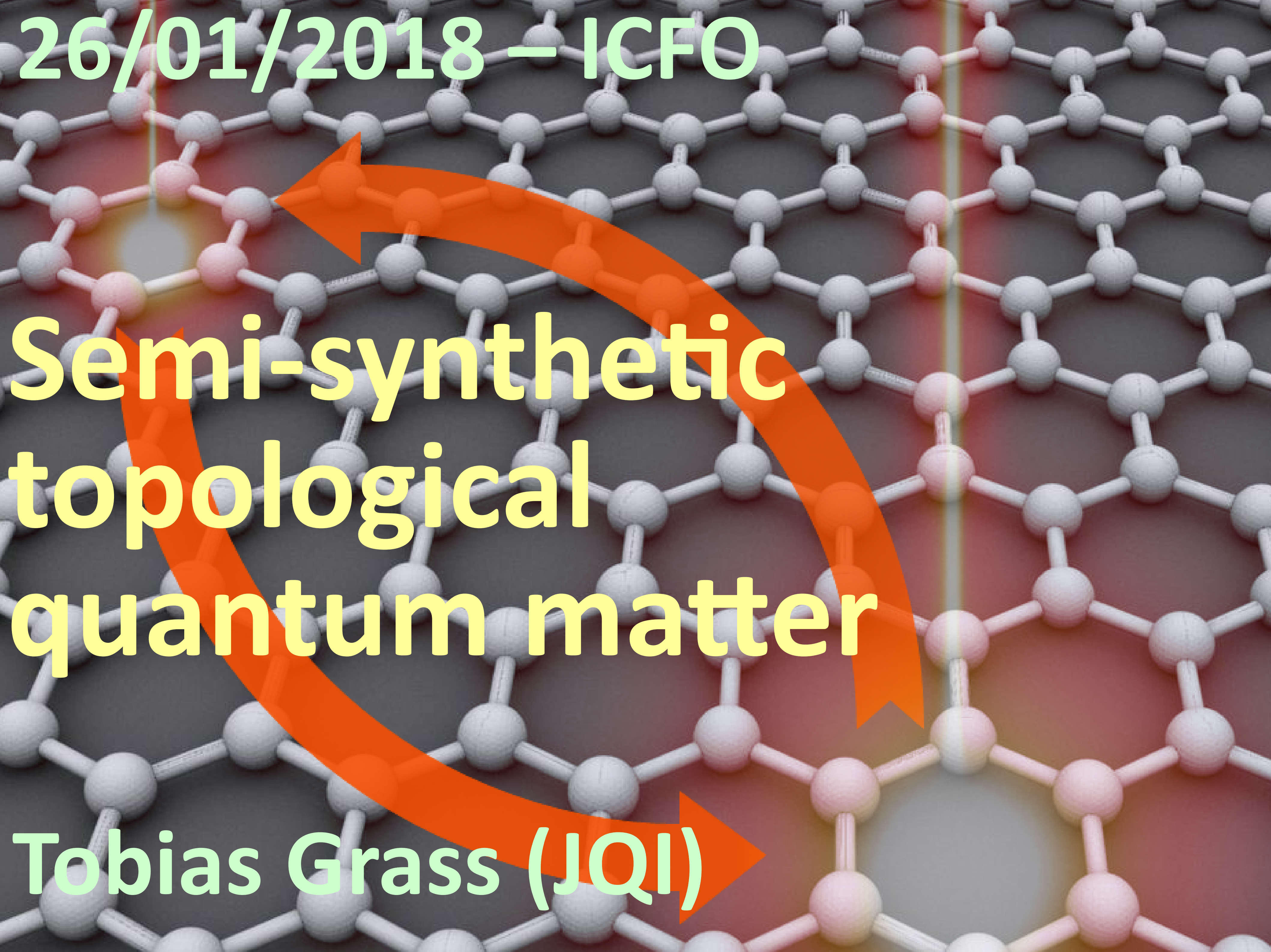
26/01/2018 – ICFO



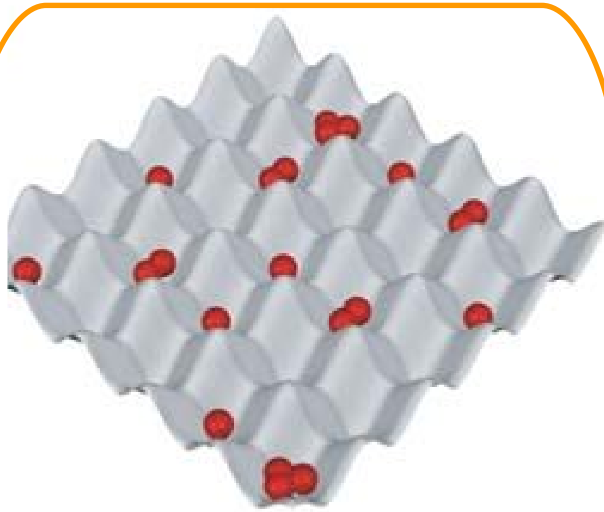
Semi-synthetic topological quantum matter

Tobias Grass (JQI)

Real matter

with relevant features intrinsic to the material

- **Solid state materials with intrinsic electronic properties:**
  - semiconductors
  - semimetals
  - metals
  - insulators ...
- **Topological features:**
  - topological insulators
  - quantum Hall samples (require external field)



Synthetic matter

for which these feature must be generated artificially

- **Quantum simulators: usually AMO systems in which light-matter interactions create some features (e.g. atomic gas in lattice potential)**
- **Topological synthetic matter: artificial gauge fields**

Real matter

with relevant features intrinsic to the material

- Solid state materials with intrinsic electronic properties:
  - semiconductors
  - semimetals
  - metals
  - insulators ...
- Topological features:
  - topological insulators
  - quantum Hall samples (require external field)

Semi-synthetic matter

Enrich real matter with artificial features

- Floquet topological insulator:

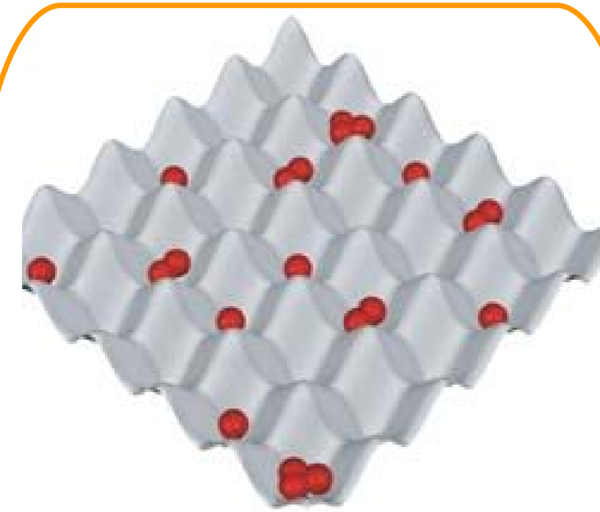
PHYSICAL REVIEW B **79**, 081406(R) (2009)

Photovoltaic Hall effect in graphene

Takashi Oka and Hideo Aoki

See also experiments at MIT [Gedik group]

- Light-induced superconduc-tivity in cuprates [Cavalleri group]
- This talk: Light-induced quantum Hall phases in graphene



Synthetic matter

for which these feature must be generated artificially

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- Topological synthetic matter: artificial gauge fields

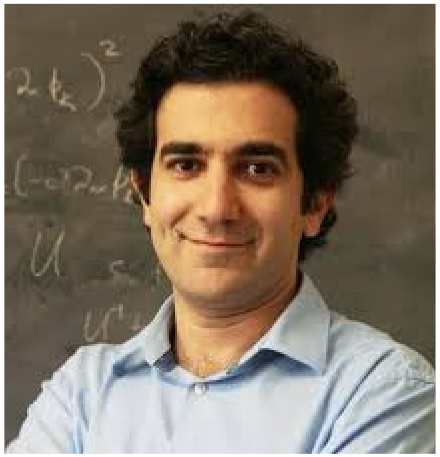
Outline

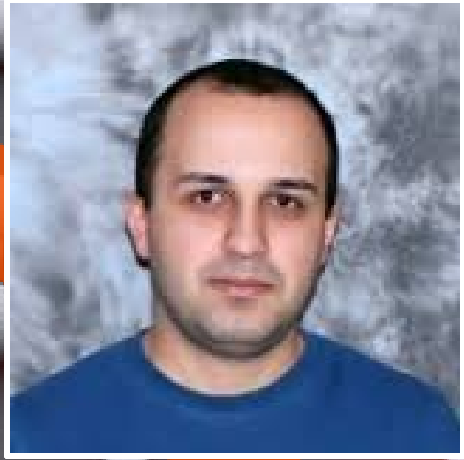
Intro: Quantum Hall, Graphene,
Light-matter coupling

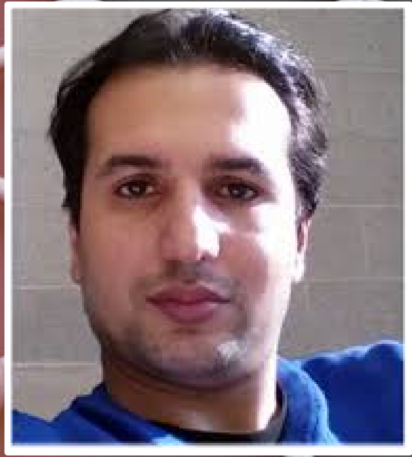
Part I: Optical driving:
Controlling FQH phases

Part II: Optical excitations:
Flux pump and braiding

Work in collaboration with:







Mohammad Hafezi

(JQI / NIST)

Michael Gullans

(Princeton)

