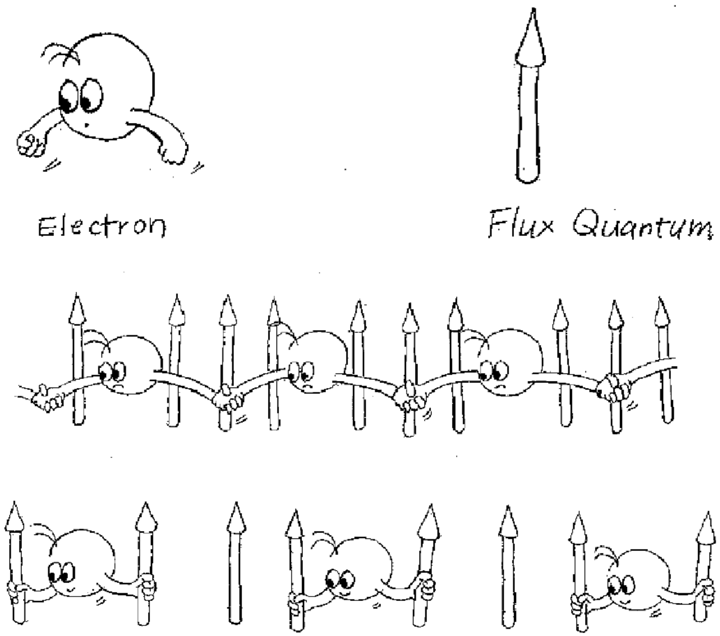
   → Jastrow factor: $J(z) = \prod_{i>j} (z_i - z_j)^m$

2. CFs fill Landau levels at modified magnetic field

   → Slater determinant ϕ of filled LLs

3. Project back into low-energy space:
   Lowest Landau level of the original system

$$\Psi_{\mathrm{CF}} = \mathcal{P}_{\mathrm{LLL}} \Phi(z) J(z)$$



(from Jain book)

Construction works for fermionic and bosonic systems with or without spin, at filling factors $\nu = \frac{n}{mn \pm 1}$ where the number *m* of attached fluxes per particle must be even for fermions or odd for bosons.

Trial states: Overview

	Spinless fermions	Spinless bosons	Two-component bosons (fully unpolarized)
Abelian Fractional Quantum Hall States	Laughlin $\nu = 1/q, \;\; q \text{ odd}$ CF states $\nu = \frac{n}{mn+1}, \;\; m \text{ even}$	Laughlin $\nu = 1/q, \;\; q \text{ even}$ CF states $\nu = \frac{n}{mn+1}, \;\; m \text{ odd}$	Halperin $\nu = \frac{2}{m+n}, \;\; m \text{ even}$ CF states $\nu = \frac{n}{n \pm 1} \notin \mathbb{N}$
Non-Abelian Fractional Quantum Hall States	Read-Rezayi $\nu = \frac{k}{k+2}$	Read-Rezayi $\nu = \frac{k}{2}$	NASS $\nu = \frac{2k}{3}$
Integer Quantum Hall States	trivial	✗	CF state $\nu = 2$

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The System

We now focus on:

- bosons
- pseudospin-1/2
- in the lowest Landau level
- with contact interactions:

$$H = \sum_{i<j} \left[g_{\uparrow\uparrow}\delta(z_{i\uparrow} - z_{j\uparrow}) + g_{\downarrow\downarrow}\delta(z_{i\downarrow} - z_{j\downarrow}) + g_{\uparrow\downarrow}\delta(z_{i\uparrow} - z_{j\downarrow}) + g_{\uparrow\downarrow}\delta(z_{i\downarrow} - z_{j\uparrow}) \right]$$

- SU(2)-symmetric: $g_{\uparrow\uparrow} = g_{\uparrow\downarrow} = g_{\downarrow\downarrow}$

Numerical studies on different geometries:

Disk	Torus	Sphere
· most realistic · edge effects	· Purely bulk physics · Complicated wave functions	· Purely bulk physics · Relatively simple wave func. · Shifted filling factors

NASS series on the torus?

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **86**, 031604(R) (2012)

Quantum Hall states in rapidly rotating two-component Bose gases

Shunsuke Furukawa and Masahito Ueda
Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **86**, 021603(R) (2012)

Non-Abelian spin-singlet states of two-component Bose gases in artificial gauge fields

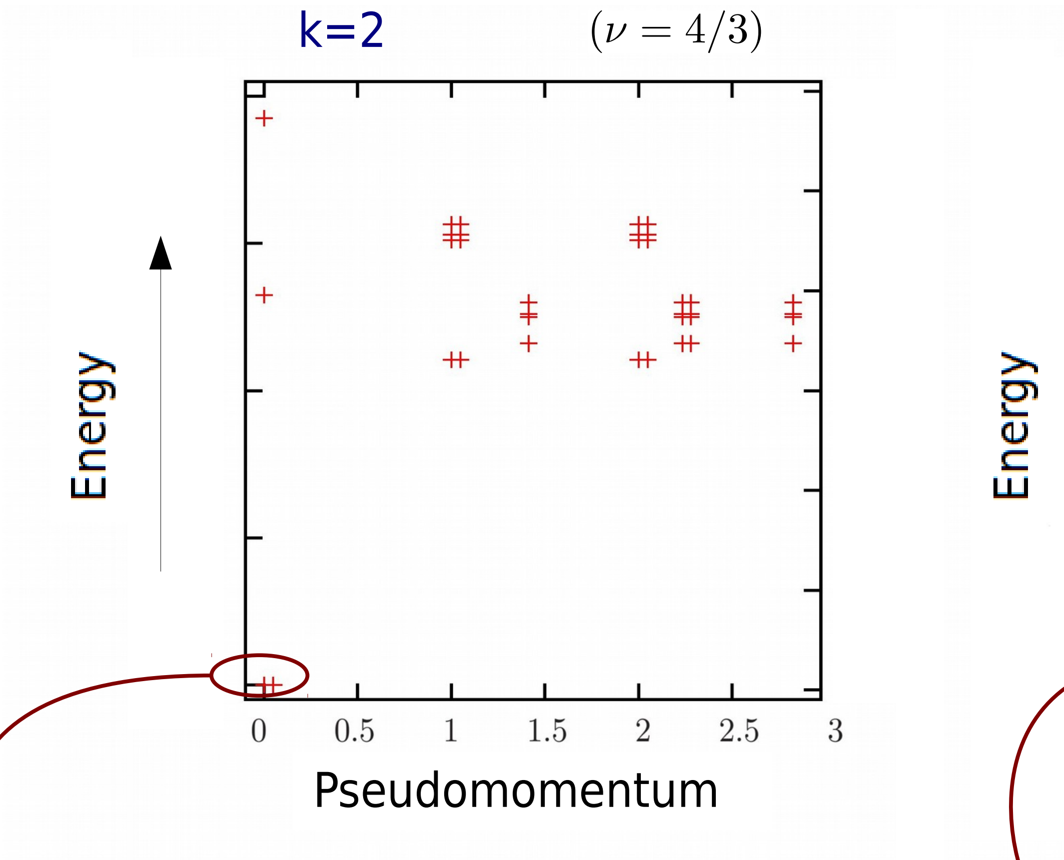
T. Graß,[1] B. Juliá-Díaz,[1] N. Barberán,[2] and M. Lewenstein[1,3]
[1]*ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain*
[2]*Departament ECM, Facultat de Física, Universitat de Barcelona, 08028 Barcelona, Spain*
[3]*ICREA-Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*