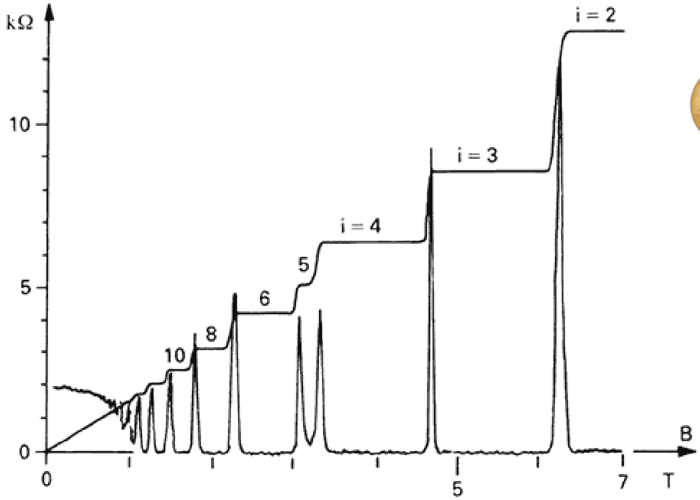
Areg Ghazaryan

Pouyan Ghaemi

(City College New York)

Quantum Hall Effect

As transport phenomenon: Quantized Hall Resistance



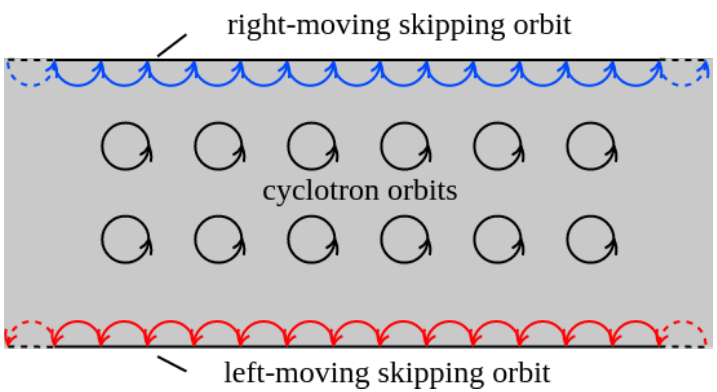
1985

Klaus v. Klitzing

Explanation in terms of topology: Protected Edge States

2016

David Thouless



Fractional Quantum Hall Effect and Anyonic Quasiparticles

$$\Psi_{\text{Laughlin}} = \prod_{i<j} (z_i - z_j)^{1/\nu} \mathrm{e}^{-\sum_i |z_i|^2/4}$$

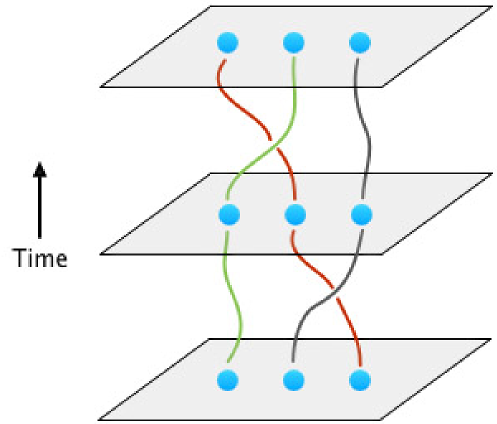
1998

Robert B. Laughlin
Prize share: 1/3

Horst L. Störmer
Prize share: 1/3

Daniel C. Tsui
Prize share: 1/3

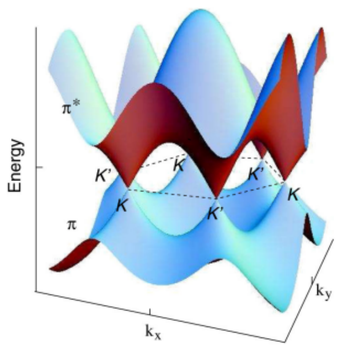
Non-Abelian Anyons and Topological Quantum Computing



Use non-Abelian anyons as robust quantum memory.
Quantum information is processed by braiding these anyons.

NO NOBEL PRIZE YET!!

Graphene in magnetic field: Landau levels



Effective Hamiltonian around Dirac point:

$$H_\xi = \xi v_{\rm F}(p_x \sigma_x + p_y \sigma_y)$$

$$\xi = \pm \text{ for } K, K'$$

Pauli matrices represent sublattice structure!

In magnetic field:

$$p_i \rightarrow \Pi_i = p_i - \frac{e}{c} A_i$$

$$\Pi_x = \frac{\hbar}{\sqrt{2} l_{\rm B}}(a^\dagger + a) \quad \text{and} \quad \Pi_y = \frac{\hbar}{i\sqrt{2} l_{\rm B}}(a^\dagger - a)$$

$$H_\xi = \xi \sqrt{2} \frac{\hbar v_{\rm F}}{l_{\rm B}} \begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix}$$

"Standard" Landau level wave functions:

$$a^\dagger \varphi_{n,m} = \varphi_{n+1,m}$$

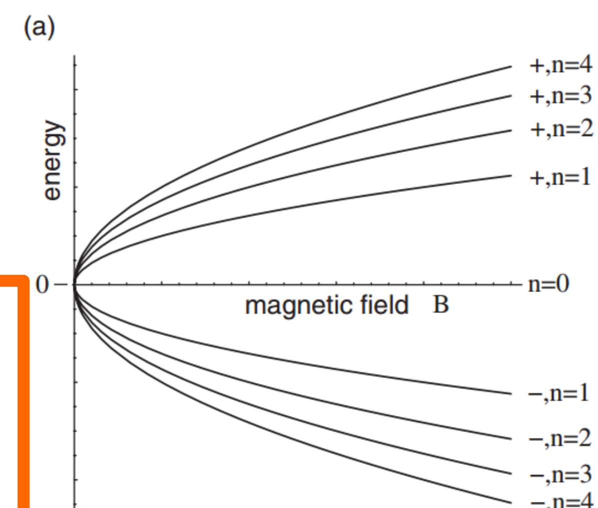
Graphene Landau level wave functions:

$$\Psi_{n=0,m} = \begin{pmatrix} 0 \\ \varphi_{0,m} \end{pmatrix} \text{ and } \Psi_{n\neq 0,m} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_{|n|-1,m} \\ \xi \text{sign}(n) \varphi_{|n|,m} \end{pmatrix}$$

At energies $\epsilon_n = \text{sign}(n) \frac{\hbar v_{\rm F}}{l_{\rm B}} \sqrt{2|n|}$

Features of relativistic Landau levels:

- Spinor wave function
- Spin and valley degeneracy: 4 bands per energy level
- Particle-hole symmetry
- Non-equidistant energy levels!



See also review article: M. Goerbig, *Electronic properties of graphene in a strong magnetic field*, Rev. Mod. Phys. **83** 1193 (2011)

Interactions between light and Landau levels

Dirac Hamiltonian:

$$H = v_{\rm F}(p_x\sigma_x + p_y\sigma_y)$$

Minimal coupling:

$$p_i \to \Pi_i = p_i - \frac{e}{c}A_i$$

Light-matter interaction:

$$H_{\rm int} \sim \sigma_\pm A_\pm(x,y,t) + \text{h.c.}$$

Example. circularly polarized plane-wave in x-direction: $$\mathbf{A}(x,y,t) \sim \exp[i(kx-\omega t)]\begin{pmatrix}1\\ \pm i\end{pmatrix} + \text{h.c.}$$

rotating frame: $$\left(\langle\tilde n-1,\tilde m|,\langle\tilde n,\tilde m|\right) H_{\rm int}\begin{pmatrix}0\\ |0,m\rangle\end{pmatrix} \sim \left(\langle\tilde n-1,\tilde m|,\langle\tilde n,\tilde m|\right)\exp\left[\pm ikx\right]\begin{pmatrix}|0,m\rangle\\ 0\end{pmatrix}$$

The transition matrix element of non-relativistic Landau levels is given by:

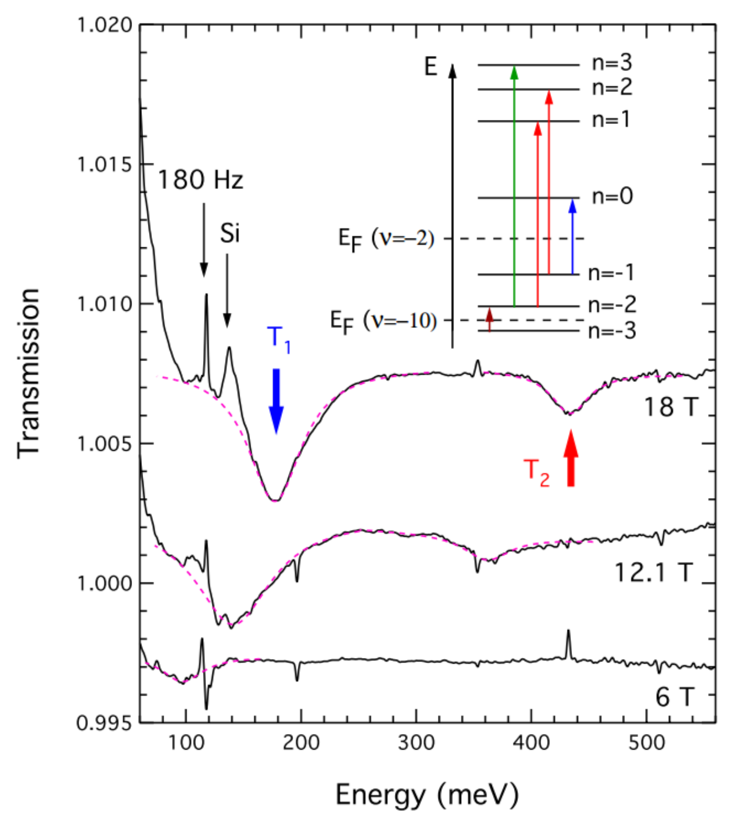
$$\int \mathrm{d}^2z\ \varphi_{\tilde n,\tilde m}(z)^*\varphi_{n,m}(z)e^{\pm ikx} = \sqrt{\frac{n!m!}{\tilde n!\tilde m!}}(\pm ik)^{\tilde n-n}L_n^{\tilde n-n}(k^2)L_m^{\tilde m-m}(k^2)$$

It is dominated by: $\tilde n = n \quad \text{and} \quad \tilde m = m$

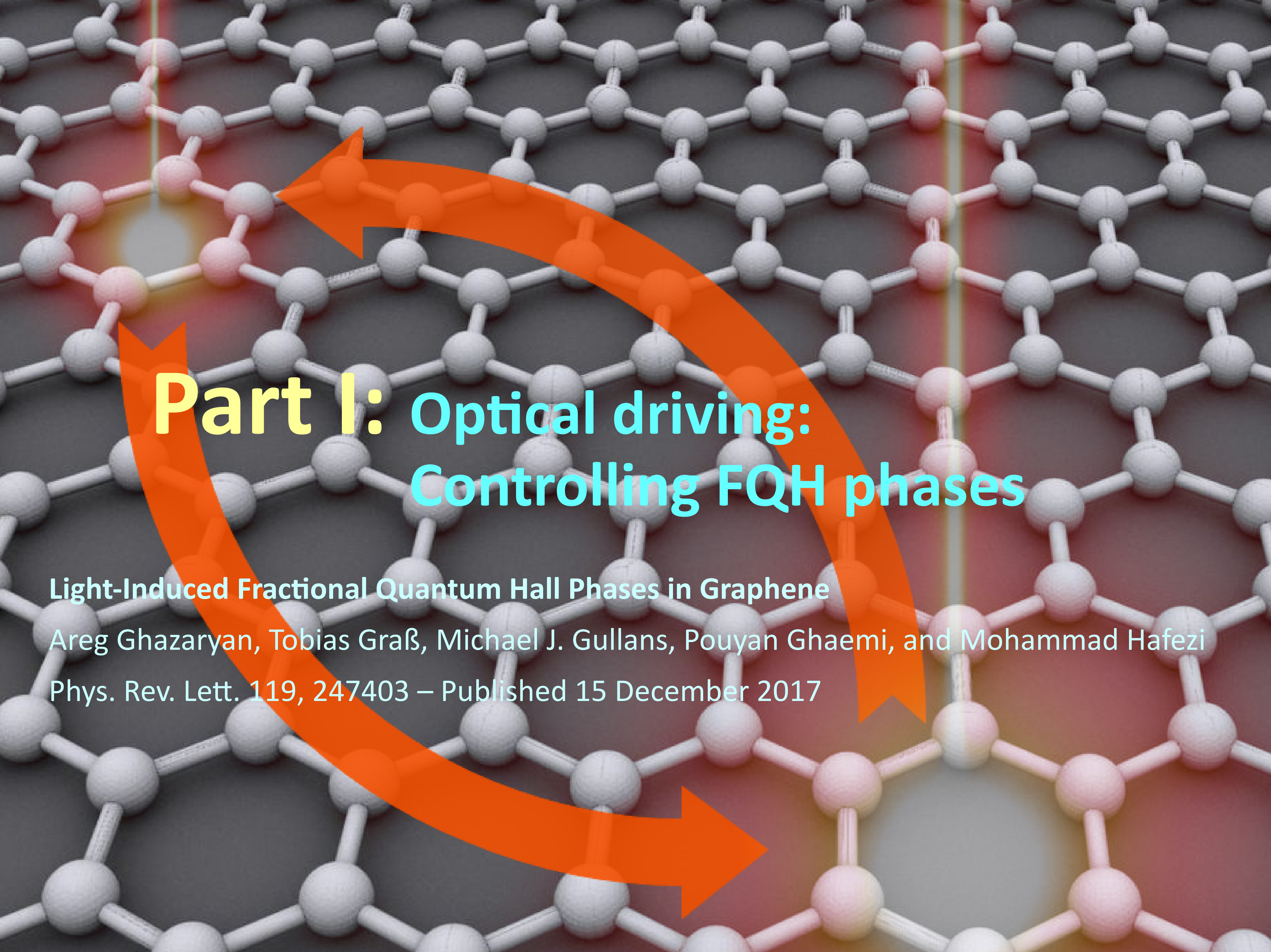
Thus, in terms of relativistic LL spinors, the optical selection rules are:

$$\tilde n = |n| \pm 1 \quad \text{and} \quad m = \tilde m$$

m-selection rule can be modified by using light with OAM, cf.
M. Gullans *et al.*, PRB 95, 235439 (2017).



Z. Jiang *et al.*, PRL 98, 197403 (2007)



Part I: Optical driving: Controlling FQH phases

Light-Induced Fractional Quantum Hall Phases in Graphene

Areg Ghazaryan, Tobias Graß, Michael J. Gullans, Pouyan Ghaemi, and Mohammad Hafezi

Phys. Rev. Lett. 119, 247403 – Published 15 December 2017

Coupled Landau levels

LL energy | Coupling

$$H_0(t) = \sum_m \left[\frac{\Delta E}{2}\left(c^\dagger_{n+1,m}c_{n+1,m} - c^\dagger_{n,m}c_{n,m}\right) + \hbar\Omega\left(c^\dagger_{n+1,m}c_{n,m}\mathrm{e}^{-i(\Delta E-\delta)t} + \text{h.c.}\right)\right]$$

In rotating frame after rotating wave approximation

$$H_0 = \sum_m \left[\hbar\delta c^\dagger_{n+1,m}c_{n+1,m} + \hbar\Omega c^\dagger_{n+1,m}c_{n,m}\right] + \text{h.c.}$$

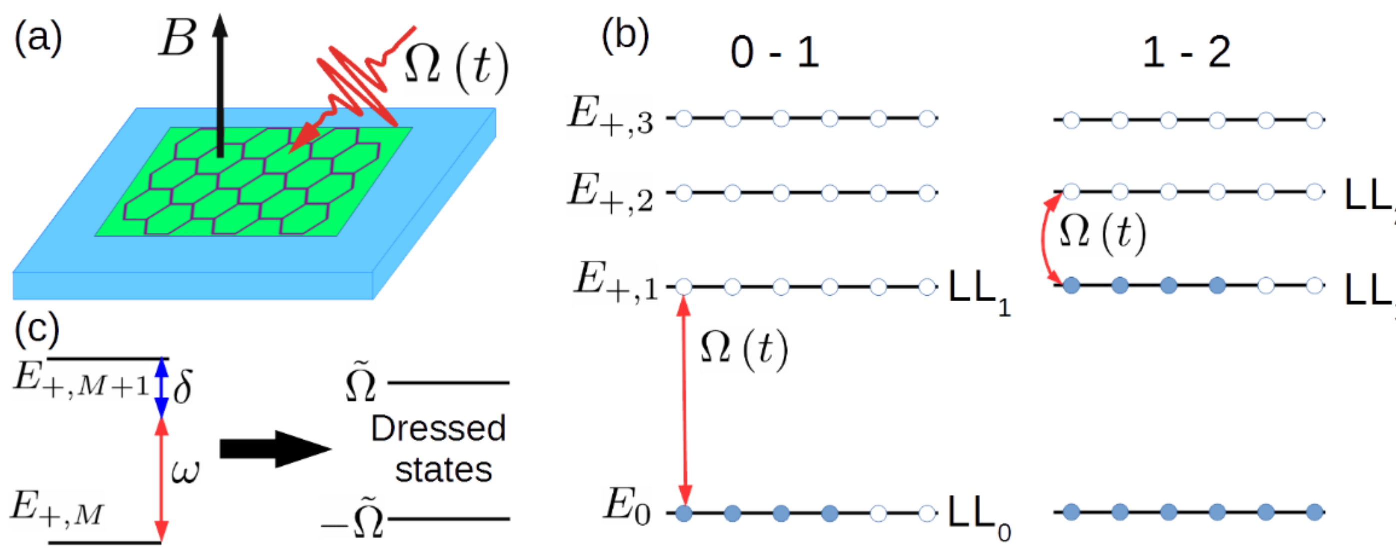
Less is more!

Strong coupling:

Lowest Landau level becomes dressed, but may not change much the physics.

Weak coupling:

Both Landau levels can be occupied: System becomes analogous to a bilayer.