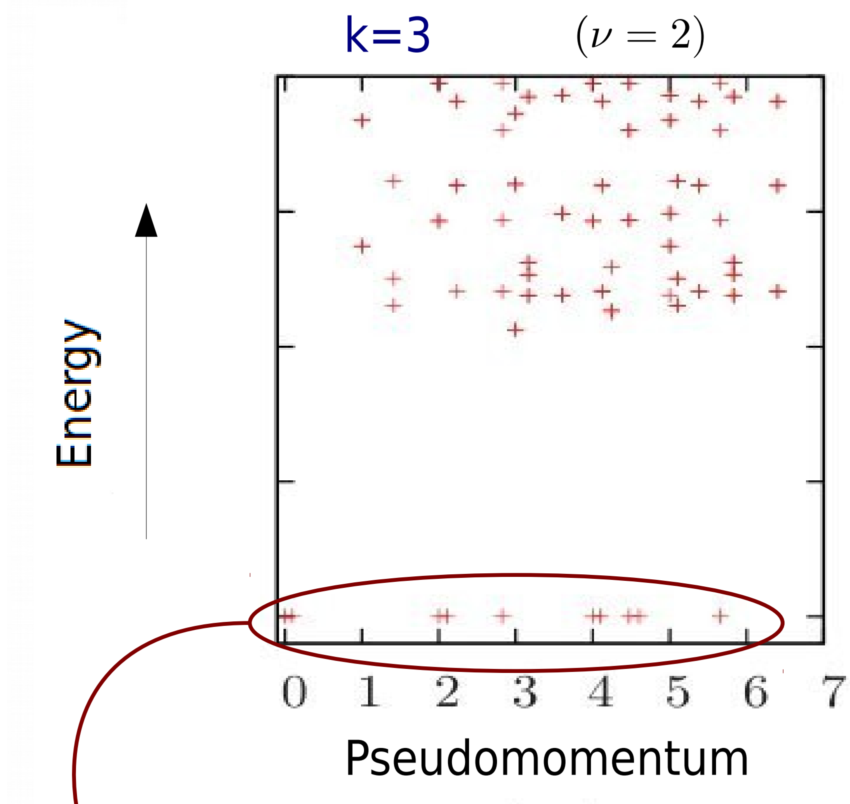
Exact diagonalization on the torus:

- Evidence of incompressible (gapped) phases at $\nu = \frac{2k}{3}$ for $k = 1, 2, 3.$
- NASS series?

NASS series on the torus?

Spectra of *(k+1)*-body contact interaction



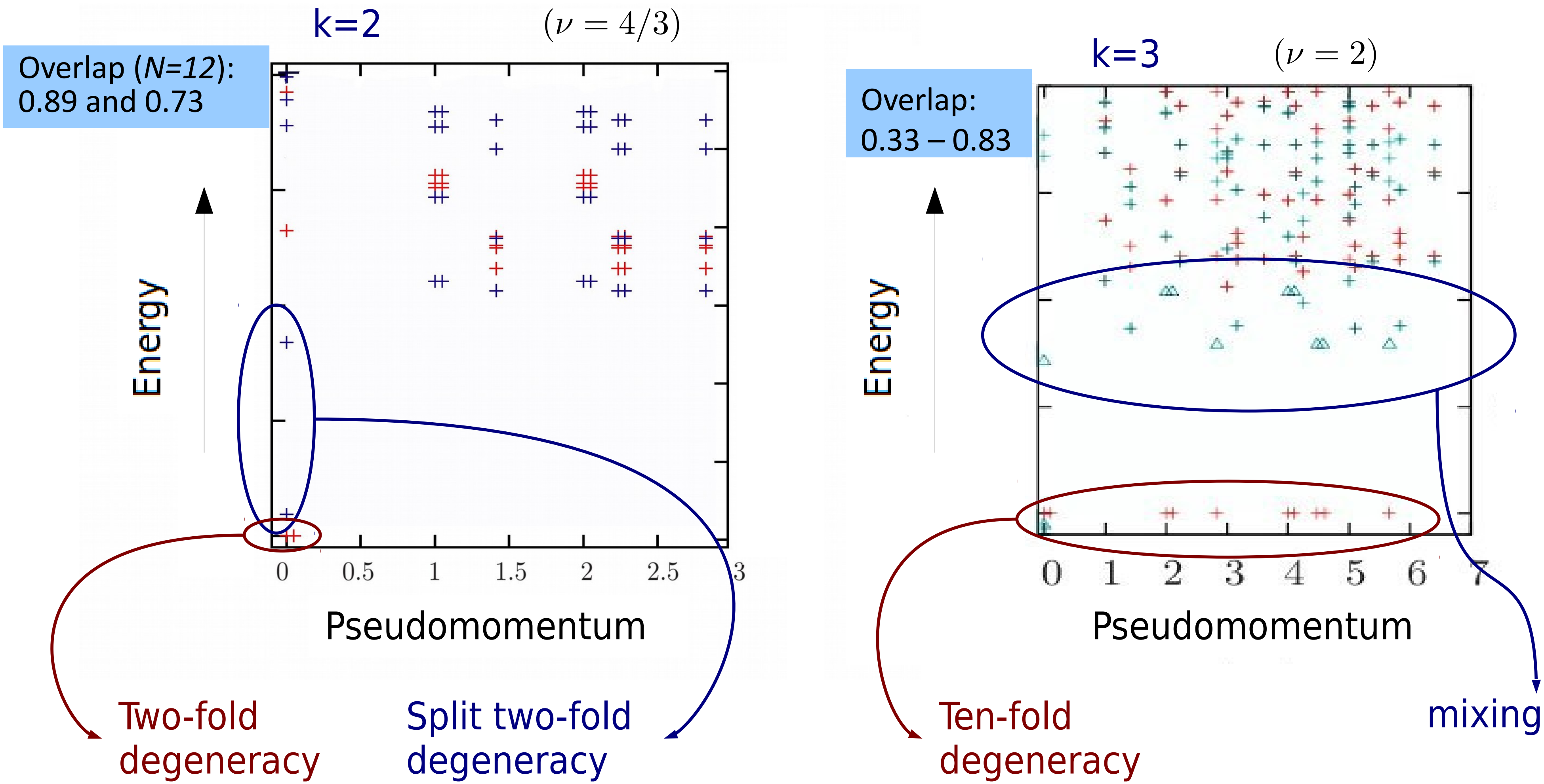


Two-fold degeneracy

Ten-fold degeneracy

NASS series on the torus?