Interactions between coupled LLs

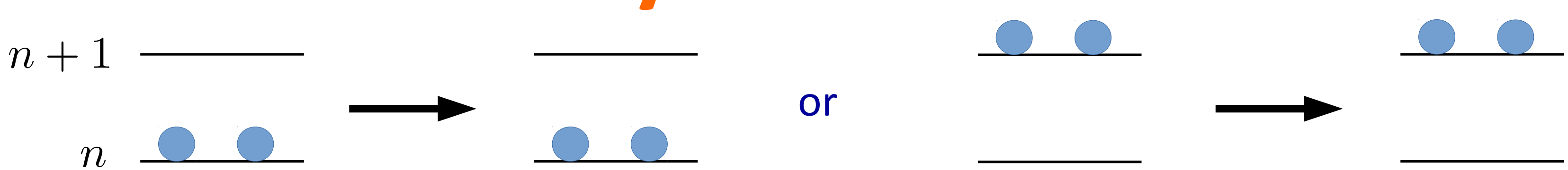
Fractional Quantum Hall Hamiltonian: $H = H_0 + V^{(\mathrm{RWA})}$

$$V^{n_1,n_2,n_3,n_4(\mathrm{RWA})}_{m_1,m_2,m_3,m_4} = \delta_{n_1+n_2-n_3-n_4} V^{n_1,n_2,n_3,n_4}_{m_1,m_2,m_3,m_4}$$

Pseudopotential expansion: $V^{n_1,n_2,n_3,n_4}_{m_1,m_2,m_3,m_4} = \sum_{m,M} V^{n_1,n_2,n_3,n_4}_{m} \langle m_1, m_2 | m, M \rangle \langle m, M | m_3, m_4 \rangle$

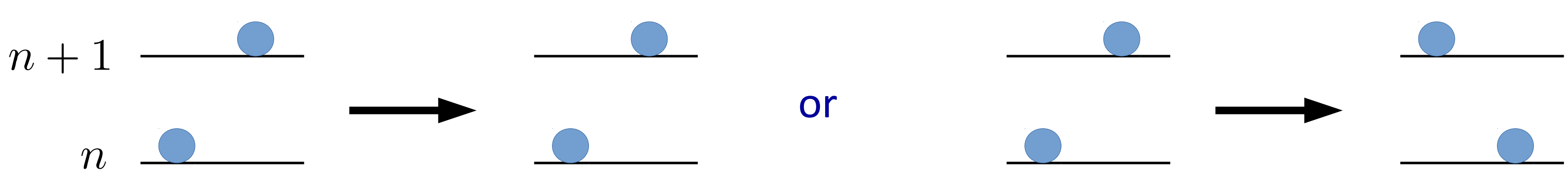
Different kinds of interaction processes:

(I) "Intra-layer" interactions



$V_m^{(\downarrow)} \equiv V_m^{n,n,n,n}$ $V_m^{(\uparrow)} \equiv V_m^{n+1,n+1,n+1,n+1}$

(II) "Inter-layer" interactions



$V_m^{\downarrow\uparrow,\uparrow\downarrow}$ $V_m^{\downarrow\uparrow,\downarrow\uparrow}$

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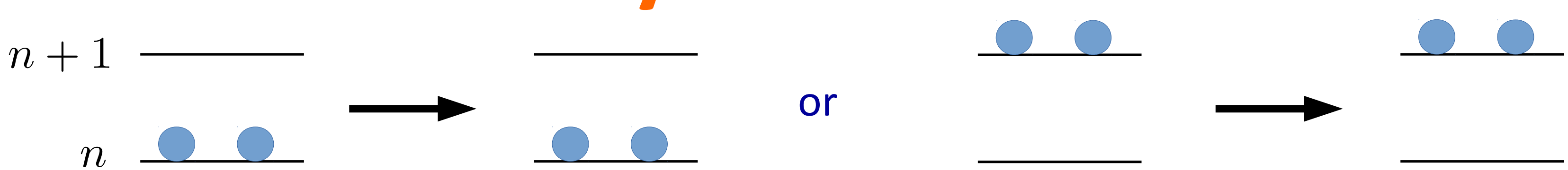


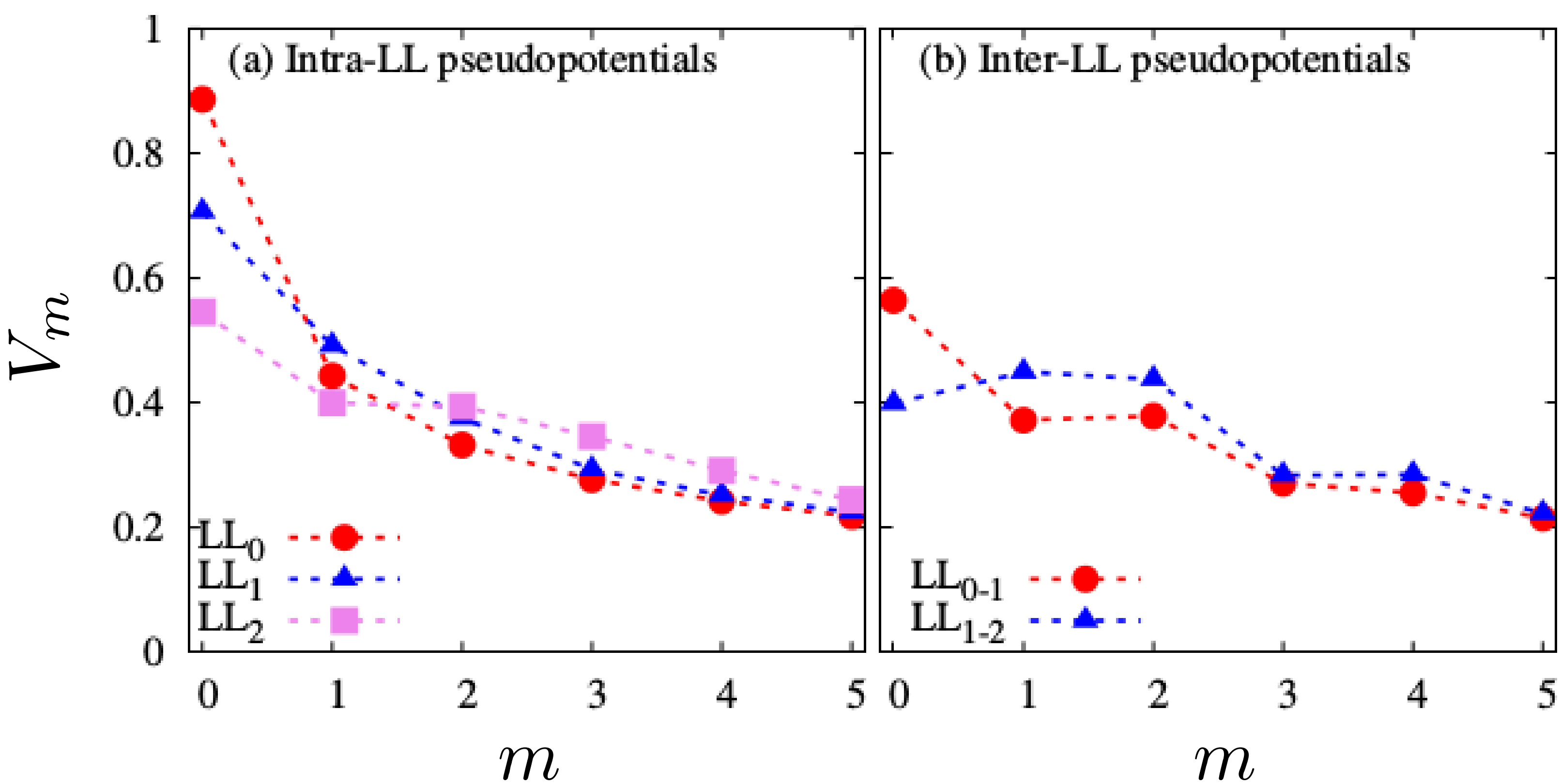
$$V_m^{(\downarrow)} \equiv V_m^{n,n,n,n} \qquad V_m^{(\uparrow)} \equiv V_m^{n+1,n+1,n+1,n+1}$$

(II) "Inter-layer" interactions

$$V_m^{(\mathrm{singlet})} \equiv \langle \uparrow\downarrow - \downarrow\uparrow | V_m | \uparrow\downarrow - \downarrow\uparrow \rangle = \frac{1}{2}\left(V_m^{\uparrow\downarrow,\downarrow\uparrow} - V_m^{\uparrow\downarrow,\uparrow\downarrow}\right) \qquad \text{requires even } m$$

$$V_m^{(\mathrm{triplet})} \equiv \langle \uparrow\downarrow + \downarrow\uparrow | V_m | \uparrow\downarrow + \downarrow\uparrow \rangle = \frac{1}{2}\left(V_m^{\uparrow\downarrow,\downarrow\uparrow} + V_m^{\uparrow\downarrow,\uparrow\downarrow}\right) \qquad \text{requires odd } m$$

Interactions between coupled LLs



(a) Intra-layer pseudopotentials for different graphene LLs

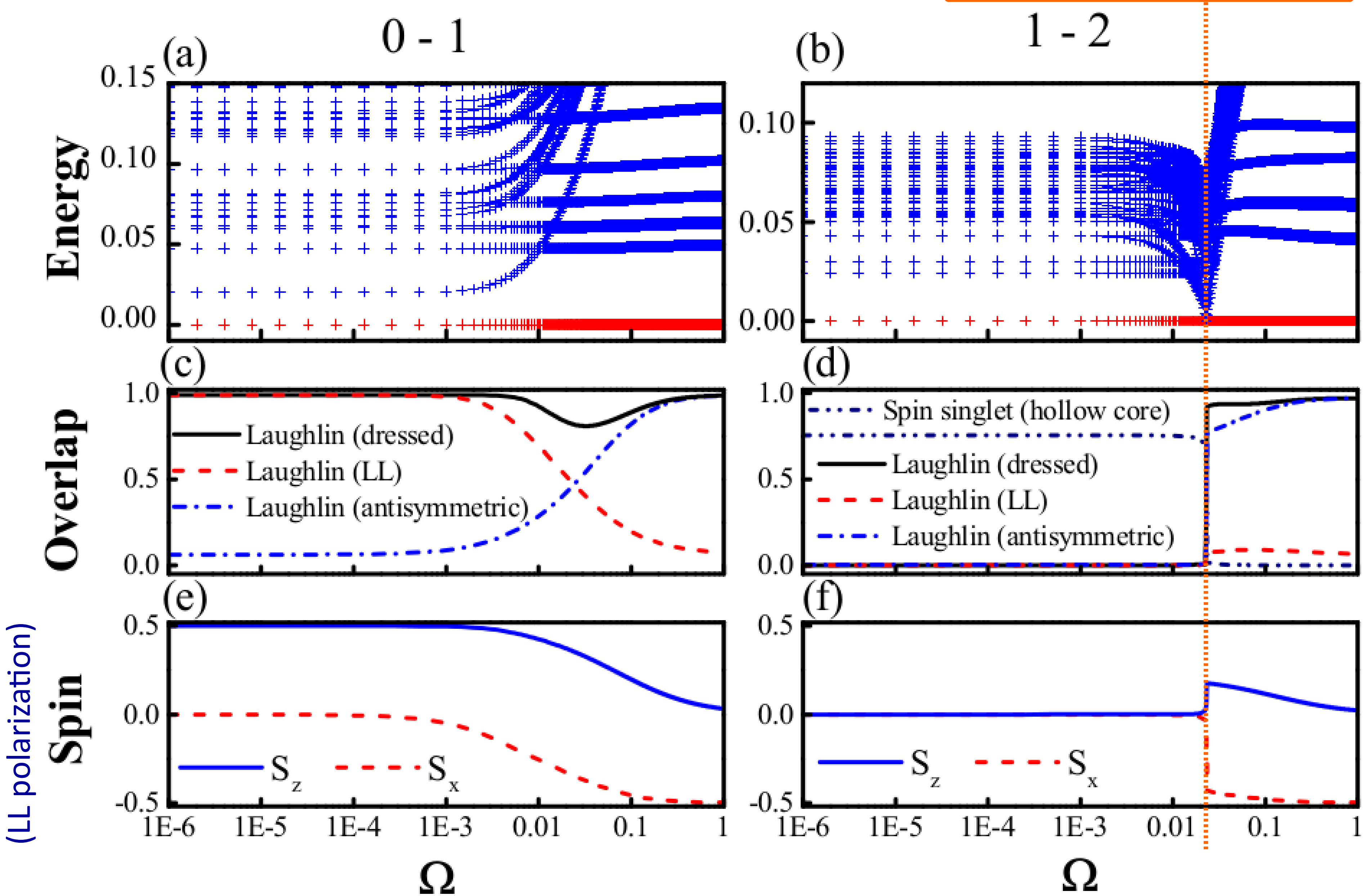
(b) Inter-layer pseudopotentials when n=0 graphene LL is coupled to n=1 (red), or n=1 is coupled to n=2 (blue).

Coupling 1-2 favors singlets at m=0 over triplets at m=1. We thus expect a tendency towards singlet ground states, maybe phases described by a hollow-core model?

LL 0-1 coupling vs. LL 1-2 coupling

Exact diagonalization results for N=8 electrons on a torus at filling ν=2/3:

Phase transition:
Polarized vs. singlet phase



LL 1-2 coupling: Singlet vs. polarized phase

	Singlet phase	Polarized phase
$\nu = 2/3$		Laughlin state of holes
$\nu = 1/2$		Composite Fermi sea (Halperin, Lee, Read)

Ω

LL 1-2 coupling: Singlet vs. polarized phase

	Singlet phase	Polarized phase
$\nu = 2/3$	- Intra-layer Pfaffian - Inter-layer Pfaffian - Fibonacci - (113)-Halperin - (330)-Halperin - CF singlet	Laughlin state of holes
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Ω

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$\nu = 1/2$	- Haldane-Rezayi - Jain CF singlet - (331)-Halperin	Composite Fermi sea (Halperin, Lee, Read)

Ω

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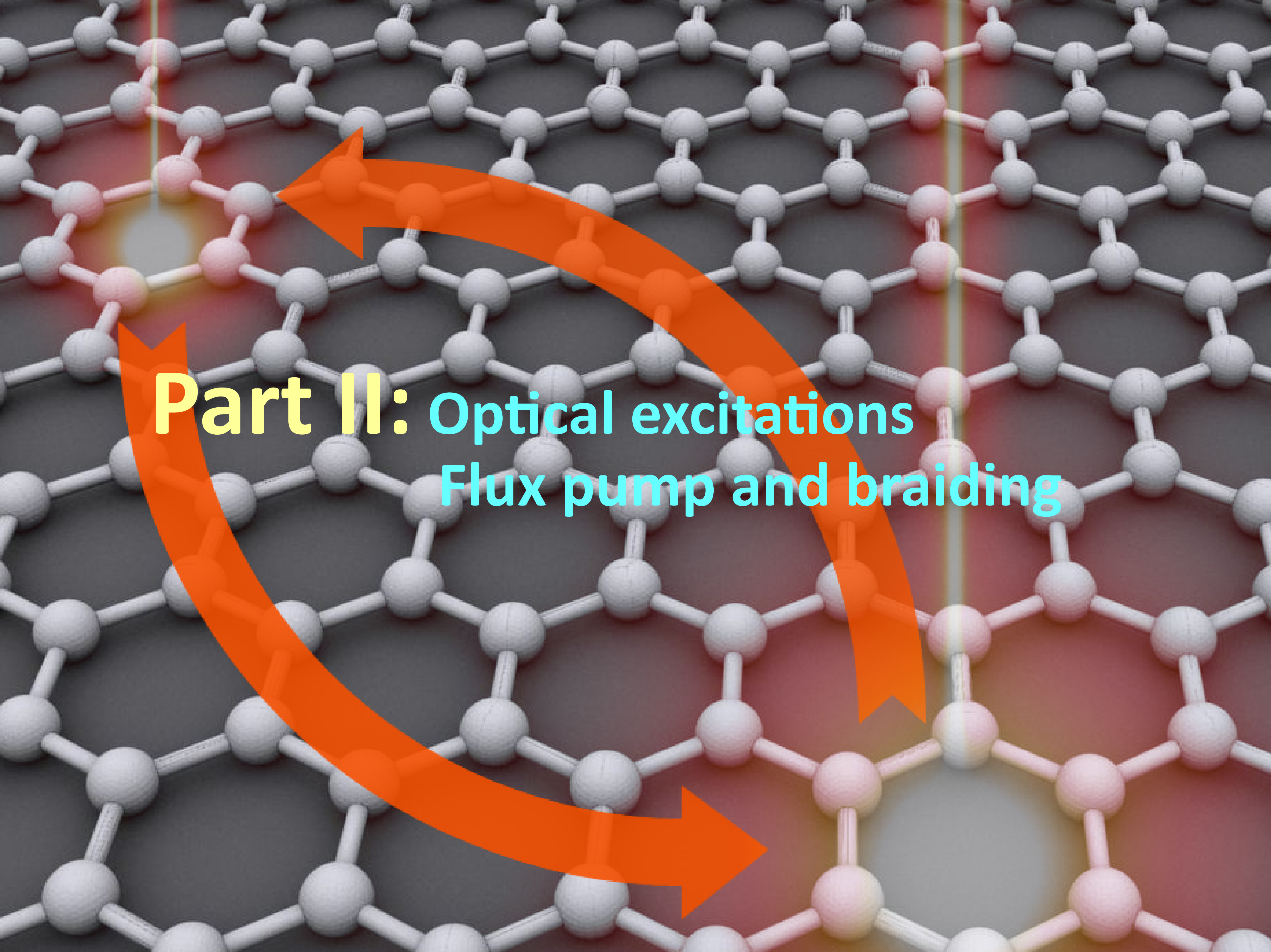
$\nu = 1/2$	**- Haldane-Rezayi** ~~- Jain CF singlet~~ ~~- (331)-Halperin~~	Composite Fermi sea (Halperin, Lee, Read)

Ω



Part II: Optical excitations
Flux pump and braiding

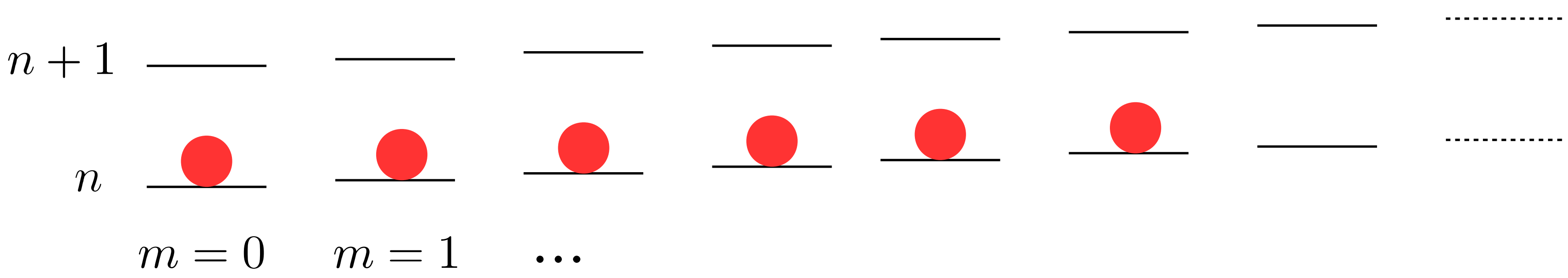
π-pulse excitations

Single-particle level: π-pulse flips spin.