Spectra of *(k+1)*-body contact interaction
versus
Spectra of two-body contact interaction



CF states on the sphere?

ED on torus:

NASS phase
at $\nu=4/3$

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Different picture on the sphere!

PHYSICAL REVIEW B **87**, 245123 (2013)

Quantum Hall effect of two-component bosons at fractional and integral fillings

Ying-Hai Wu and Jainendra K. Jain

Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA

CF states on the sphere?

Overlaps on the sphere:

- with NASS state: 0.918
- with CF state: 0.985

for *N=12* at filling *ν=4/3.*

BUT: Filling factor is biased on the sphere.

$$\nu = \frac{N}{N_{\mathrm{V}}} + \delta$$

Direct competition between NASS and CF is not possible on the sphere.
(Neither on small disks!)

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?

To solve the competition, one might study:

- Overlaps on the torus
- System on a disk → Quantum simulation!

PHYSICAL REVIEW B **87**, 245123 (2013)

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Cold atom quantum simulation

All ingredients of the Hamiltonian are available:

- ✓ Synthetic magnetic fields
- ✓ 2-body contact potential

But how could a quantum simulation distinguish between different states?

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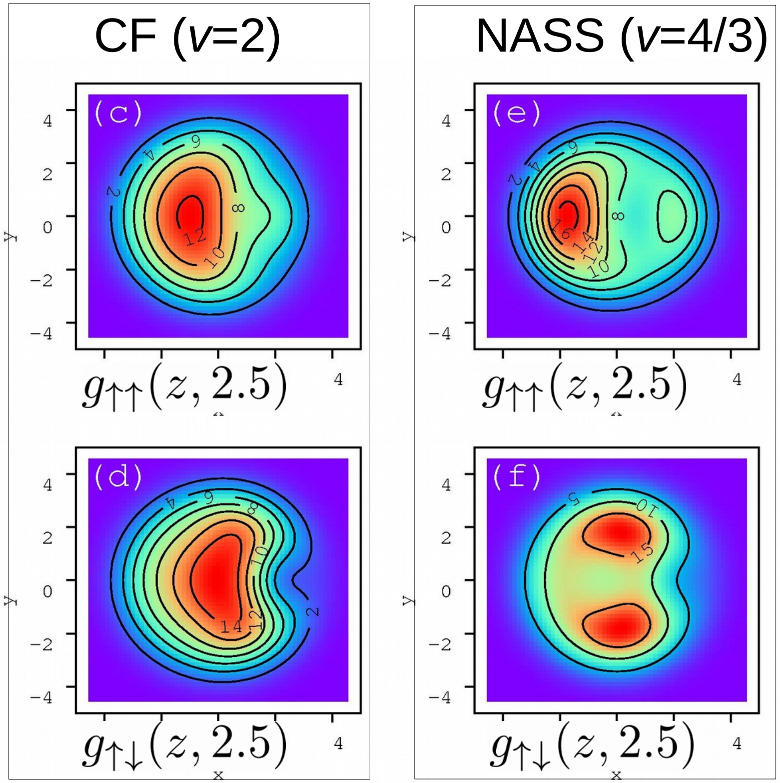
Example:

$$N_\uparrow = 4$$
$$N_\downarrow = 4$$
$$L = 16$$



Correlation functions: $C(z_1, z_2) = \langle \Psi | \hat{\psi}^\dagger(z_1) \hat{\psi}^\dagger(z_2) \hat{\psi}(z_1) \hat{\psi}(z_2) | \Psi \rangle$

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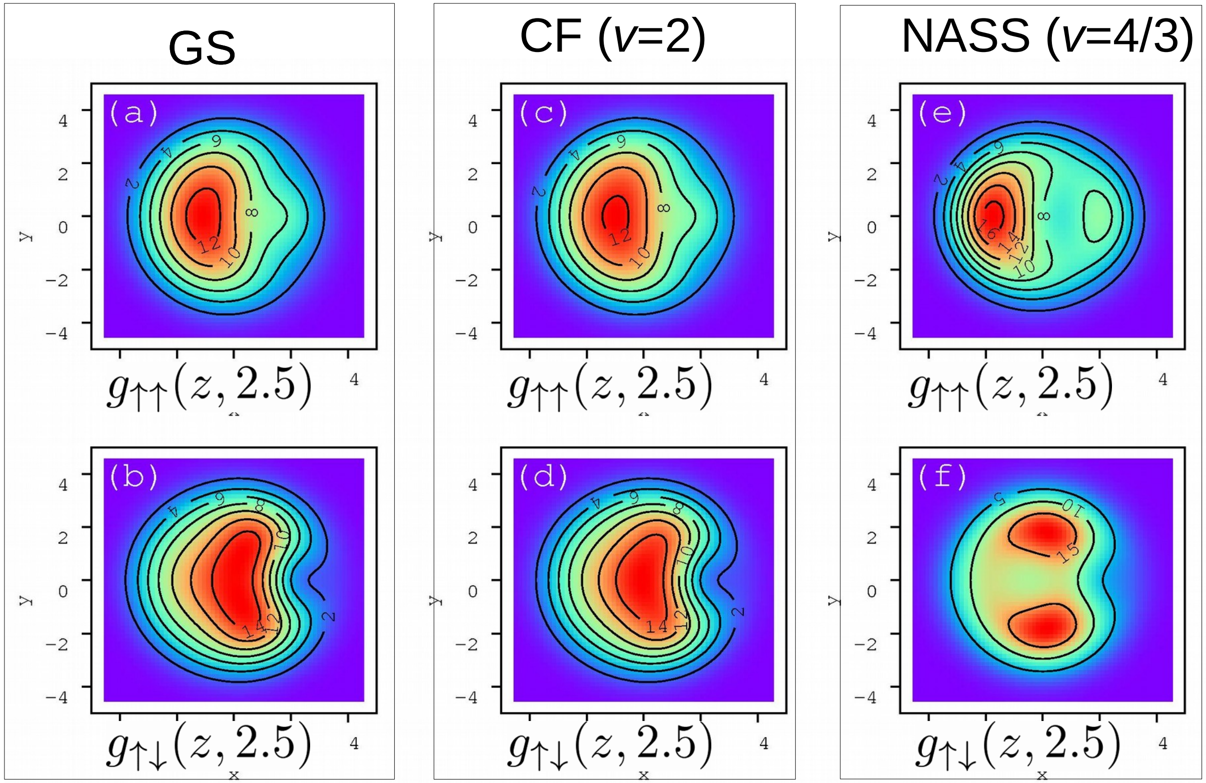

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What happens at $\nu=2$?