In terms of Landau levels: $\varphi_{n,m} \to \varphi_{n+1,m} = a^\dagger \varphi_{n,m}$

Pulse with OAM: $\varphi_{n,m} \to \varphi_{n+1,m+1} = a^\dagger b^\dagger \varphi_{n,m}$

Action onto an IQH phase:



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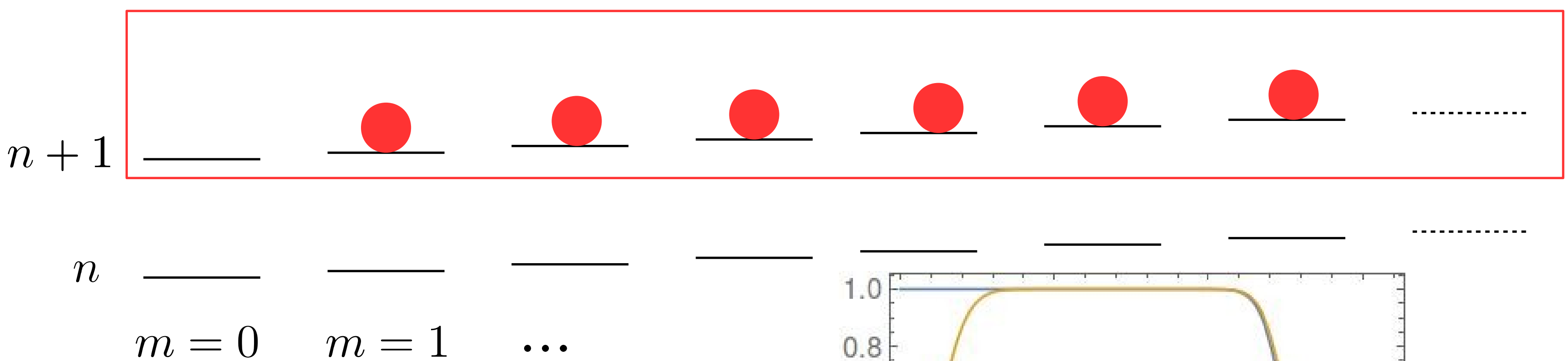
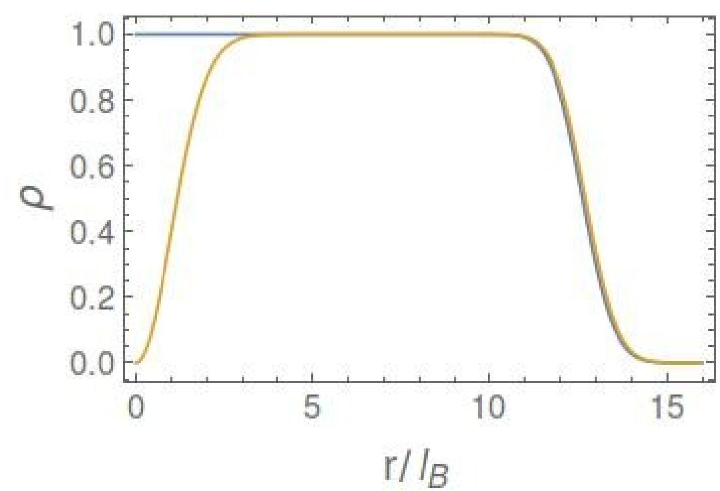


IQH with hole in the center



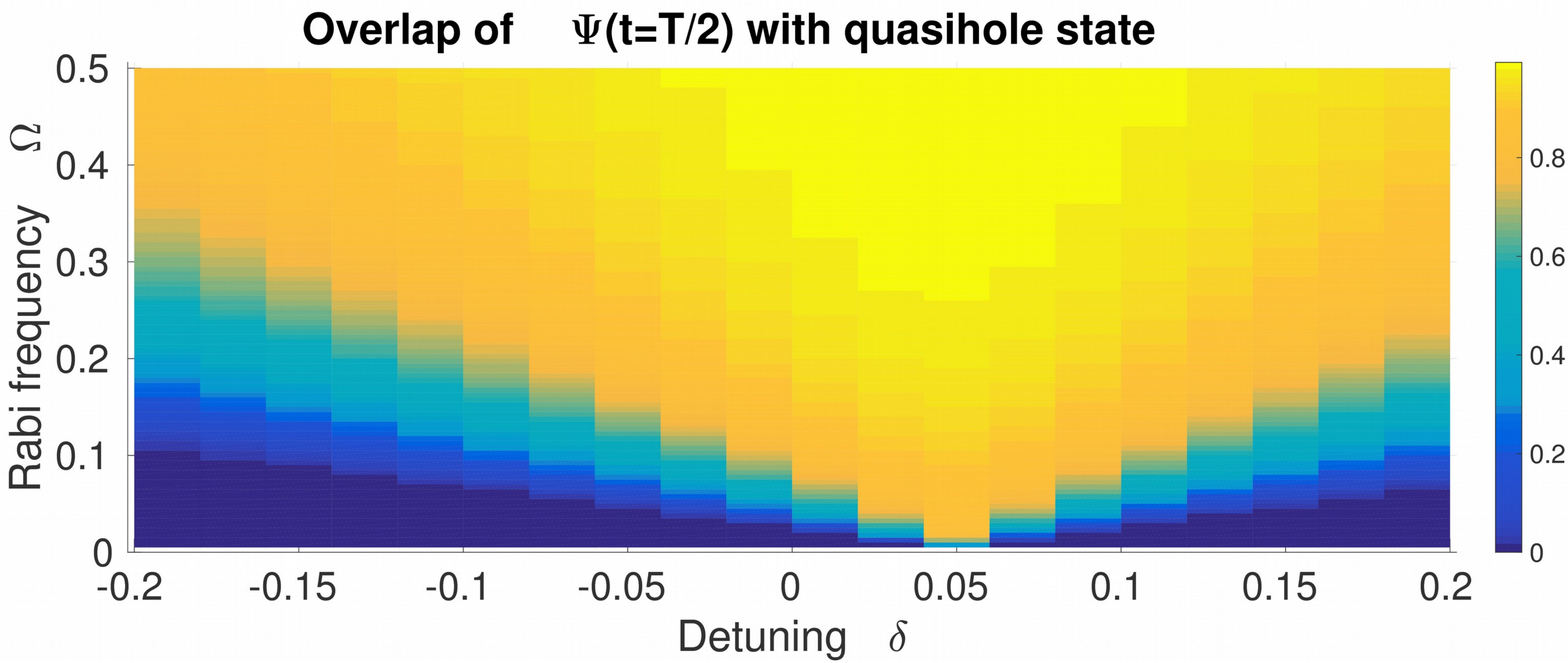
Density of IQH and hole, for *N*=100 within *n*=0 LL.

π-pulse: Many-body excitations

Action of a pulse on many-body wave function in the LLL:

$$\Psi \to \prod_{i=1}^{N} a_i^\dagger b_i^\dagger \Psi = \prod_{i=1}^{N} a_i^\dagger \left(\prod_{i=1}^{N} z_i \Psi \right)$$

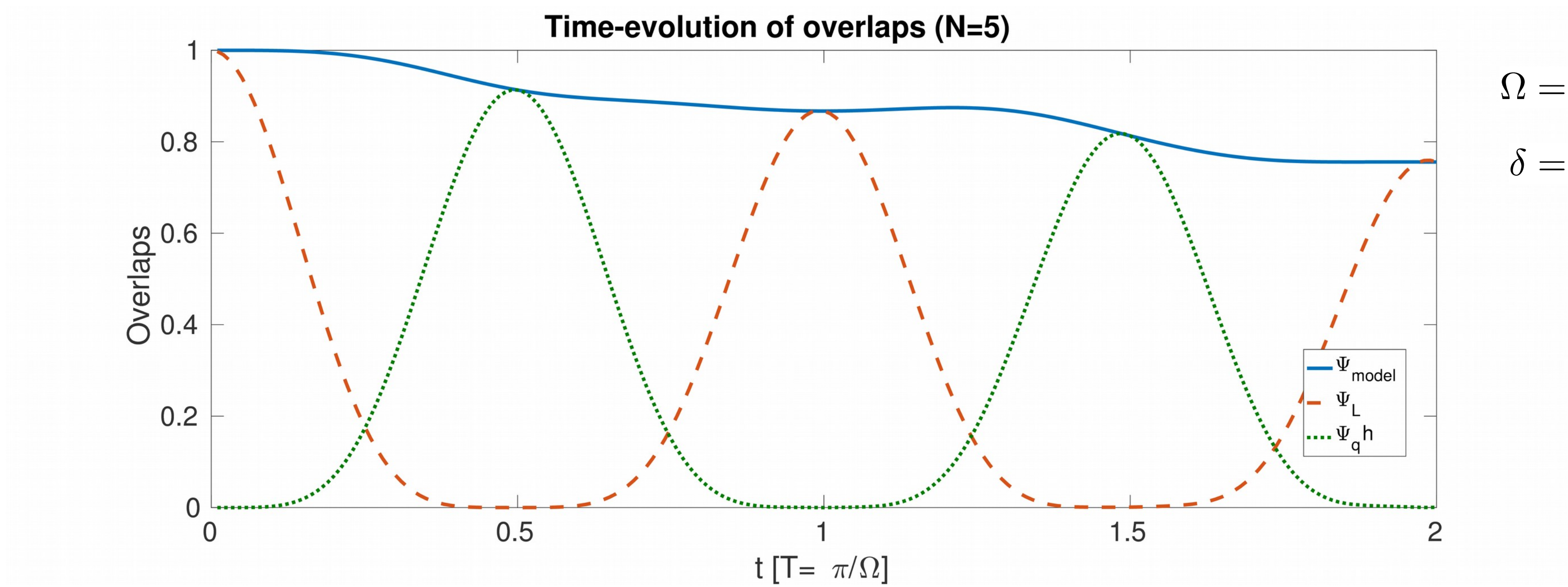
Fidelity of pi-pulse in the presence of Coulomb interactions (*N*=5):



Time evolution in the light field

can be modeled as superposition of initial state, quasihole state, and edge-like excitations:

$$\Psi_{\mathrm{model}}(t) = \sum_{s=0}^{N} \sqrt{\binom{N}{s}} \cos(\Omega t)^{N-s} \sin(\Omega t)^{s} \Psi^{(s)}, \qquad \Psi^{(s)} \sim \sum_{\{k_1,\ldots,k_s\}} \frac{1}{\sqrt{\binom{N}{s}}} \prod_{j=1}^{s} a_{k_j}^{\dagger} b_{k_j}^{\dagger} \Psi_{\mathrm{L}}.$$

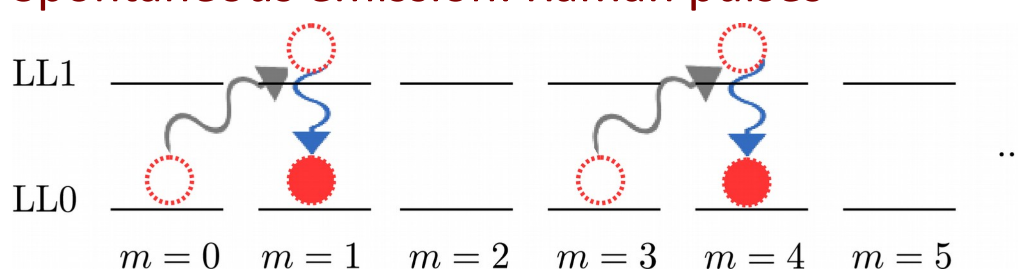


$$\Omega = 0.1 e^2/(\epsilon l_B)$$

$$\delta = 0.04 e^2/(\epsilon l_B)$$

Orthogonality catastrophe:
Fine tuning?