PRL **110,** 046801 (2013) PHYSICAL REVIEW LETTERS week ending 25 JANUARY 2013

Integer Quantum Hall Effect for Bosons

T. Senthil[1] and Michael Levin[2]
[1]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[2]*Department of Physics, Condensed Matter Theory Center, University of Maryland, College Park, Maryland 20742, USA*

Effective field theory: Possibility of an interacting integer quantum Hall effect for two-component bosons at $\nu=2$.

PRL **111,** 090401 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Integer Quantum Hall State in Two-Component Bose Gases in a Synthetic Magnetic Field

Shunsuke Furukawa and Masahito Ueda
Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku,

PHYSICAL REVIEW B **87,** 245123 (2013)

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Ying-Hai Wu and Jainendra K. Jain
Pennsylvania State University, University Park, Pennsylvania 16802, USA

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **88,** 161106(R) (2013)

Microscopic model for the boson integer quantum Hall effect

N. Regnault[1,2] and T. Senthil[3]
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[2]*Laboratoire Pierre Aigrain, ENS and CNRS, 24 rue Lhomond,*
[3]*Department of Physics, Massachusetts Institute of Technology, Cambrid*

PHYSICAL REVIEW B **89,** 045114 (2014)

Quantum Hall phases of two-component bosons

T. Graß,[1] D. Raventós,[2] M. Lewenstein,[1,3] and B. Juliá-Díaz[1,2]
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What happens at $\nu=2$?

Torus:
Grass, Julia-Diaz, Barberan, Lewenstein (PRA, 2012)
Regnault & Senthil (PRB, 2013)
Furukawa & Ueda (PRL, 2013)

→ no NASS phase
→ unique, gapped GS

Sphere:
Furukawa & Ueda (PRL, 2013)
Wu & Jain (PRB, 2013)

Entanglement spectra:
→ edge physics of iIQHE

Overlap: with CF state
0.888 ($N=14$)

Disk:
Wu & Jain (PRB, 2013)
Grass, Raventos, Julia-Diaz, Lewenstein (PRB, 2014)

Edge spectrum agrees with IQH theory.

Overlap with CF state:
0.970 (N=8, L=16)

Edge spectrum at *ν=2*

Effective edge Hamiltonian of singlet state

[J.E. Moore, F.D.M. Haldane, PRB **55** 7818 (1997)]

$$H_{\mathrm{edge}} \propto v_s\left(S_z^2 + \sum_l l b_l^\dagger b_l\right) + v_c \sum_l l c_l^\dagger c_l$$

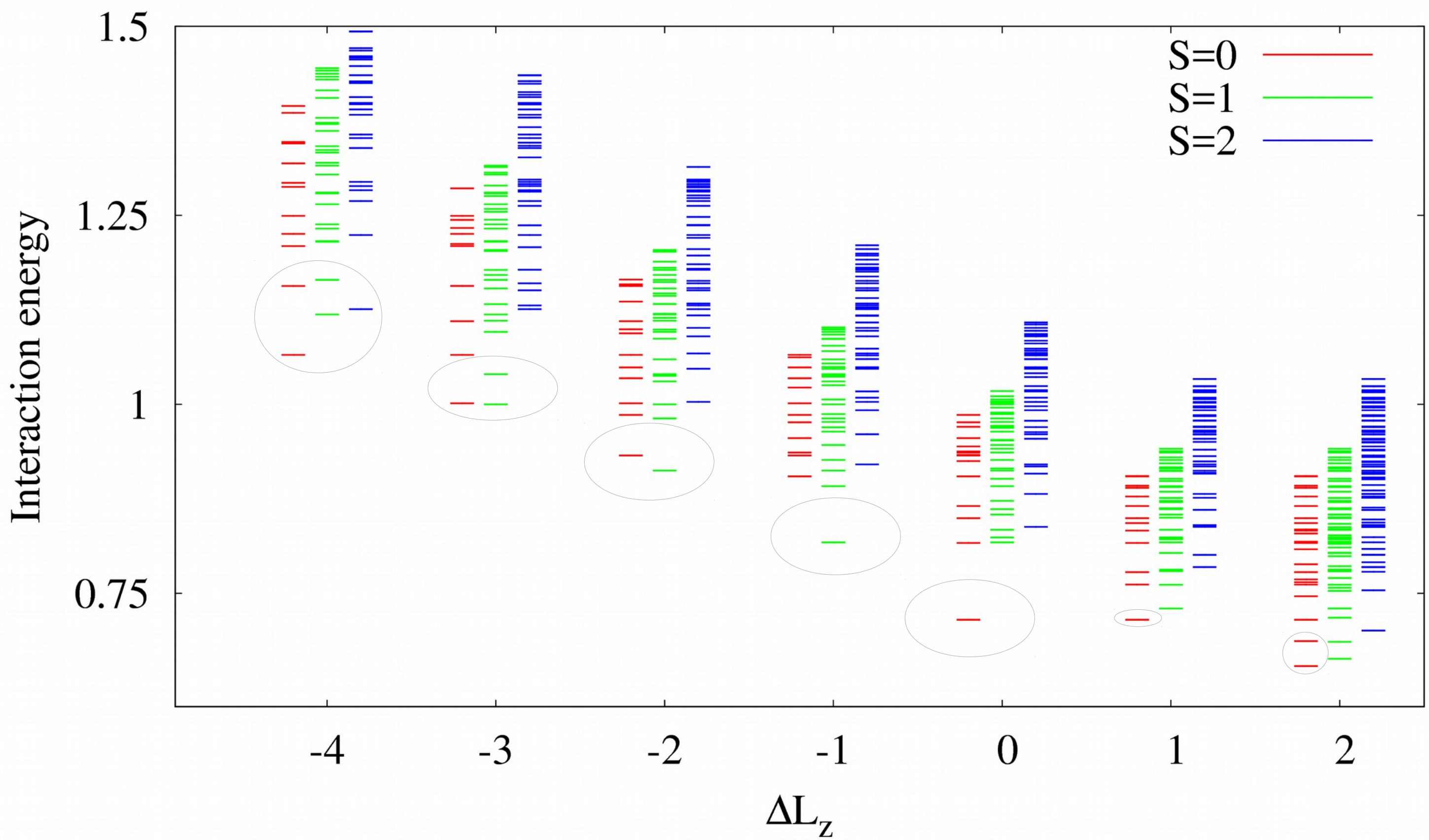
TABLE I. Number of modes of H_{edge} with $v_s < 0$ and $v_c > 0$.