Spontaneous emission: Raman pulses



Trapping quasiholes with light

Potential from AC Stark shift of a Gaussian light beam:

$$V_{\rm opt}^{(\xi,w)}(z) = \frac{I(z)}{\Delta} = \left(\frac{l_{\rm B}}{w}\right)^2 V_{\rm opt,0}\exp[(z-\xi)^2/w^2]$$

Center of the beam

width of the beam
~ microns

For AMO systems: magnetic length of the order of microns → delta-potential supports quasihole excitations

VOLUME 87, NUMBER 1 PHYSICAL REVIEW LETTERS 2 JULY 2001

$\frac{1}{2}$-Anyons in Small Atomic Bose-Einstein Condensates

B. Paredes,* P. Fedichev, J. I. Cirac, and P. Zoller

New Journal of Physics
The open access journal at the forefront of physics

PAPER

Fractional quantum Hall states of a few bosonic atoms in geometric gauge fields

B Juliá-Díaz[1,2,4], T Graß[2], N Barberán[1] and M Lewenstein[2,3]

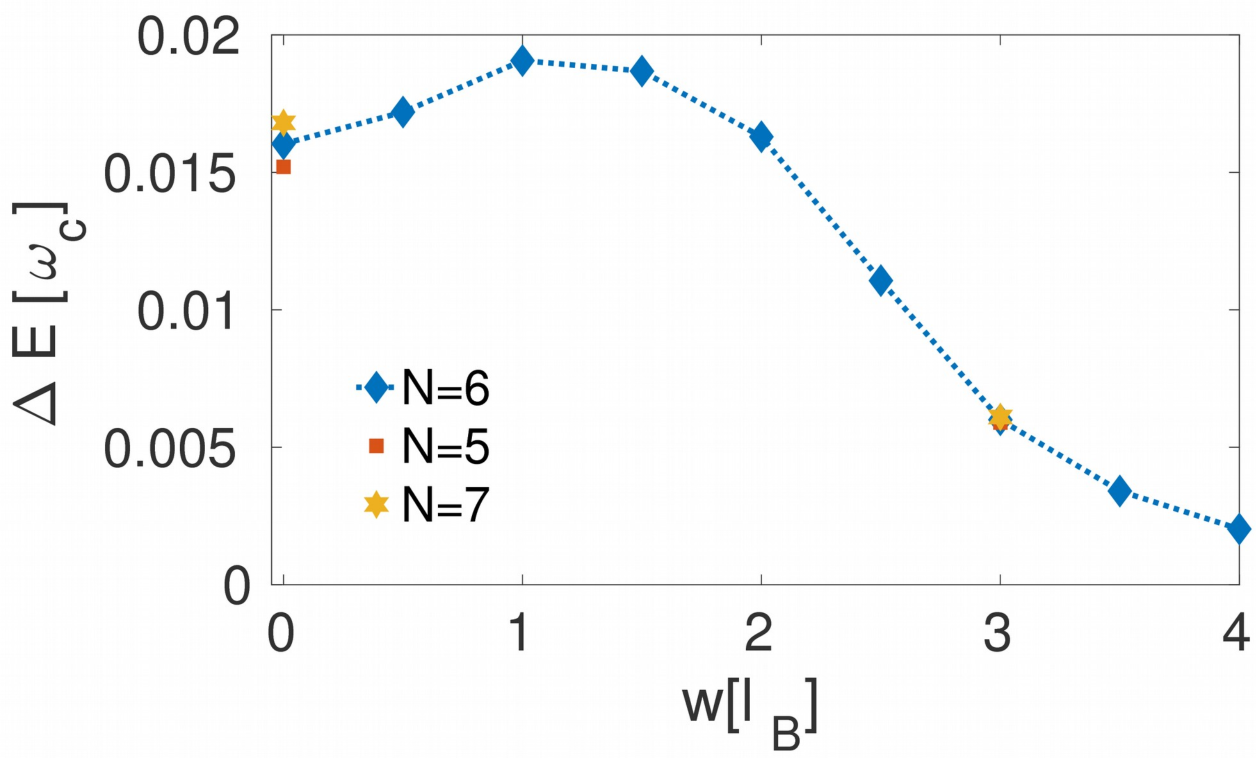
Published 1 May 2012 • IOP Publishing and Deutsche Physikalische Gesellschaft
New Journal of Physics, Volume 14, May 2012
Focus on Bose Condensation Phenomena in Atomic and Solid State Physics

Electronic systems: $w \gg l_{\rm B} = 26{\rm nm}/\sqrt{B[\text{ in T}]}$

Can broad potentials still trap quasiholes?

Trapping quasiholes with light

Even for broad potentials, the quasihole state is favored (high overlaps), but the energy gap to other states becomes small:



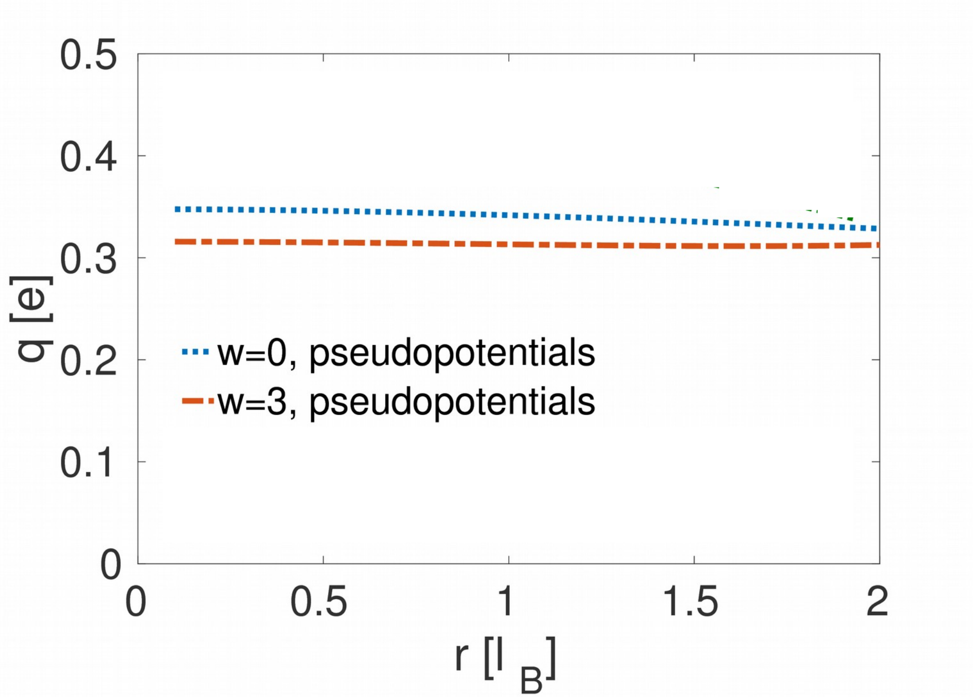
ED for system at filling 1/3 on the torus.

Moving quasiholes with light

Berry phase for moving the quasihole adiabatically on closed loop: $\gamma = \oint \mathrm{d}\xi \langle \Psi(\xi) | \nabla_\xi | \Psi(\xi) \rangle$

Berry phase is related to the charge *q* of the quasihole: $\frac{q}{e} = \gamma \frac{l_\mathrm{B}^2}{A}.$

We can extract Berry phase from ground states at different quasihole positions:



ED for system on disk in a Laughlin-like phase: Broadness of potential does not spoil the charge of the quasihole.

Moving quasiholes with light

Berry phase for moving the quasihole adiabatically on closed loop: $\gamma = \oint \mathrm{d}\xi \langle \Psi(\xi) | \nabla_\xi | \Psi(\xi) \rangle$

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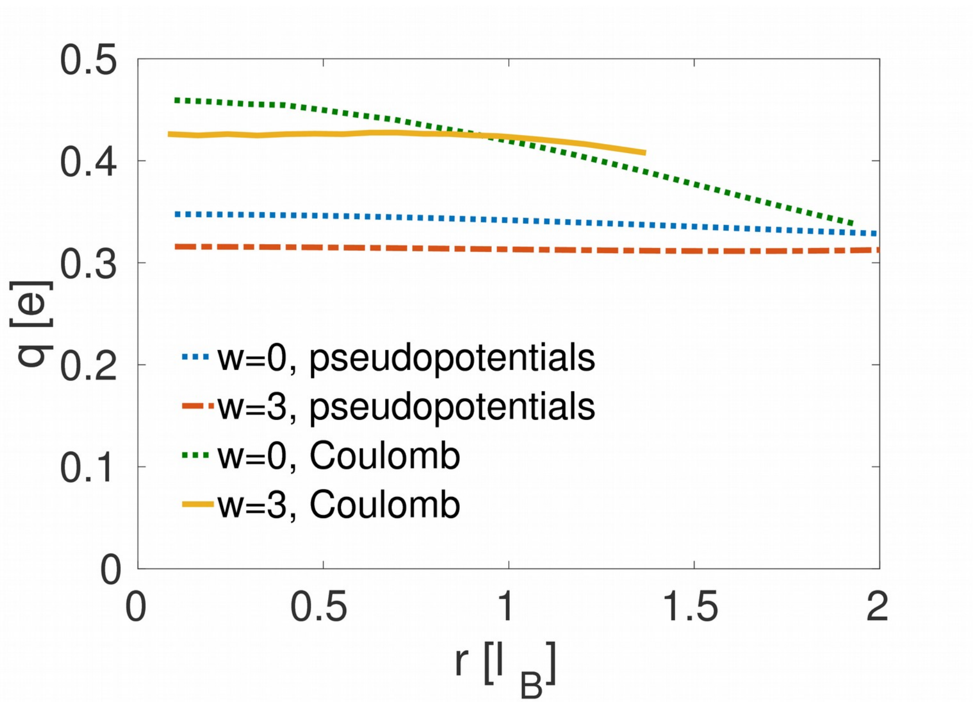