ΔL_z	-4	-3	-2	-1	$+1$	$+2$	$+3$	$+4$
Number of singlets	2	1	1	0	1	2	3	5
Number of triplets	2	2	1	1	0	0	0	0
Number of quintets	1	0	0	0	0	0	0	0

Numerical results on a disk

[T. Grass *et al.*, PRB **89** 045114 (2014)]

$N_\uparrow = 4$
$N_\downarrow = 4$
$L_z = 16 + \Delta L_z$



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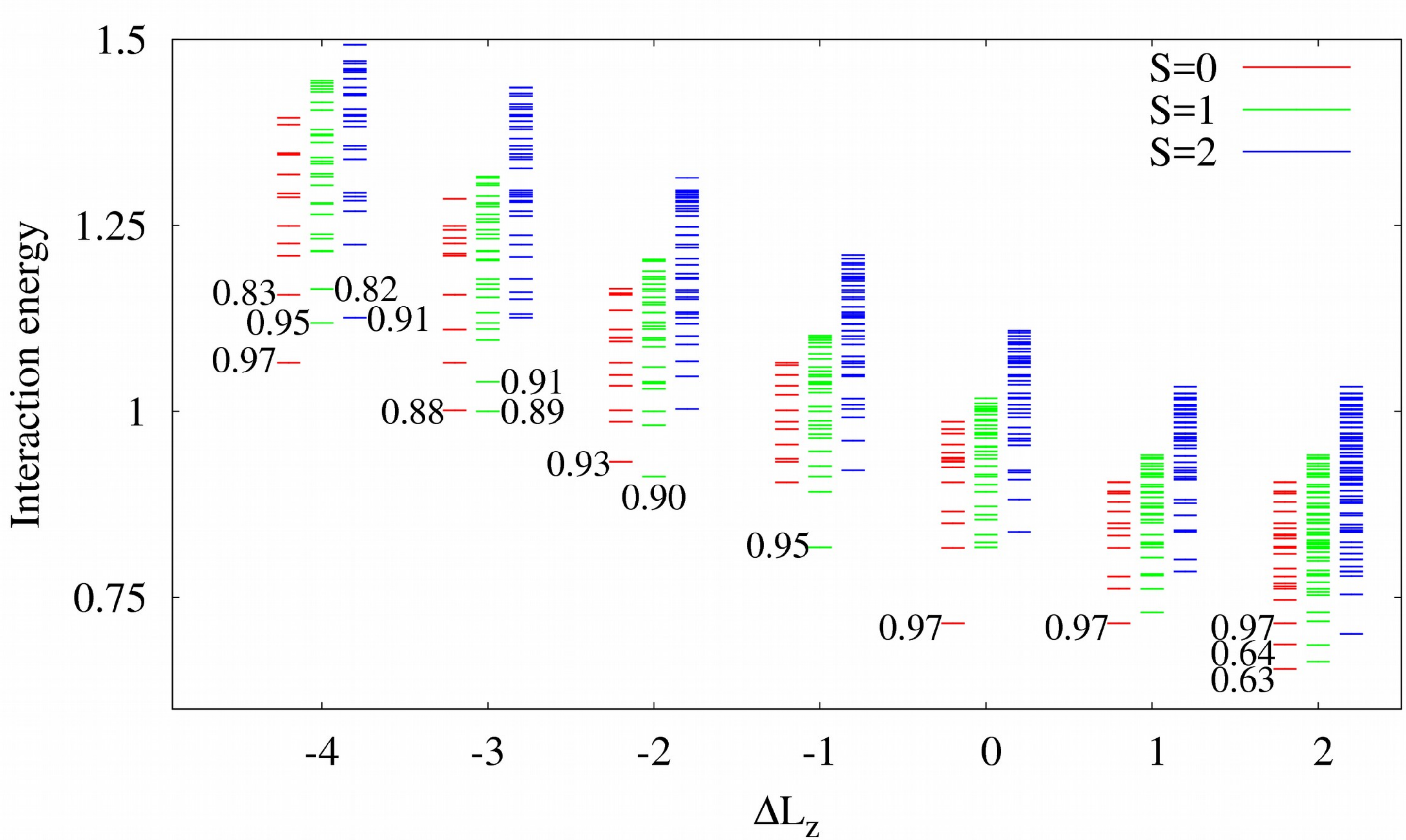
Numerical results on a disk

[T. Grass *et al.*, PRB **89** 045114 (2014)]

$$N_\uparrow = 4$$
$$N_\downarrow = 4$$
$$L_z = 16 + \Delta L_z$$

Explicit construction of edge states provides the same counting, and good overlaps:

- Backward states: Excite CFs in flux-reversed LLs
- Forward states: Symmetric polynomial



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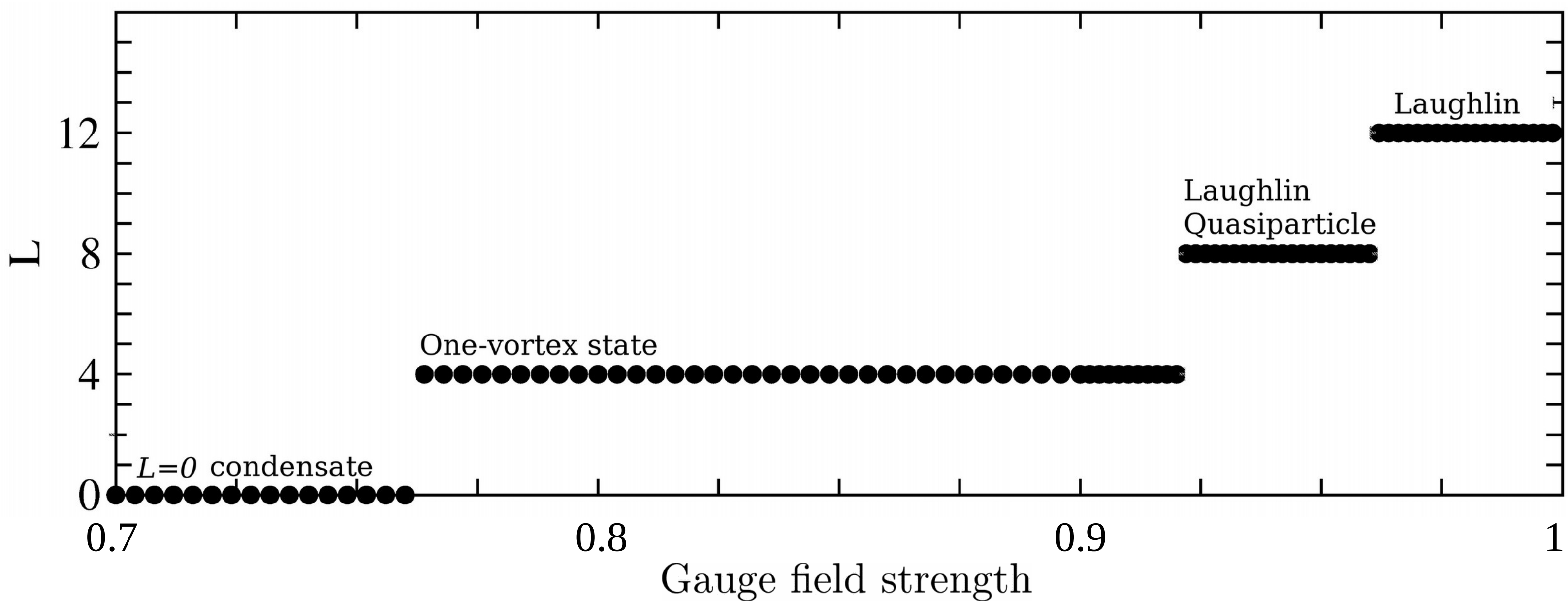
3. *Integer* quantum Hall physics of spin-1/2 bosons:

- Interactions are crucial
- Edge spectrum as a fingerprint

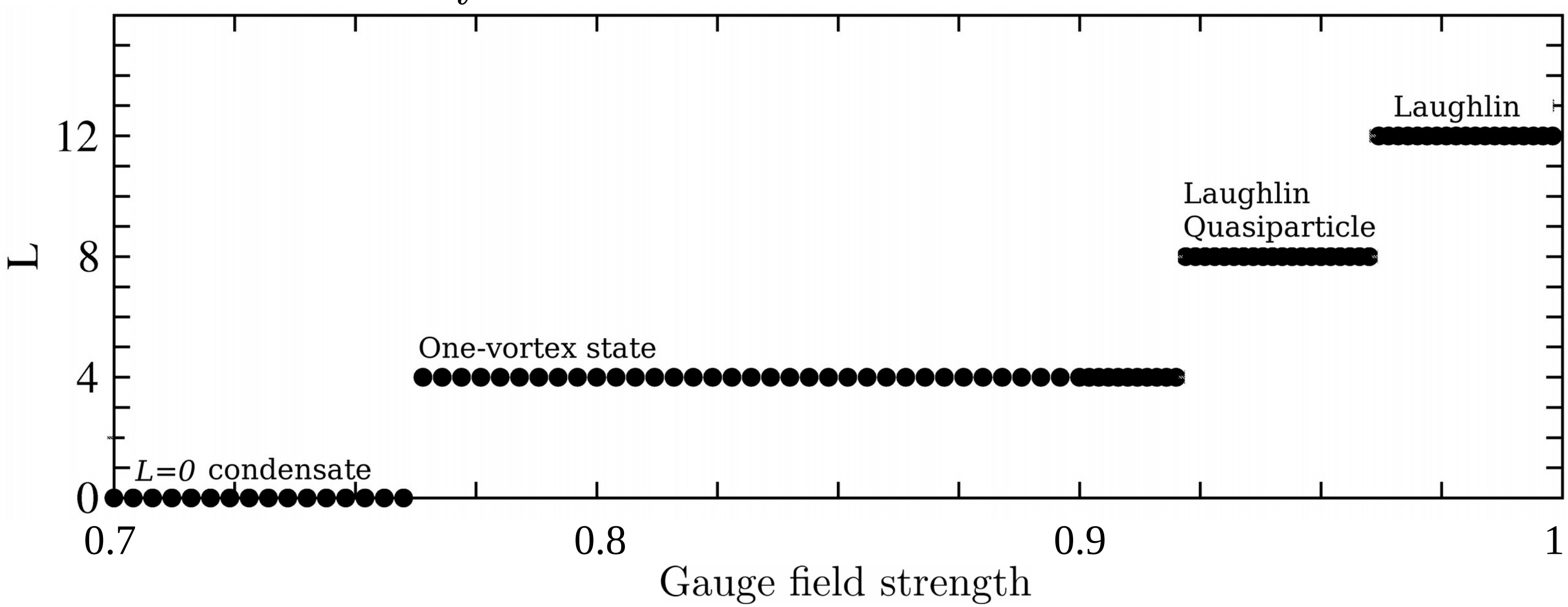
4. Anyon braiding in small systems

One-component Bose gas



One-component Bose gas

Pierce holes $\Psi \to \prod_i (z_i - \xi)\Psi$ for laser potential $V_{\mathrm{L}} \propto \delta(\xi)$.