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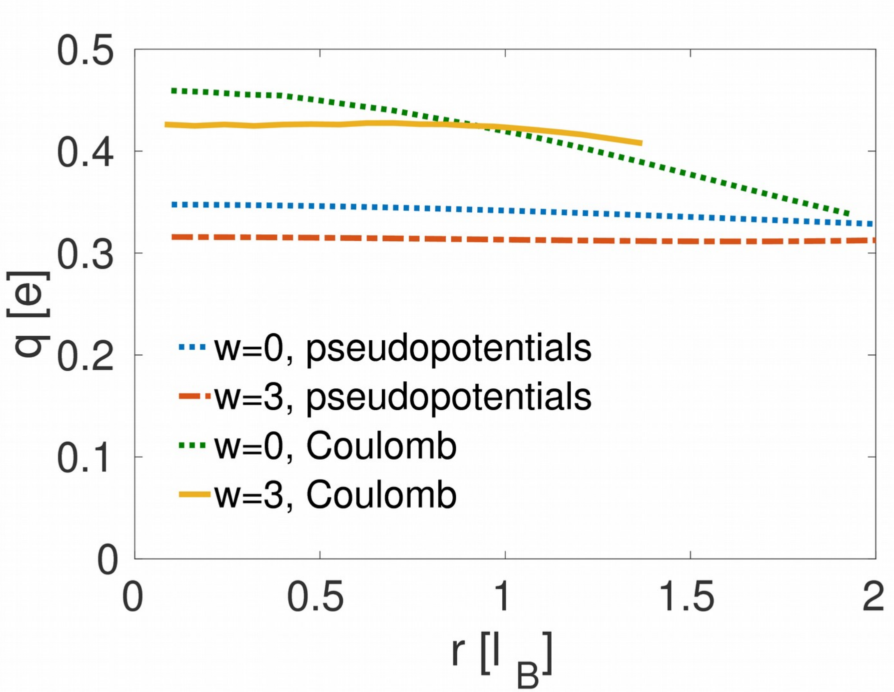
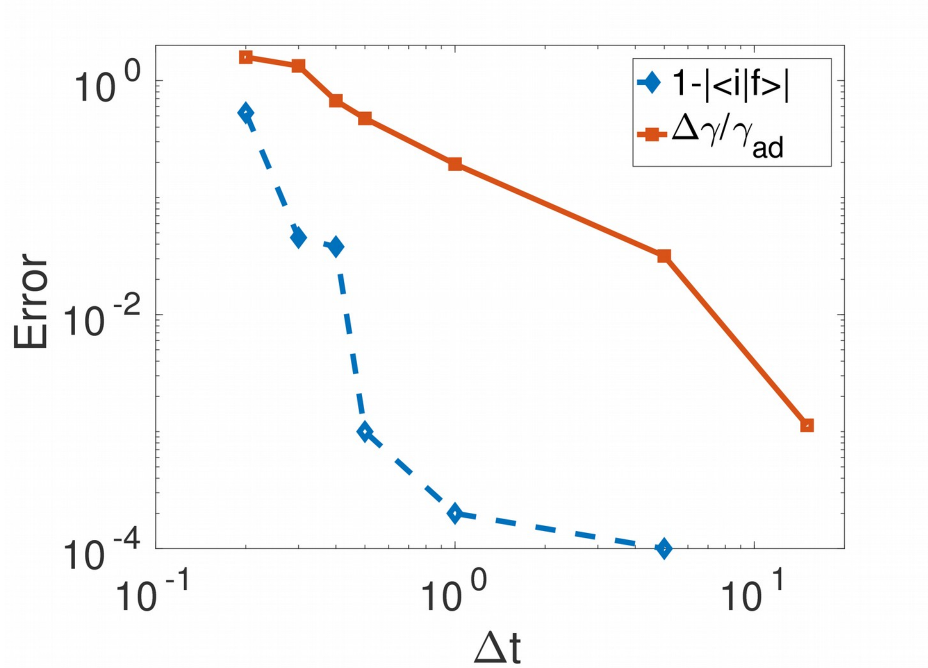
Berry phase is related to the charge *q* of the quasihole: $\frac{q}{e} = \gamma \frac{l_\mathrm{B}^2}{A}.$

We can extract Berry phase from ground states at different quasihole positions:



ED for system on disk in a Laughlin-like phase: Broadness of potential does not spoil the charge of the quasihole.

Or we can extract Berry phase from time evolution while the potential moves:



Discretized in 200 step. Error as a function of step duration. Phase error (red curve) is large.

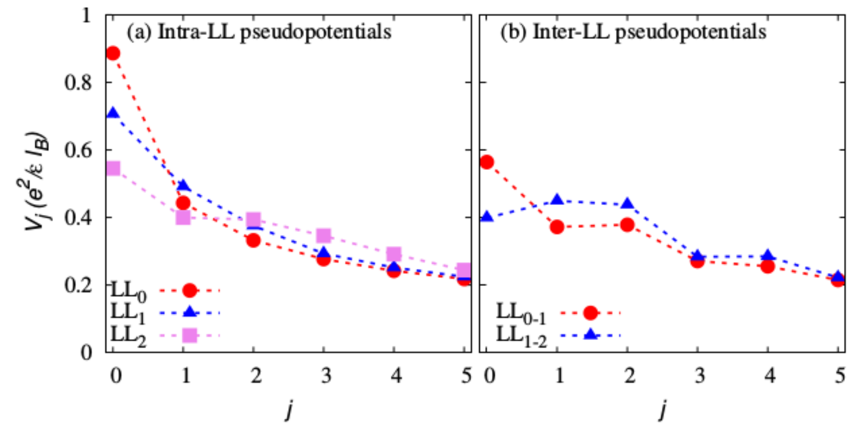
Summary

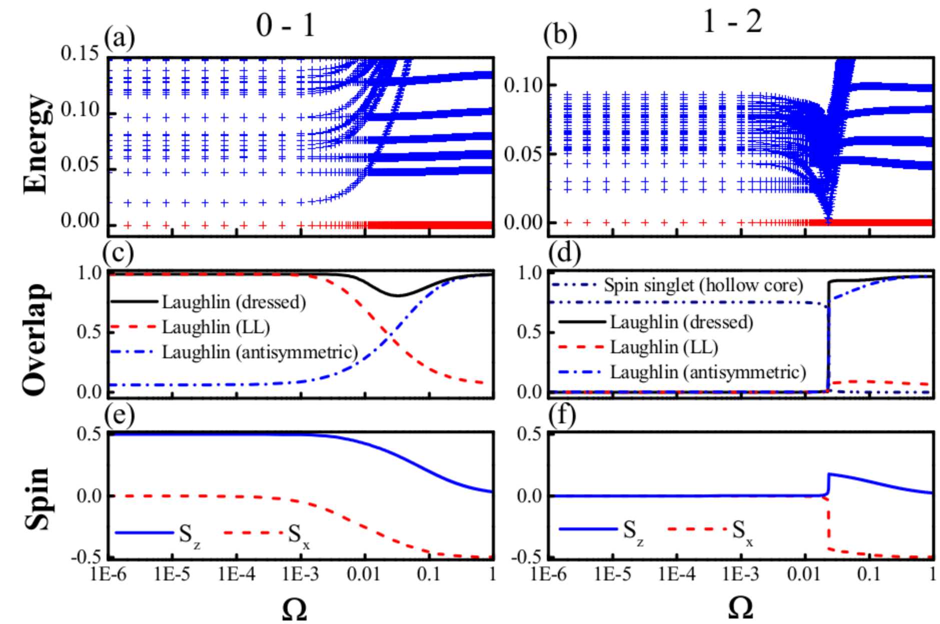
Part I: Optical driving.

Areg Ghazaryan, Tobias Graß, Michael J. Gullans, Pouyan Ghaemi, and Mohammad Hafezi, Phys. Rev. Lett. 119, 247403 (2017)

- Synthetic bilayer:
  Interactions are potentially very different from interactions in real bilayers.

- Transition to exotic FQH phases:
  - non-Abelian Fibonacci phase at filling 2/3
  - Haldane-Rezayi phase at filling ½ ?



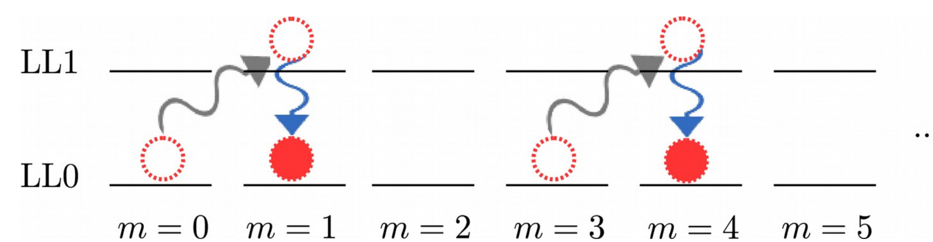


Part II: Optical excitations.

to appear on arXiv soon

- Light pulse with OAM produces (quasi)holes.

- Despite their broadness, laser beams can pin quasiholes.



Thank you!