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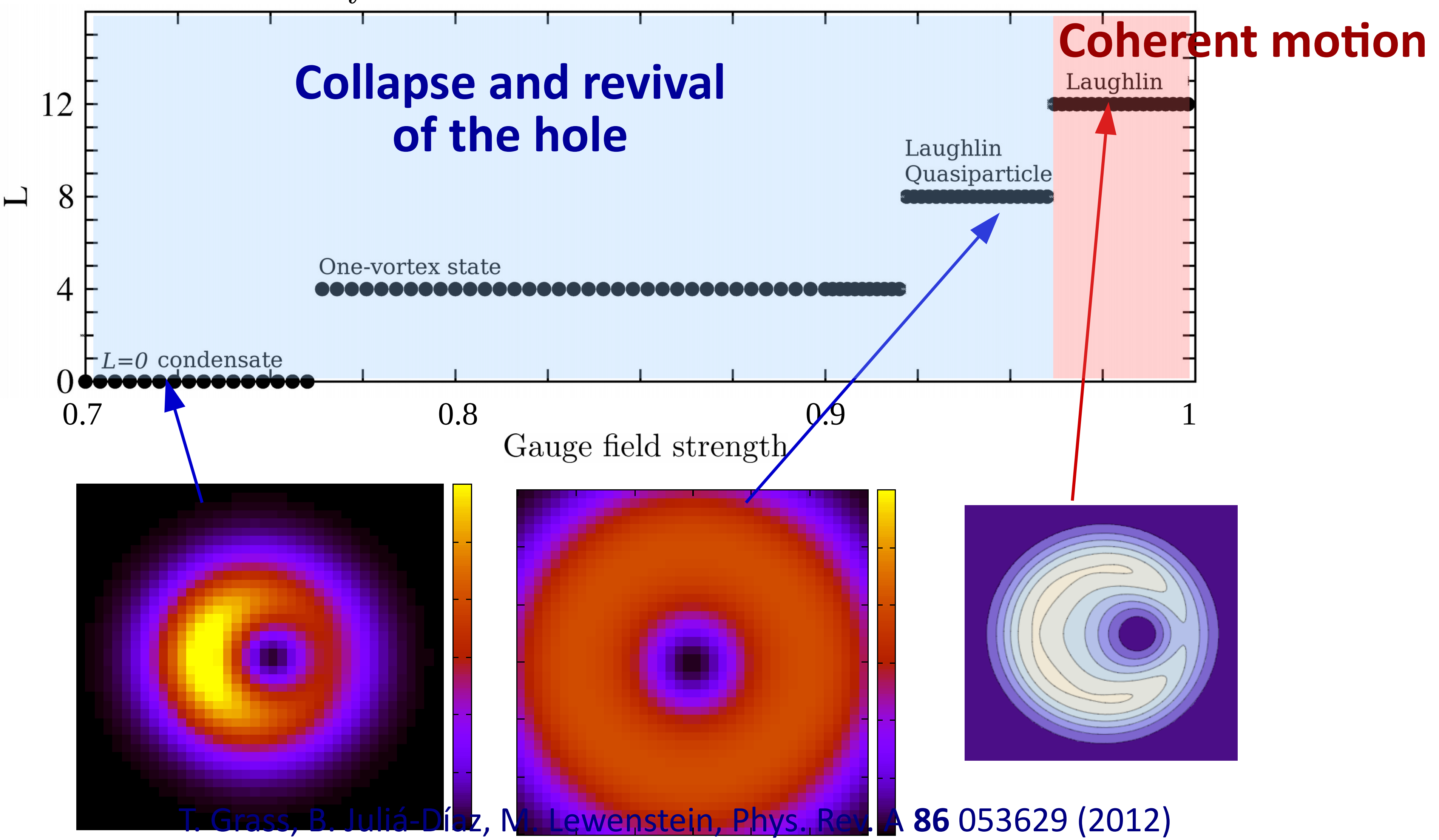
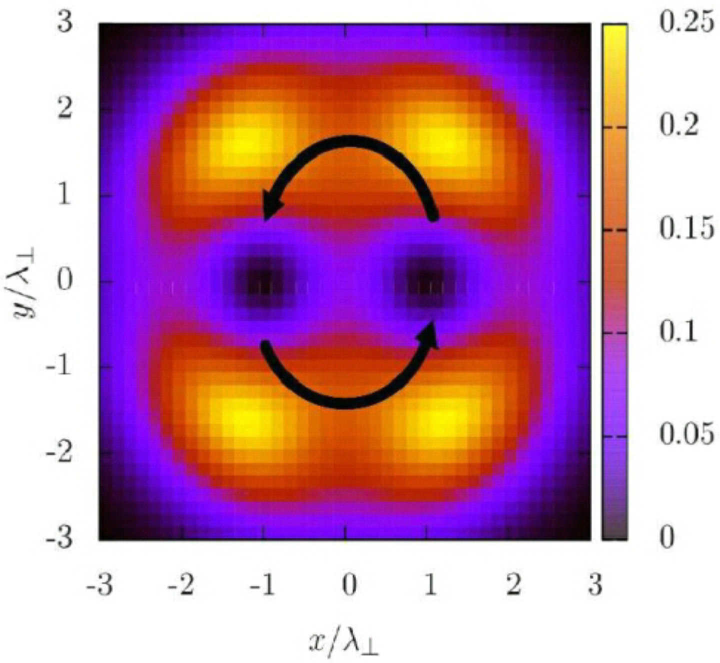


T. Grass, B. Juliá-Díaz, M. Lewenstein, Phys. Rev. A **86** 053629 (2012)

Quasiholes in FQH states

Holes in fractional quantum Hall systems:

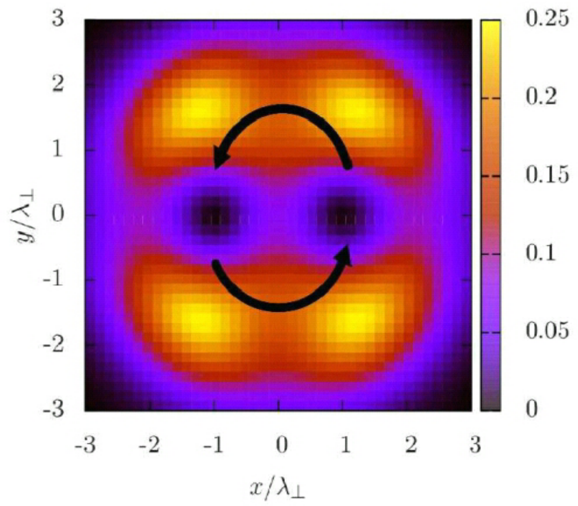
- Candidates for anyonic quasi-particle excitations
- Fractional quantum statistics
- Atomic samples:
  - *Creation by a laser beam*
  - *Control of the position*
  - *Ramsey interferometry*



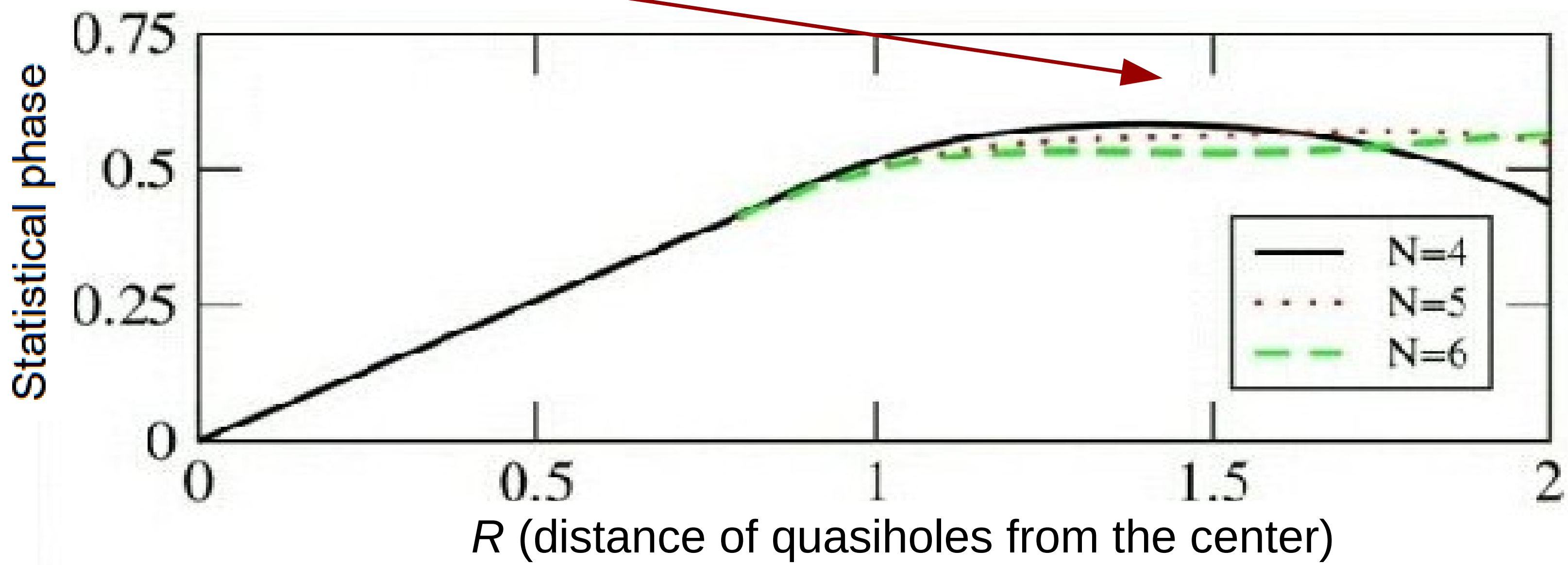
B. Paredes, P. Fedichev, J. I. Cirac, and P. Zoller. PRL **87**, 010402 (2001)
B. Juliá-Díaz, T. Grass, N. Barberán, M. Lewenstein, NJP **14**, 055003 (2012)
T. Grass, B. Juliá-Díaz, M. Lewenstein, PRA **89**, 013623 (2014)

Anyon braiding

Interchange two quasiholes and observe the phase difference: $\Psi \rightarrow e^{i\varphi}\Psi$



Fractional statistical phase becomes robust even in small Laughlin systems



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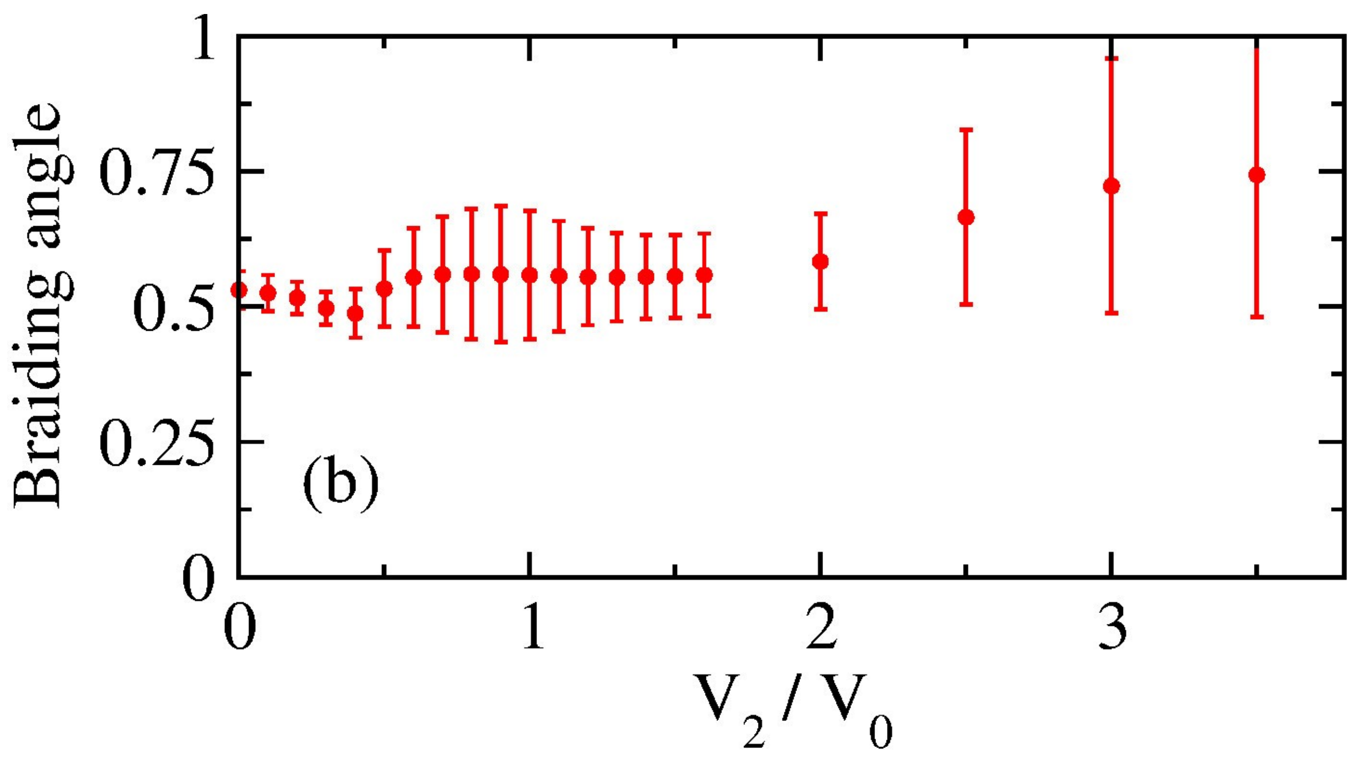
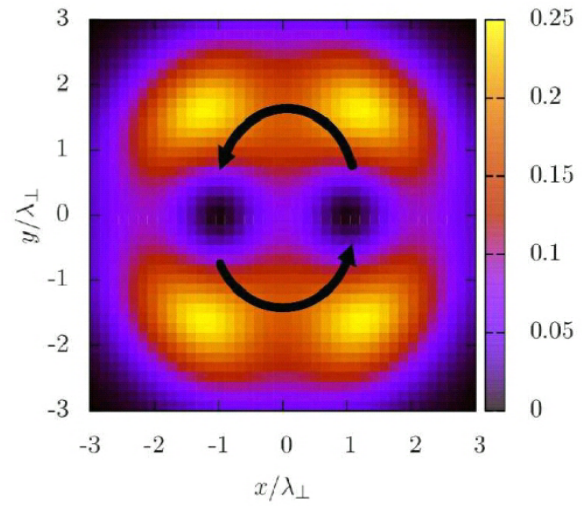
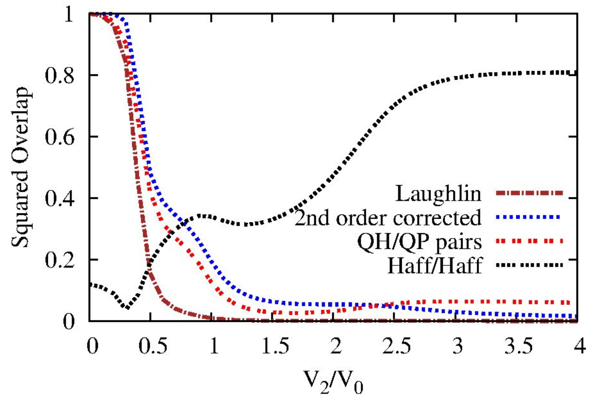
Anyon braiding

To test the robustness of braiding phase, we switch on an additional, tunable d-wave interaction:







T. Grass, B. Juliá-Díaz, M. Lewenstein, PRA **89**, 013623 (2014)

Outline

1. Quantum Hall Physics – in general:

- Single-particle physics: Landau levels
- Many-body effects: Trial states

2. Fractional quantum Hall physics of spin-1/2 bosons:

- Abelian vs. Non-Abelian phases
- Numerical results: ambiguous
- Prospects of a quantum simulation

3. *Integer* quantum Hall physics of spin-1/2 bosons:

- Interactions are crucial
- Edge spectrum as a fingerprint

4. Anyon braiding in small systems

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